



ISSN: 2350-0328

International Journal of Advanced Research in Science,
Engineering and Technology

Vol. 5, Issue 2 , February 2018

Numerical Solution of Thermal Radiation and Joule-Heating Effects on an Unsteady MHD with Heat and Mass Transfer with Chemical Reaction

Adel A. Megahed, Ali A.Halool, Hamed.A. El Mky

Department of Mathematics and Engineering Physics, Faculty of Engineering, Cairo University, Egypt.

. Department of Physics and Engineering Mathematics, Higher Institute for Engineering in 15 May,
Helwan, Cairo, Egypt.

Department of Mathematics, Faculty of Science, Aswan University, Egypt.

ABSTRACT: The radiation and chemical reaction effects on an unsteady two-dimensional magnto-hydrodynamics free convection flow with heat and mass transfer from a vertical porous stretching surface with joule-heating. The fluid is assumed to be viscous, electrically conducting and incompressible. The non-linear partial differential equations, governing the flow field under consideration, have been transformed by a similarity transformation into a system of non-linear ordinary differential equations and then solved numerically using the Runge-Kutta integration scheme with a modified version of the Newton-Raphson shooting method. Resulting non-dimensional velocity, temperature and concentration profiles are then presented graphically for different values of the parameters of physical including the Prandtl number Pr , Eckert number Ec , Schmidt number Sc , Chemical Reaction G , unsteadiness parameter A , mixed convection parameter λ , suction parameter f_0 , thermal radiation parameter R and magnetic parameter M .

KEYWORDS: heat and mass transfer, thermal radiation, joule heating, chemical reaction.

Nomenclature

C	Concentration the fluid
C_∞	Concentration in fluid far away from the stretching surface
C_w	Concentration at the surface
C_0	Reference concentration
k	Coefficient of thermal conductivity of the fluid
Pr	Prandtl number
Re_x	local Reynolds number
q_r	Radiation heat flux
q_T	local heat transfer coefficient
q_C	local mass transfer coefficient
t	Time
T	Fluid temperature
T_w	Surface temperature
T_0	Reference temperature
T_∞	Temperature for away from the stretching surface
u, v	Velocity components along x and y axes
u_w	Velocity of the moving sheet
x, y	Cartesian coordinates along the sheet and normal to it respectively.
V_w	Velocity of suction
k	Thermal diffusivity of the fluid



c, α	Positive constants
η	Similarity variable
g	Gravity field
ν	Kinematic viscosity
β	Volumetric coefficient of thermal expansion
σ	Electrical conductivity
c_p	Specific heat at constant pressure
D	Diffusivity coefficient
k_I	Chemical reaction
B_0	Uniform magnetic field
ρ	Density
σ_s	Stefan-Boltzman constant
A	Unsteadiness parameter
λ	Mixed convection parameter
M	Magnetic field parameter
Pr	Prandtl number
k^*	Absorption coefficient.
R	thermal radiation parameter
Ec	Eckert number
Sc	Schmidt number
ψ	Stream function
G	Chemical reaction parameter
f_0	Suction parameter
F'	Dimensionless velocity
θ	Dimensionless temperature
ϕ	Dimensionless concentration
C_f	Skin friction
Nu_x	local Nusselt number
Sh_x	local Sherwood number

Superscripts

Differentiation with respect to η

I. INTRODUCTION

Boundary layer flow, heat and mass transfer over a linearly stretched surface has received considerable attention in recent years. This is because of the various possible engineering and metallurgical applications such as hot rolling, wire drawing, metal and plastic extrusion, continuous casting, glass fiber production, crystal growing and paper production. These are discussed in a book published by Gebhart et al. [1]. A.M. Rashad [2], focused on the study of unsteady magneto hydrodynamics boundary-layer flow and heat transfer for a viscous laminar incompressible electrically conducting and rotating fluid due to a stretching surface embedded in a saturated porous medium with a temperature-dependent viscosity in the presence of a magnetic field and thermal radiation effects. But EL-Kabeir et al. [3], studies the unsteady two-dimensional flow of an electrically-conducting incompressible fluid over a continuous moving vertical stretching sheet embedded in a fluid-saturated porous medium and the flow is permeated by a uniform transverse magnetic field. The unsteady heat transfer problems over a stretching surface, which is started impulsively from rest or is stretched with a velocity that depends on time, are considered. Abo-Eldahab et al. [4], investigated the effects of Heat Transfer on MHD Flow Over an Unsteady Stretching Surface in a Micropolar Fluid and a Porous Medium with Prescribed Surface Heat Flux .Elbashbeshy and Bazid [5], presented an exact similarity solution for unsteady momentum and heat transfer flow whose motion is caused solely by the linear stretching of a horizontal



stretching surface . EL-Hakim et al.[6] presented Group method analysis for the effect of radiation on MHD coupled heat and mass transfer natural convection flow for water vapor over a vertical cone through porous medium. E. M.A.Elbashbeshy, D.A.Aldawody[7], examined the effects of Thermal Radiation and Magnetic Field on Unsteady Mixed Convection Flow and Heat Transfer Over a Porous Stretching Surface. M.A. EL-Hakim, A.M. Rashad [8], carried out an Effect of radiation on nonDarcy free convection from a vertical cylinder embedded in a fluid-saturated porous medium with a temperature-dependent viscosity.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. In order to study the thermal stratification effects over the above-mentioned problem, an attempt has been made have to analyze the nonlinear hydro magnetic flow with heat and mass transfer over a vertical stretching surface with chemical reaction and thermal stratification effects. Among some of the interesting problems which were studied are the analyses of laminar forced convection mass transfer with homogeneous chemical reaction by J.D. Goddard, A. Acrivos [9]. Lakshmisha et al. [10] presented numerical solutions for the unsteady three-dimensional flow with heat and surface mass transfer over a stretching surface. Gorla and Sidawi [11] presented numerical solutions for the problem of steady three-dimensional free convection flow over a stretching surface with suction and blowing. Chamkha [12] studied the problem of the hydro magnetic steady three-dimensional free convective flow on a vertical stretching surface with heat generation or absorption. Eldabe et al.[13] presented a Numerical study of viscous dissipation effect on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a porous medium.

In recent years, numerous investigations have been conducted on the magneto-hydrodynamic flows and heat transfer because of its important applications in metallurgical industry. In the presence of a transverse magnetic field, the flow and heat transfer over a stretching surface have been investigated by [14-18].

All the above mention studies consider the steady state problem.Abo- El_dahab et al. [19] studied the joule-heating effect on hydrody magnetic three – dimensional flow over a stretching surface with heat and mass transfer. Recently, Abo-Eldahab et al. [20] studied the Effects of Heat and Mass Transfer on MHDFlow Over a Vertical Stretching Surface with Free Convection and Joule-Heating. The aim of the present work is to study the effects of thermal radiation,Chemical Reaction and magnetic field on unsteady mixed convection flow, heat and mass transfer over a vertical stretching surface with viscous dissipation in the presence of wall suction.

