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Relations for Moments of Kumaraswami Power Function Distribution Based on Ordered Random Variables and a Characterization

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ABSTRACT: In this paper, recurrence relations for single and product moments of generalized order statistics from Kumaraswami power function distribution have been obtained. These relations are incorporated as a particular case of recurrence relations for order statistics and k^{th} records. Further, characterization result for this distribution has been obtained from recurrence relations of single moments of generalized order statistics.

KEYWORDS: Generalized order statistics, order statistics, upper record, recurrence relations, Kumaraswami Power function distribution, characterization.

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I. INTRODUCTION

The concept of generalized order statistics *(gos)* was given by Kamps (1995), which is given as below:
Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed (iid) random variables (rv) with absolutely continuous distribution function (df) $F(x)$ and probability density function (pdf) $f(x)$, $x \in (\alpha, \beta)$.

Let $n \in N$, $k > 0$, $\tilde{m} = (m_1, m_2, \ldots, m_{n-1}) \in \Re^{n-1}$, $M_r = \sum_{j=r}^{n-1} m_j$, $1 \le r \le n-1$, be the parameters such that $\gamma_r = k + n - r + M_r > 0$, for all $r \in \{1, 2, \ldots, n-1\}$. Then $X(1,n,\tilde{m},k), X(2,n,\tilde{m},k), \ldots, X(r,n,\tilde{m},k)$, $r = 1, 2, \ldots, n$ are called *gos* if their joint pdf is given by

$$f_{X(1,n,\tilde{m},k),\ldots,X(n,n,\tilde{m},k)}(x_1, x_2, \ldots, x_n)$$

$$= k\left(\prod_{j=1}^{n-1} \gamma_j\right)\left(\prod_{i=1}^{n-1} [\bar{F}(x_i)]^{m_i} f(x_i)\right)[\bar{F}(x_n)]^{k-1} f(x_n) \tag{1.1}$$

on the cone $F^{-1}(0+) < x_1 \le \ldots \le x_n < F^{-1}(1)$ of $\Re^n$ where $\bar{F}(x) = 1 - F(x)$ denotes the survival function

Choosing the parameters appropriately, models such as ordinary order statistics $(m = 0, k = 1, i.e. \gamma_i = n - i + 1)$, k^{th} record values $(m = -1, k \in N, i.e. \gamma_i = k)$, sequential order statistics $[\gamma_i = (n - i + 1)\beta_i; \beta_1, \beta_2, \ldots, \beta_n > 0]$, order statistics with non-integral sample size $[\gamma_i = (\beta - i + 1); \beta > 0]$, Pfeifer record values $[\gamma_i = \beta_i; \beta_1, \beta_1, \ldots, \beta_1 > 0]$ and progressive type

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II censored order statistics $(m \in \mathrm{N}, k \in \mathrm{N})$ can be obtained as particular cases of *gos*. For simplicity we have assumed that $m_1 = m_2 = \cdots = m_{n-1} = m$.

The *pdf* of $r-th$ *gos* is given by Kamps (1995)

$$f_{X(r,n,m,k)}(x) = \frac{C_{r-1}}{(r-1)!}\left(\bar{F}(x)\right)^{\gamma_r - 1} g_m^{r-1}\left(F(x)\right) f(x), \; \alpha \le x \le \beta \tag{1.2}$$

And the joint *pdf* of $X(r,n,\tilde{m},k)$ and $X(\mathrm{s},n,\tilde{m},k)$, $1 \le r < s \le n$ is given by

$$f_{X(r,n,m,k),X(s,n,m,k)}(x,y) = \frac{C_{s-1}}{(r-1)!(s-r-1)!}\left(\bar{F}(x)\right)^{m} g_m^{r-1}\left(F(x)\right)$$

$$\times\left[h_m\left(F(y)\right) - h_m\left(F(x)\right)\right]^{s-r-1}\left(\bar{F}(y)\right)^{\gamma_s - 1} f(x) f(y), \quad \alpha \le x \le \beta \tag{1.3}$$

where

$$C_{r-1} = \prod_{i=1}^{r} \gamma_i, \qquad \gamma_i = k + (n-i)(m+1),$$

$$h_m(x) = \begin{cases} -\dfrac{1}{m+1}(1-x)^{m+1}, & m \ne -1 \\ -\ln(1-x), & m = -1 \end{cases}$$

and

$$g_m(x) = h_m(x) - h_m(0), \quad x \in (0,1)$$

Recurrence relations are quite useful in computing the moments.The result given in present paper can be used to compute the moments of ordered random variables if the parent population follows Kumaraswami power function distribution. Several authors derived the recurrence relations for *gos* for different distributions. See, Kamps and Gather (1997), Ahsanullah (2000), Ahmad and Fawzy (2003), Al-Hussaini *et al.*(2005), Ahmad (2007), Khan *et al.* (2007), Kumar (2011), Kumar and Khan (2013), Khan *et al.*(2015a, 2015b), Khan and Khan (2016).

A random variable X is said to be Kumaraswami-Power function distribution (Kw-PFD) Abdul Moniem (2017) with the following probability density function *pdf*

$$f(x) = \frac{ab\theta}{\lambda}\left(\frac{x}{\lambda}\right)^{a\theta-1}\left[1-\left(\frac{x}{\lambda}\right)^{a\theta}\right]^{b-1}; \; a,b,\theta > 0, 0 \le x \le \lambda \tag{1.4}$$

and its distribution function *df* is given by

$$F(x) = 1-\left[1-\left(\frac{x}{\lambda}\right)^{a\theta}\right]^{b}; \; a,b,\theta > 0, 0 \le x \le \lambda \tag{1.5}$$`

the corresponding survival function (SF) are

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$$\bar{F}(x)=\left[1-\left(\frac{x}{\lambda}\right)^{a\theta}\right]^{b};\ a,b,\theta>0, 0\le x\le\lambda \tag{1.6}$$

In view of (1.4) and (1.6), we get

$$\bar{F}(x)=\frac{1}{ab\theta}\left[\lambda^{a\theta}x^{1-a\theta}-x\right]f(x) \tag{1.7}$$

Relation (1.7) is used to obtain the recurrence relations for moments of generalized order statistics.

The Kumaraswami Power function distribution was introduced by Abdul-Moniem (2017), adding the new shape and scale parameters (Abdul-Moniem (2017)).Which is the generalization of Kumaraswami distribution. The Kumaraswami power function distribution reduces to Kumaraswami distribution for $\lambda=1$ and $\theta=1$ The Kumaraswami distribution was appreciated for its use in hydrological phenomena and for other purposes. For more detail and its application see Kumaraswami (1980), Sundar and Subbiah (1989), Fletcher and Ponnambalam (1996), Seifi*et al.* (2000), Ganji*et al.*(2006), Courard-Hauri (2007) and Sanchez *et al.*(2007).

This paper is organized in four sections. In section 2, we have produced recurrence relation for single moments of *gos* for Kw-PFD. In section 3, we have deduced the recurrence relation for product moments of *gos* from Kw-PFD. In section 4, the characterization result based on the recurrence relation for single moments of *gos* for Kw-PFD is deduced.

II. Recurrence relation for single moments for Kw-PFD from *gos*

In this section, recurrence relation for single moments of Kw