II. MATHEMATICAL FORMULATION

Consider an unsteady two-dimensional MHD free convection laminar boundary layer flow of a viscous incompressible and electrically conducting fluid with heat and mass transfer under the influence of thermal radiation over a vertical porous stretching surface and moving with velocity $u_w = cx/(1-\alpha t)$ where c and α are constants and with temperature distribution $T_w = T_\infty + [T_0 c x / 2\nu(1-\alpha t)^2]$, where T_0 is a reference temperature such that $0 \leq T_0 \leq T_w$ and the concentration distribution $C_w = C_\infty + [C_0 c x / 2\nu(1-\alpha t)^2]$, C_0 is a reference concentration such that $0 \leq C_0 \leq C_w$. Introducing the Cartesian coordinate system, the x-axis is taken along the stretching surface in the vertically upward direction and the y-axis is taken as normal to the surface. Two equal and opposite forces are introduced along the x-axis, so that the surface is stretched keeping the origin fixed. A uniform magnetic field of strength B_0 is applied normal to the stretching surface which produces magnetic effect in the x-axis . The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. This work is an extension of [7]. Under the above assumptions, the governing boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty) - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{J^2}{\rho c_p \sigma} \tag{3}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_1 (C - C_\infty) \tag{4}$$

With the boundary condition

$$\left. \begin{aligned} u = U_w, v = -V_w, T = T_w, C = C_w \text{ at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ at } y \rightarrow \infty \end{aligned} \right\} \rightarrow \tag{5}$$

Where $V_w = -\sqrt{\frac{cv}{1-\alpha t}}$ is the velocity of suction ($V_w > 0$), u and v are the velocity components along x and y axes, respectively, t is the time, ν is the kinematics viscosity, g is the gravity field, β is the volumetric coefficient of thermal expansion, σ is the electrical conductivity, B_0 is the uniform magnetic field, c_p is the specific heat at constant pressure, ρ is the density, k is the coefficient of thermal conductivity of the fluid, D is the diffusivity coefficient and k_1 is the chemical reaction (reaction rat). T, T_∞ and T_w are the temperature, the temperature for away from the stretching surface and surface temperature respectively. C, C_∞ and C_w are the species concentration in fluid, species concentration in fluid far away from the stretching surface and species concentration at the surface. q_r , the radiation heat flux. By using

Rossel and diffusion approximation for radiation $q_r = \frac{-4\sigma_s}{3k^*} \frac{\partial T^4}{\partial y}$ where σ_s and k^* are the Stefan-Boltzman constant and absorption coefficient. Let the temperature within the flow is such that T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher order we get $T^4 = 4T_\infty^3 T - 3T_\infty^4$. Now eq. (3) becomes:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p} + \frac{16\sigma_s T_\infty^3}{3\rho c_p k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{J^2}{\rho c_p \sigma} \rightarrow \tag{6}$$

The continuity equation is satisfied if we choose a stream function $\psi(x, y)$ such that $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ we used the similarity variable:

$$\left. \begin{aligned} \eta = \sqrt{\frac{c}{v(1-\alpha t)}} y, \psi(x, y) = \sqrt{\frac{cv}{(1-\alpha t)}} x f(\eta), \quad J = \sigma(E + v \underline{\Lambda B}) \\ T = T_\infty + T_0 \left[\frac{cx}{2v(1-\alpha t)^2} \right] \theta(\eta), \quad C = C_\infty + C_0 \left[\frac{2cx}{3v(1-\alpha t)^2} \right] \phi(\eta) \end{aligned} \right\} \rightarrow (7)$$

From the eq.(7) in eqs. (1,2,4 and 6) the governing equations finally reduce to:

$$F''' - \frac{1}{2} A \eta F'' + F'' F - F'^2 - (A + M) F' + \lambda \theta = 0 \quad \rightarrow (8)$$

$$\theta'' + \left(\frac{\text{Pr}}{1 + R} \right) \left[F \theta' - F' \theta - \frac{A}{2} (\eta \theta' + 4\theta) + M Ec F'^2 \right] = 0 \quad \rightarrow (9)$$

$$\frac{1}{Sc} \phi'' + \left(F - \frac{1}{2} A \eta \right) \phi' - (2A + G) \phi - F' \phi = 0 \quad \rightarrow (10)$$

And the boundary conditions (5) are transformed to:

$$\left. \begin{aligned} F(0) = -\sqrt{\frac{1-\alpha}{c\nu}} V_w = f_0, \quad F'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{at } \eta = 0 \\ F'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0 \quad \text{at } \eta \rightarrow \infty \end{aligned} \right\} \rightarrow (11)$$

where the parameter that measures the unsteadiness is $A = \alpha/c$, the mixed convection parameter is $\lambda = g\beta T_0/2\nu c$, $Ec = 2\nu c x/T_0 c_p$ is the Eckert number, $Sc = \nu/D$ is the Schmidt number, $M = \sigma B_0^2 (1-\alpha)/c\rho$ is the magnetic field parameter, $Pr = \mu c_p/k$ is the Prandtl number, $G = (1-\alpha)/c_p$ is the Chemical Reaction, $f_0 = -\sqrt{1-\alpha/c\nu} V_w$ is the suction parameter, $R = 16\sigma_s T_\infty^3/3kk^*$ is the thermal radiation parameter and $Re_x = \frac{x u_w}{\nu}$ is the local Reynolds number.

III. NUMERICAL METHOD FOR SOLUTION

The transformed equations (8) - (10) subject to the boundary conditions (11) form a nonlinear two-point boundary value problem, which has been solved numerically using the Runge-Kutta integration scheme with a modified version of the Newton-Raphson shooting method. First of all, the higher order non-linear differential equations (8) - (10) are converted into simultaneous linear differential equation of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta fourth order method. The step size $\Delta\eta = 0.001$ is used to obtain the numerical solution with six decimal accuracy as criterion of convergence. The above mentioned third order and second order equations are written in terms of first order equations as follows:

$$F' = z, z' = p$$

$$p' = \left(\frac{1}{2} A \eta - F \right) p + z^2 + (A + M) z - \lambda \theta \quad \rightarrow (12)$$

$$\theta' = q$$

$$q' = \left(\frac{\text{Pr}}{1 + R} \right) \left[\frac{A}{2} (\eta q + 4\theta) + z \theta - F q - M Ec z^2 \right] \quad \rightarrow (13)$$

$$\phi' = r$$

$$r' = Sc \left[(2A + G) \phi + z \phi - \left(F - \frac{1}{2} A \eta \right) r \right] \quad \rightarrow (14)$$

With boundary conditions

$$F(0) = 0, F'(0) = 1, \theta(0) = 1, \phi(0) = 1 \quad \rightarrow (15)$$

In order to integrate equations (12)-(14) as initial value problem we require a value for $p(0)$ i.e. $F''(0)$ and $q(0)$ i.e. $\theta'(0)$ but no such values are given in the boundary. The suitable guess values for $F''(0)$ and $\theta'(0)$ are chosen and

then integration is carried out. We take the series of values for $F''(0)$, $\theta'(0)$ and apply the fourth order Runge-Kutta method with different step-sizes ($\eta = 0.01, 0.001, \text{etc.}$) so that the numerical results obtained are independent of $\Delta\eta$. The above procedure is repeated until we get the results up to the desired degree of accuracy 10^{-6} .

IV. RESULTS AND DISCUSSION

The computations have been carried out for various governing flow parameters where some observations on this system can clearly be noticed. Eq. (8) is coupled to Eqs. (9) and (10) through the parameter A and so the velocity F' is not affected by the Eckert number Ec , the Schmidt number Sc and the chemical reaction parameter G . Also Eqs. (9) and (10) are coupled to each other only through the unsteadiness parameter A . Thus the temperature is not affected by Schmidt number Sc and the chemical reaction parameter G and the concentration is not affected by the Eckert number. The results are given for various values of the Prandtl number Pr , Schmidt number Sc , suction parameter f_0 , chemical reaction parameter G , Eckert number Ec , thermal radiation parameter R , mixed convection parameter λ , magnetic field parameter M and the unsteadiness A . The parameters effects on the velocity, temperature and concentration profiles are displayed graphically in Figures 1 – 21.

The dimensionless velocity, temperature and concentration profiles for different values of unsteadiness parameter A with constant f_0 , Sc , Ec , Pr , R , λ , M and G are shown in Figs. 1 to 3 respectively. It is clear that the velocity, temperature and concentration of the fluid decreasing with the increase unsteadiness parameter. Figure 4 present the behavior of the temperature profiles various values of the Eckert number Ec with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $f_0 = 0$, $A=0.2$ and $Sc=0.6$. From this figure it is shown that the temperature of the fluid decreases greatly with the increase of the Eckert number Ec . The effect of the chemical reaction parameter G with constant f_0 , sc , Ec , Pr , R , λ , M and A , and Schmidt number Sc with constant f_0 , G , Ec , Pr , R , λ , M and A on the concentration of the fluid is illustrated in Figures 5 and 6, it is obvious that the concentration of the fluid decreases accordingly the chemical reaction parameter G and Schmidt number Sc increases. The velocity profiles, temperature profiles and concentration profiles for different values of the Prandtl number Pr with $A=0.2$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$, and $Sc=0.6$ are depicted in Figs. 7 to 9.

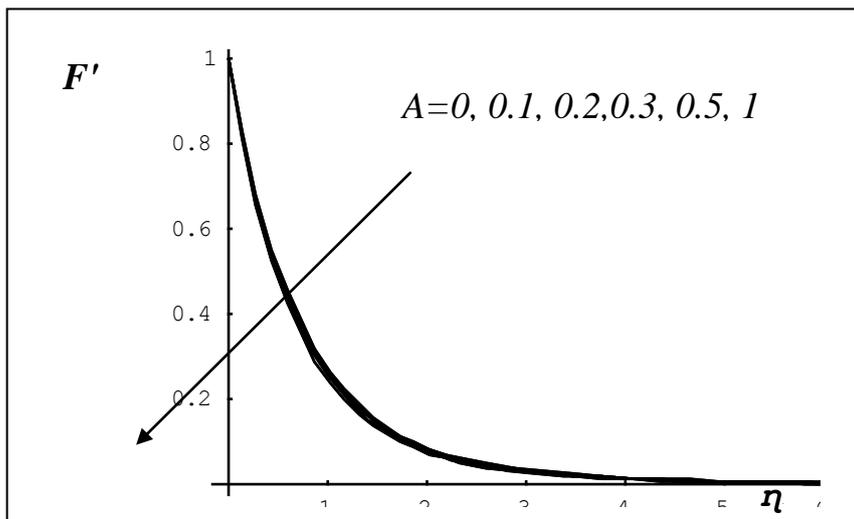
Fig 7 and 9 demonstrate that the velocity and concentration of the fluid decreases with the increase of the Prandtl number Pr . Fig. 8 Shows that the temperature of the fluid increases with the increase of the Prandtl number Pr . Figs. 10 to 12 presents velocity profiles, temperature profiles and concentration profiles for various values of magnetic field parameter M when $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $A=0.2$, $R=0.8$, $Ec=0.72$, $f_0 = 0$, and $Sc=0.6$. The velocity of the fluid decreases with increasing values of magnetic field parameter M , while the temperature and concentration of the fluid decreases with increasing values of magnetic field parameter M . The effect of the thermal radiation parameter R with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $M=1$, $A=0.2$, $Ec=0.72$, $f_0 = 0$, and $Sc=0.6$ on the velocity, temperature and concentration of the fluid are illustrated in Figs. 13 to 15. It is found that the velocity and temperature increases as the thermal radiation parameter R increases, this is agreement with the physical fact that the thermal boundary layer thickness increases with increasing R . But the concentration decreases as thermal radiation parameter R increases.

Figs. 16-18 presents the behavior of the velocity, temperature and concentration of the fluid profiles various values of the suction parameter f_0 with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $A=0.2$, and $Sc=0.6$. From the Fig. 16 it is shown that with increasing suction parameter, velocity of the fluid is found to decrease, i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. This effect acts to decrease the wall shear stress. Increase in suction causes progressive thinning of the boundary layer. The physical explanation for such a behavior is as follows; in case of suction the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of the viscosity. This effect acts to decrease the wall shear stress.

From the Figs. 17 and 18 the temperature and concentration in the boundary layer are also decreases with increasing suction parameter ($f_0 > 0$). The thermal boundary thickness decreases with suction parameter ($f_0 > 0$), which causes an

increase in the rate of heat transfer. The physical explanation for such a behavior is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness in case of suction.

Figs. 19 to 21 presents the velocity, temperature and concentration profiles for various values of mixed convection parameter λ when $Pr=0.72$, $G=-0.2$, $A=0.2$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0=0$ and $Sc=0.6$. Fig.19 The velocity increases with increasing values of mixed convection parameter λ , while from Figs.20 and 21 the temperature and concentration decreases with increasing values of mixed convection Parameter λ , at $\lambda=0$ gives the result of forced convection case.



**Fig1: Velocity profiles for various values of A with $Pr=0.72$,
 $G=-0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0=0$ and $Sc=0.6$.**

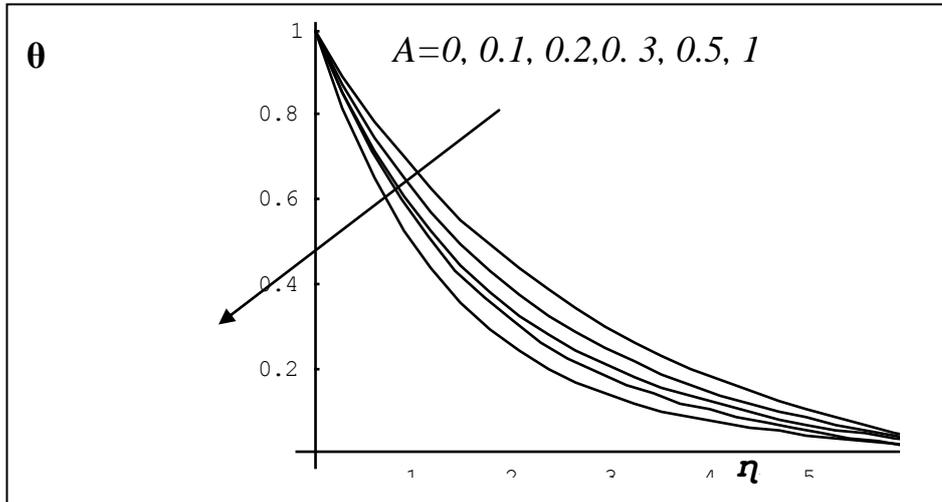


Fig.2: Temperature profiles for various values of A with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

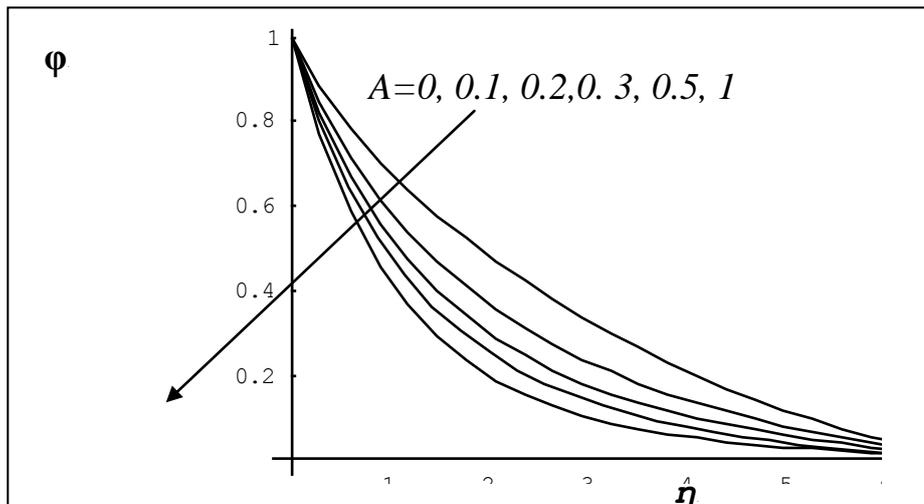


Fig 3: Concentration profiles for various values of A with $Pr =0.72$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $f_0 = 0$, $Ec=0.72$ and $Sc=0.6$.

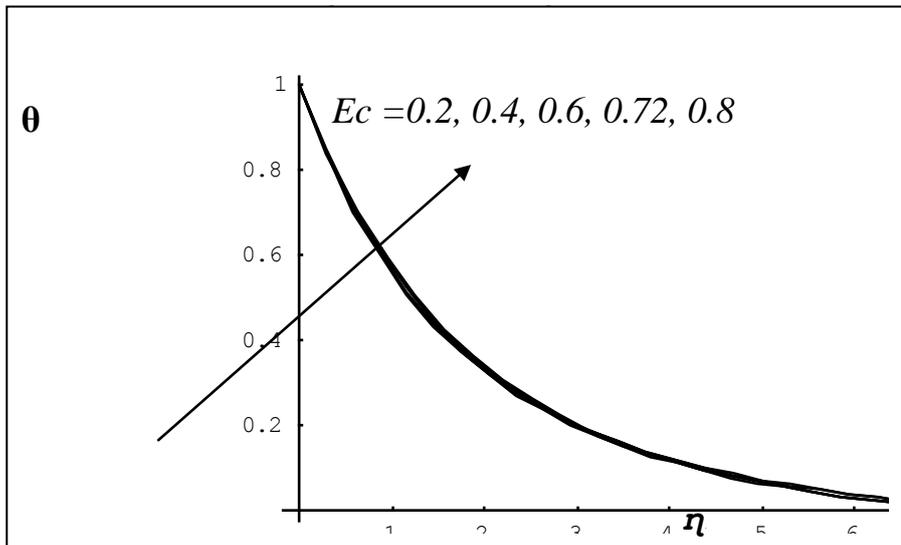


Fig.4: Temperature profiles for various values of E_c with $Pr=0.72$, $G= -0.2, \lambda=0.1, M=1, R=0.8, f_0 = 0, A=0.2$ and $Sc=0.6$.

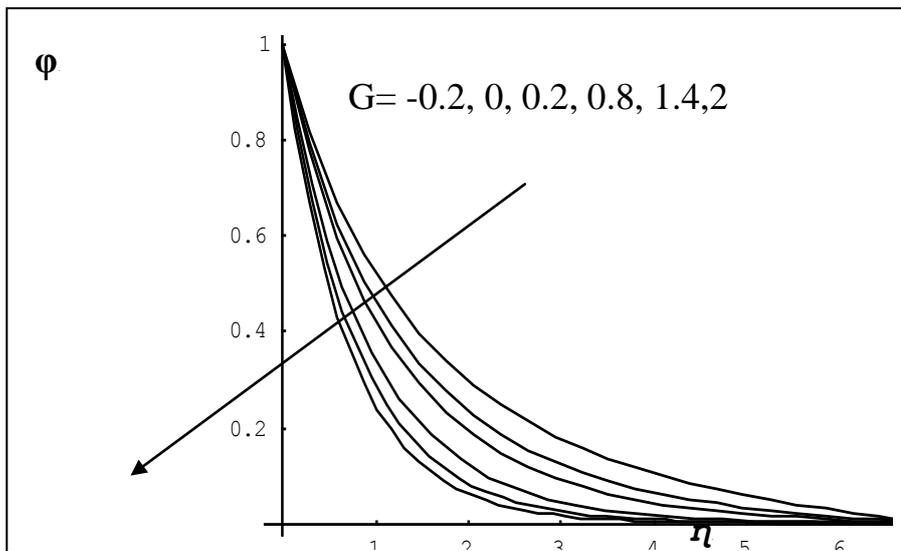


Fig.5: Concentration profiles for various values of G with $Pr=0.72, A=0.2, \lambda=0.1, M=1, R=0.8, Ec=0.72, f_0 = 0$ and $Sc=0.6$.

$Sc= 0.1, 0.3, 0.6, 1, 1.6, 2.2$

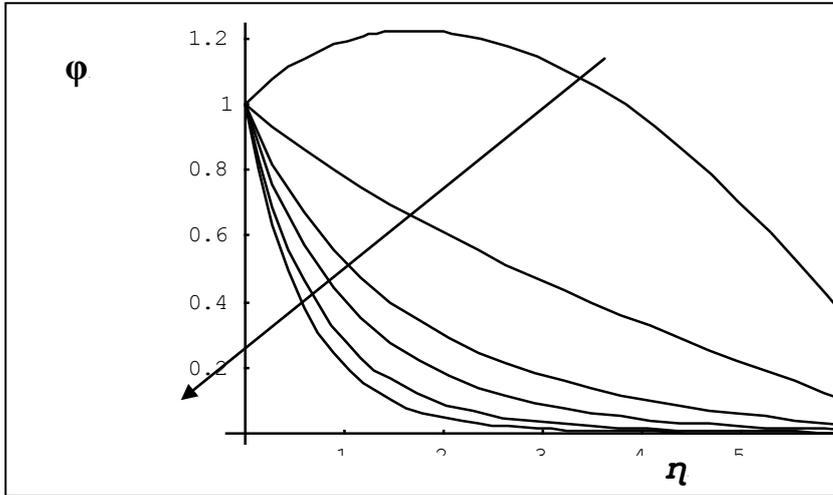


Fig.6: Concentration profiles for various values of Sc with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $A=0.2$.

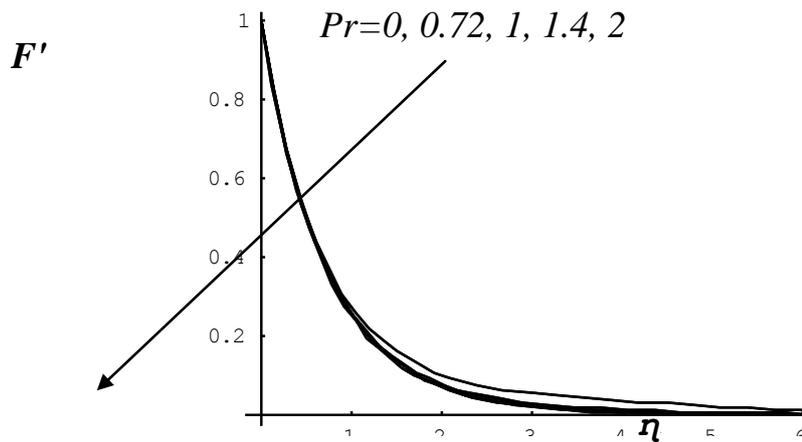


Fig.7: Velocity profiles for various values of Pr with $A=0.2$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

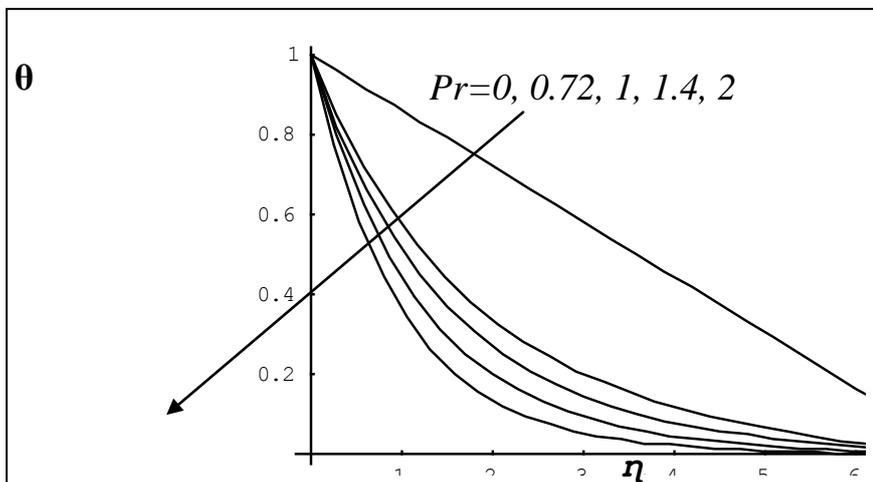


Fig.8: Temperature profiles for various values of Pr with $A=0.2$, $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

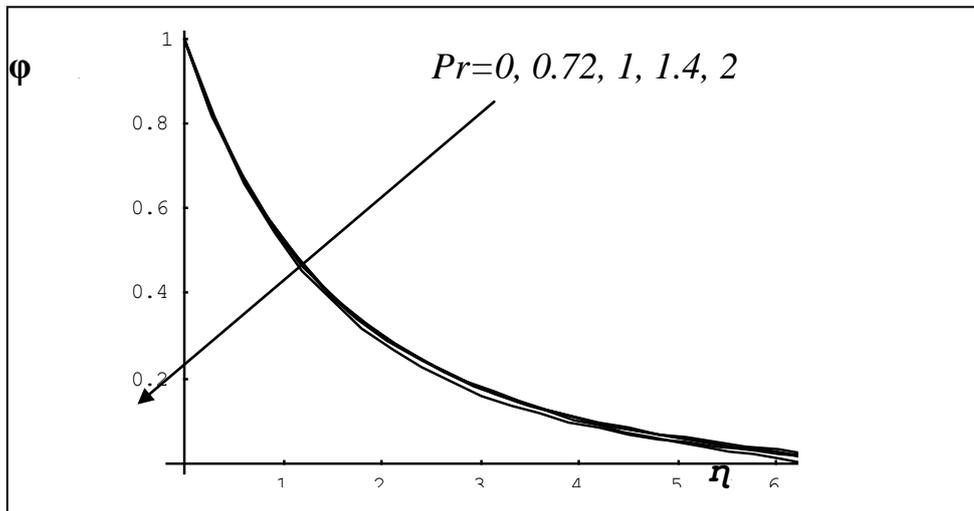


Fig.9: Concentration profiles for various values of Pr with $A=0.2$, $G=-0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0=0$ and $Sc=0.6$.

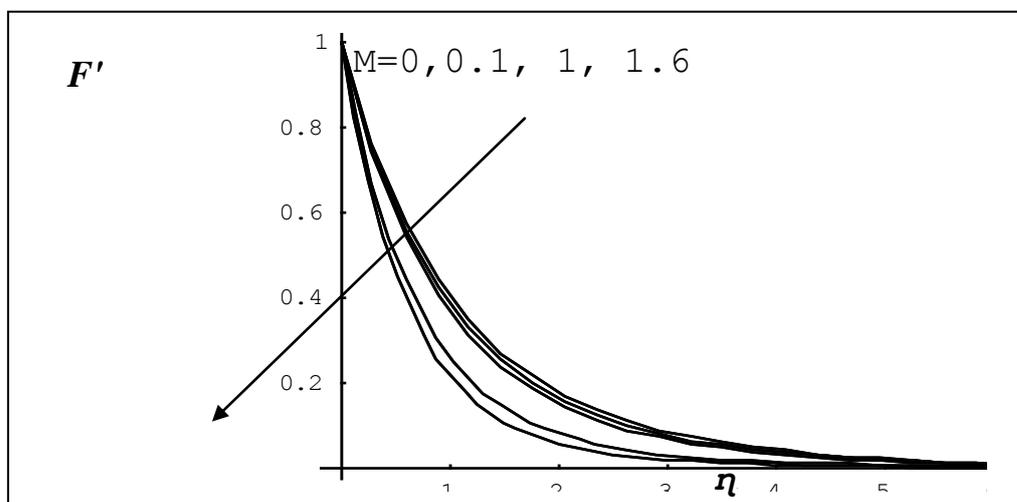


Fig.10: Velocity profiles for various values of M with $Pr=0.72$, $G=-0.2$, $\lambda=0.1$, $A=0.2$, $R=0.8$, $Ec=0.72$, $f_0=0$ and $Sc=0.6$.

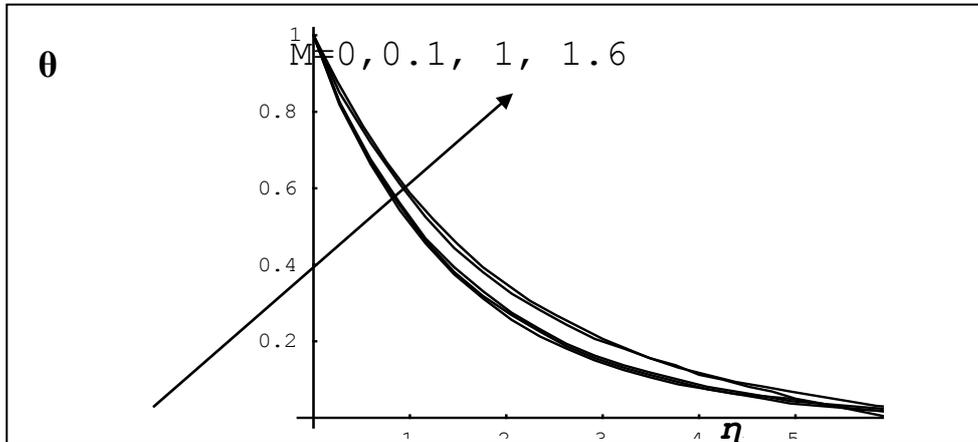


Fig.11: Temperature profiles for various values of M with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $A=0.2$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

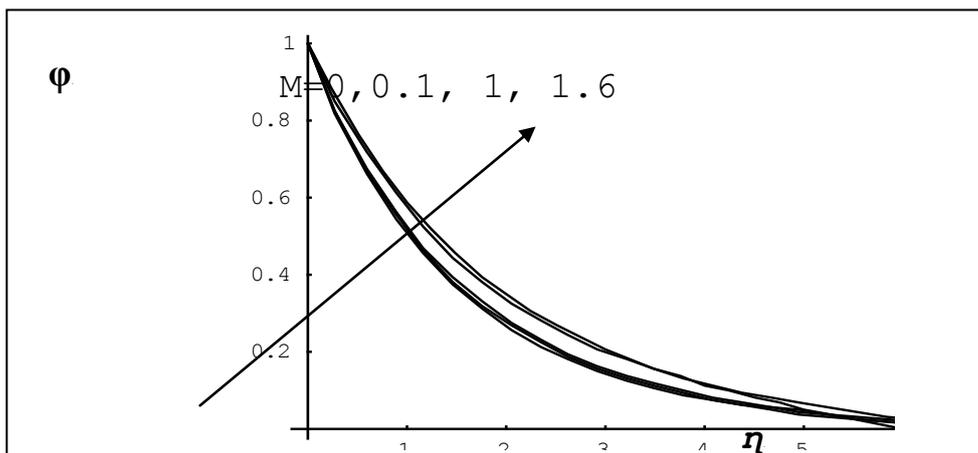


Fig.12: Concentration profiles for various values of M with $Pr=0.72$, $G= -0.2$, $\lambda=0.1$, $A=0.2$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

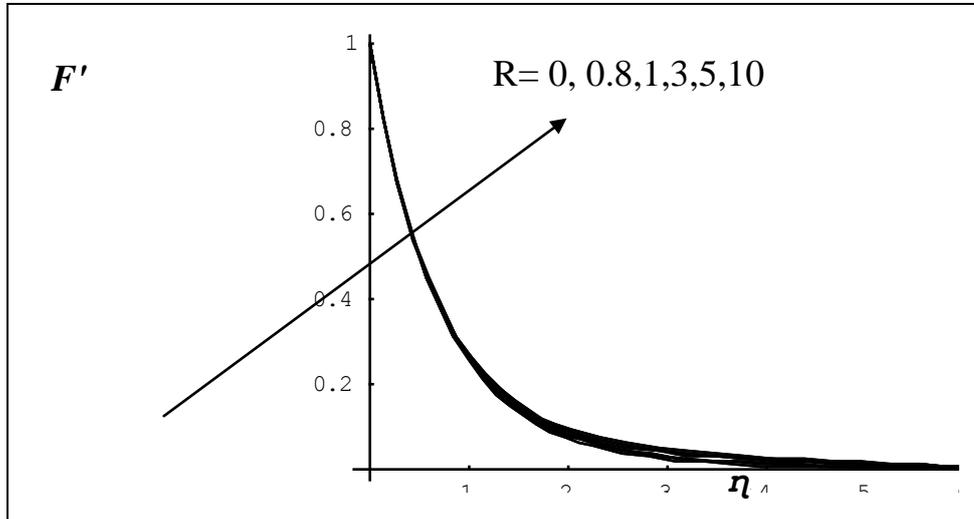


Fig.13: Velocity profiles for various values of R with $Pr=0.72$, $G=0.2$, $\lambda=0.1$, $M=1$, $A=0.2$, $Ec=0.72$, $f_0 = 0$, and $Sc=0.6$.

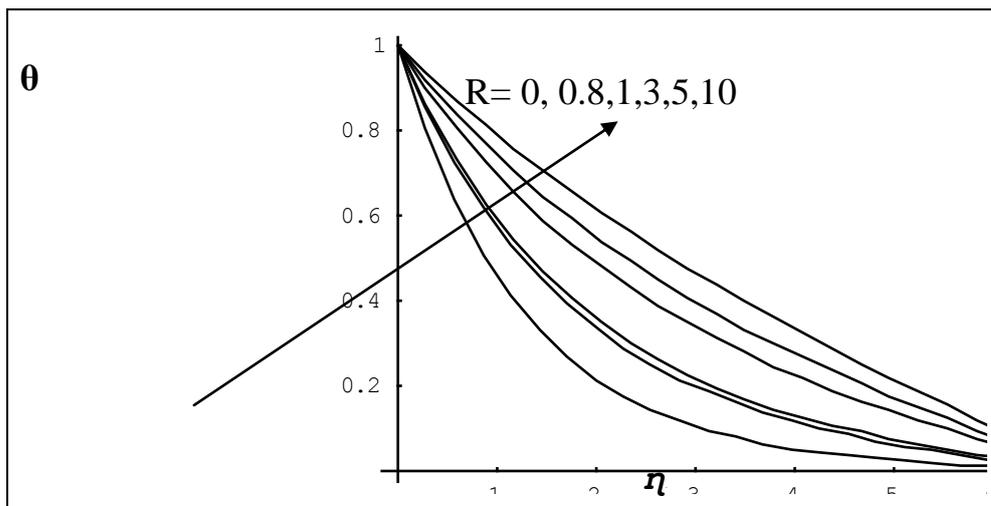


Fig.14: Temperature profiles for various values of R with $Pr=0.72$, $G = -0.2$, $\lambda=0.1$, $M=1$, $A=0.2$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

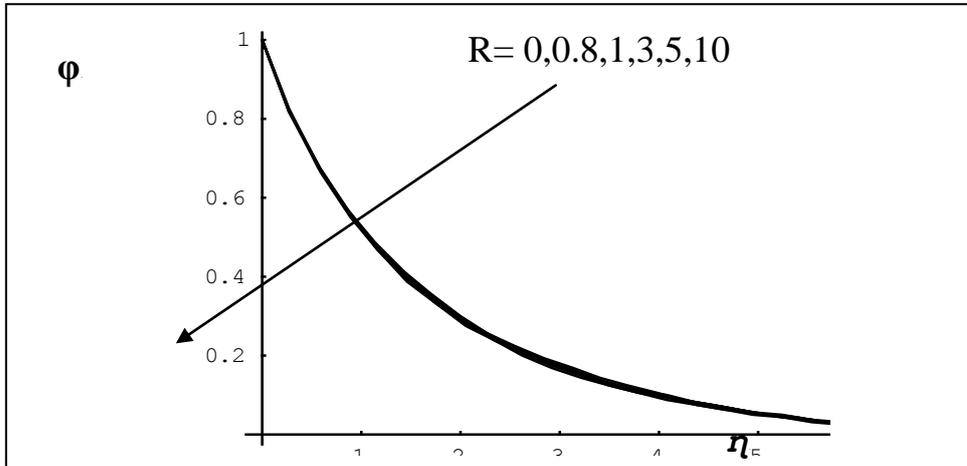


Fig.15: Concentration profiles for various values of R with Pr=0.72, G= -0.2, $\lambda=0.1$, M=1, A=0.2, Ec=0.72, $f_0 = 0$ and Sc=0.6.

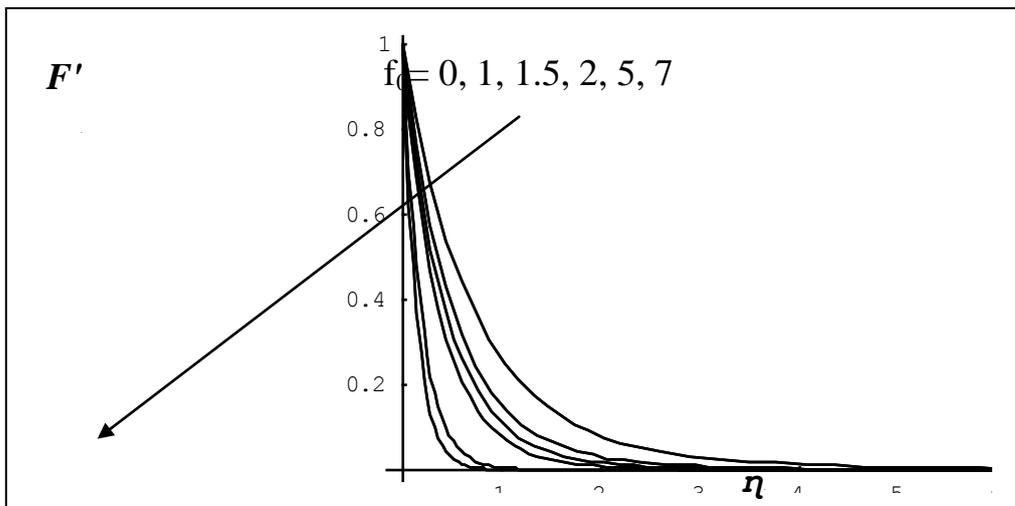


Fig.16: Velocity profiles for various values of f_0 with Pr=0.72, G= -0.2, $\lambda=0.1$, M=1, R=0.8, Ec=0.72, A=0.2 and Sc=0.6.

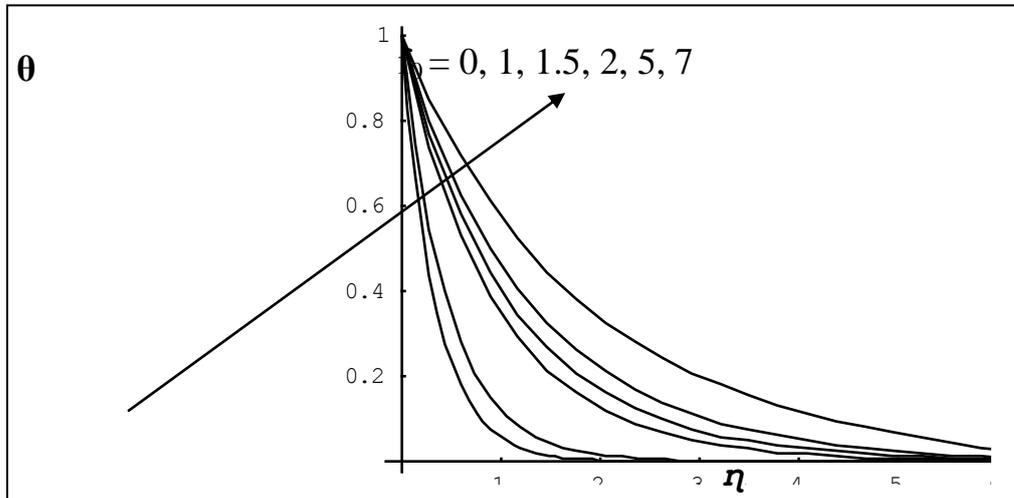


Fig.17: Temperature profiles for various values of f_0 with $Pr=0.72$,
 $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $A=0.2$ and $Sc=0.6$.

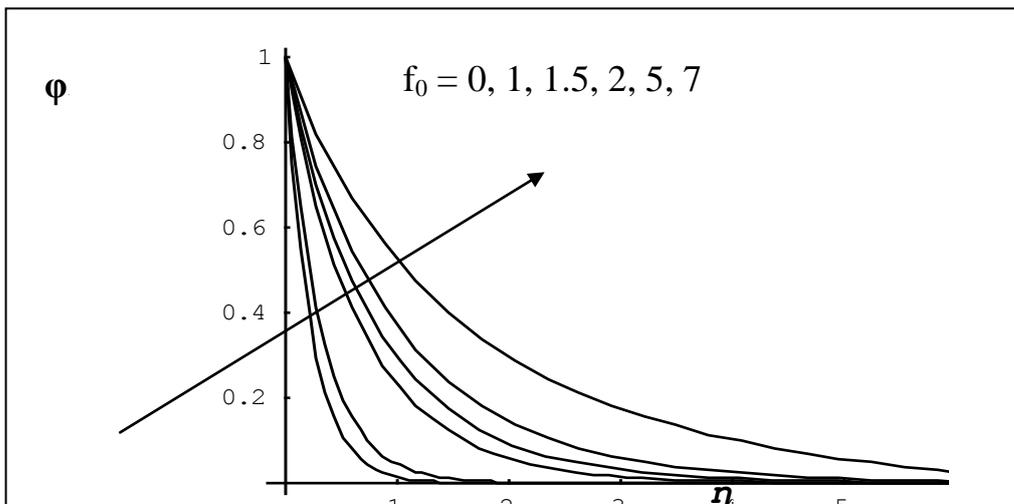


Fig.18: Concentration profiles for various values of f_0 with $Pr=0.72$,
 $G= -0.2$, $\lambda=0.1$, $M=1$, $R=0.8$, $Ec=0.72$, $A=0.2$ and $Sc=0.6$.

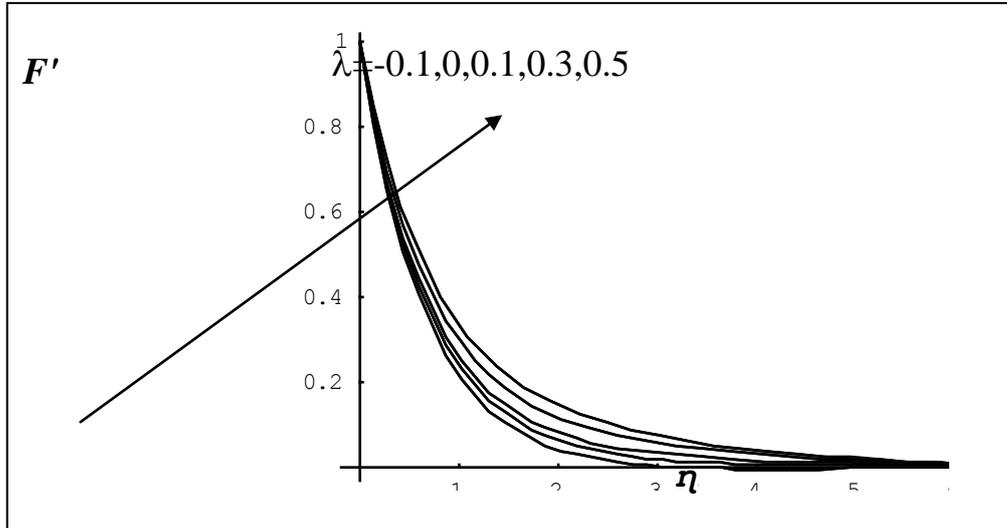


Fig.19: Velocity profiles for various values of λ with $Pr=0.72$, $G= -0.2$, $A=0.2$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

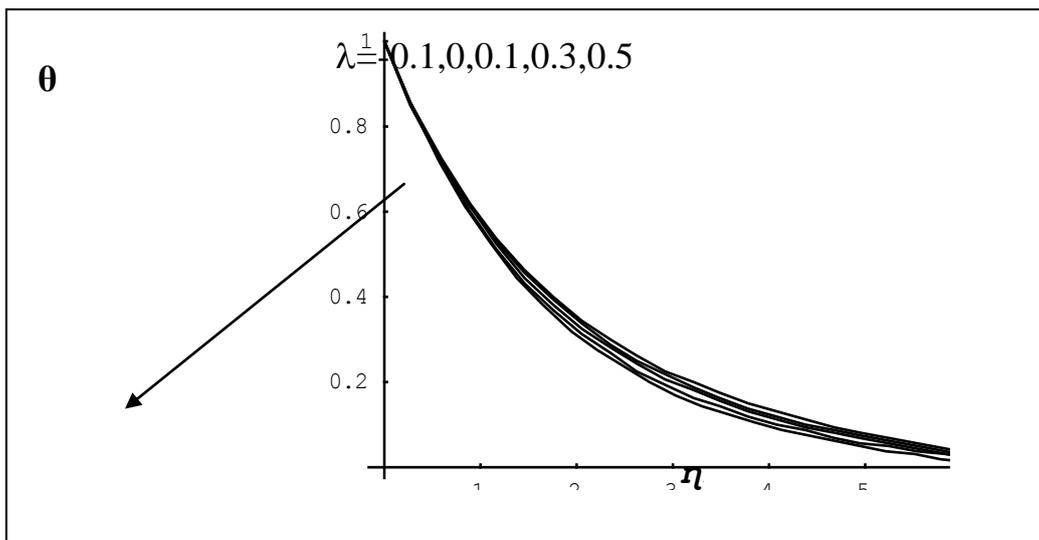


Fig.20: temperature profiles for various values of λ with $Pr=0.72$, $G= -0.2$, $A=0.2$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

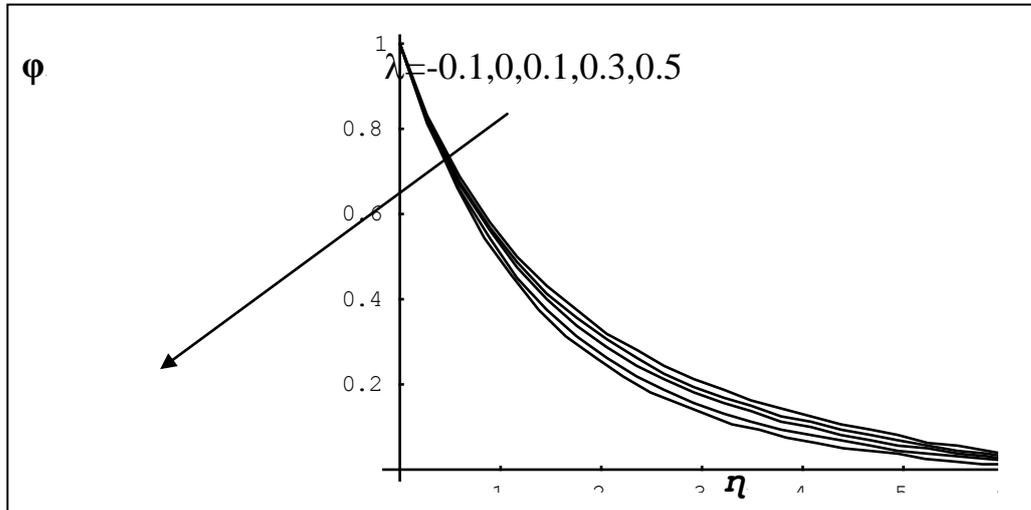


Fig21: Concentration profiles for various values of λ with $Pr=0.72$, $G= -0.2$, $A=0.2$, $M=1$, $R=0.8$, $Ec=0.72$, $f_0 = 0$ and $Sc=0.6$.

V.CONCLUSION

The purpose of the study in this paper is to present numerical solutions of unsteady mixed convection flow and heat transfer over a porous stretching surface taking into consideration, the effects of unsteadiness parameter A , mixed convection parameter λ_1 , mixed concentration parameter λ_2 , chemical reaction G , Eckert number Ec , Schmidt number Sc , magnetic parameter M , radiation parameter R , suction parameter f_0 and Prandtl number Pr . With the help of similarity transformations, the governing time dependent boundary layer equations for momentum and thermal are reduced to couple ordinary differential equations which are then solved numerically using shooting method. The numerical solution indicated that:

- 1- The velocity increases with an increase in the value of mixed convection parameter λ_1 and radiation parameter R while decreases with increase of magnetic parameter M , Prandtl number Pr , suction parameter f_0 and unsteadiness parameter A .
- 2- The temperature decreases with an increase in the value of mixed convection parameter λ_1 , Prandtl number Pr , suction parameter f_0 and unsteadiness parameter A while increases with increase of Eckert number Ec , magnetic parameter M and radiation parameter R .
- 3- The concentration increases with an increase in the value of magnetic parameter M and Prandtl number Pr while decreases with increase of unsteadiness parameter A , chemical reaction G , Schmidt number Sc , suction parameter f_0 , convection parameter λ_1 and radiation parameter R .

REFERENCES

- [1]. B. Gebhart, Y. Jaluria, R.L. Mahajan, B. Sammakia, Buoyancy, Induced Flows and Transport, Springer-Verlag, Berlin, 1988.
- [2]. A.M. Rashad, Effects of radiation and variable viscosity on unsteady MHD flow of a rotating fluid from stretching surface in porous medium, Journal of the Egyptian Mathematical Society (2014) 22, 134–142.
- [3]. S.M.M. EL-Kabeir, A.M. Rashad, Rama Subba Reddy Gorla, Unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux, Mathematical and Computer Modelling 46 (2007) 384–397.
- [4]. E. M. Abo-Eldahab, Ali A. Hallool and M.M. El-Kady, Effects of Heat Transfer on MHD Flow Over an Unsteady Stretching Surface in a Micropolar Fluid and a Porous Medium with Prescribed Surface Heat Flux, Int. J. of Appl. Mathematics and Physics, 4(1), (2012): 67-77.
- [5]. E.M.A. Elbasheshy and M.A.A. Bazid. Heat transfer over an unsteady stretching surface. *Heat Mass Transfer*, 41(2004):1-4.
- [6]. M.A. EL-Hakiem, S.M.M. EL-Kabeir, A.M. Rashad, Group method analysis for the effect of radiation on MHD coupled heat and mass transfer natural convection flow for water vapor over a vertical cone through porous medium, Int. J. Appl. Math. Mech. 3 (2) (2007) 35-53.



ISSN: 2350-0328

**International Journal of Advanced Research in Science,
Engineering and Technology**

Vol. 5, Issue 2 , February 2018

- [7]. E. M.A.Elbashbeshy, D.A.Aldawody, Effects of Thermal Radiation and Magnetic Field on Unsteady Mixed Convection Flow and Heat Transfer over a Porous stretching Surface, *Int. J. of Nonlinear Science* 9(2010)4, 448-454.
- [8]. M.A. EL-Hakiem, A.M. Rashad, Effect of radiation on non-Darcy free convection from a vertical cylinder embedded in a fluid-saturated porous medium with a temperature-dependent viscosity, *J. Porous Media* 10 (2) (2007) 209–218.
- [9]. J.D. Goddard, A. Acrivos, Quart., Analyses of laminar forced convection mass transfer with homogeneous chemical reaction. *Mech. Appl. Math.* 20(1967) 473–496.
- [10]. Lakshmisha KN, Venkateswaran S, Nath G., three dimensional unsteady flows with heat and mass transfer over a continuous stretching surface, *J. Heat Transfer* Vol.110, pp.590–595, 1988.
- [11]. Gorla RSR, Sidawi L, Free convection on a vertical stretching surface with suction and blowing, *Appl. Sci. Res.* Vol.52, pp.247–257, 1994.
- [12]. Chamkha AJ (1999) Hydromagnetic three-dimensional free convection on a vertical stretching surface with heat generation or absorption, *Int. J. Heat Fluid Flow* Vol.20, pp.84–92, 1999.
- [13]. N.T.M. Eldabe, Sallam N. Sallam, Mohamed Y. Abou-zeid, Numerical study of viscous dissipation effect on free convection heat and mass transfer of MHD non-Newtonian fluid flow through a porous medium, *J.Egyptian Math. Soc.* 20 (2012)139-151.
- [14]. P. Chandran, N. C. Sacheti and A. K. Singh. Hydro-magnetic flow and heat transfer past a continuously moving porous boundary. *Int. Commun. Heat Mass Transfer.*, 23(1996):889-898.
- [15]. I. Pop and T. Y. Na. A note on MHD over a stretching permeable surface. *Mech. Res. Commun.*, 25(1998):263-269.
- [16]. S. Mukhopadhyay, G. C. Layek and Sk. A. Samad. Study of MHD flow over a heated stretching sheet with variable viscosity. *Int. J. Heat Mass Transfer.*, 48(2005):4460-4466.
- [17]. H. I. Andersson, K. H. Bech and B. S. Dandapat. MHD flow of a power law fluid over a stretching sheet. *Int. J. Non-Linear Mech.*, 27(1992):929-936.
- [18]. C. H. Chen. Effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power law fluid past a stretched sheet with surface heat flux. *Int. J. Therm. Sci.*, 47(2008):954-961.
- [19]. Emad M. Abo El-dahab, M. Abd El-aziz, Ali A. Gaber, Joule-heating effect on hydrodynamic three – dimensional flow over a stretching surface with heat and mass transfer, *Int. J. Appl. Math. and Phys.* 1(2008), 27-38.
- [20]. E. M. Abo El-dahab, M. M. El-Kady and Ali A. Hallool, Effects of Heat and Mass Transfer on MHD Flow Over a Vertical Stretching Surface with Free Convection and Joule-Heating, *Int. J. of Appl. Math. and Physics*, 3(2) (2011): 249-258.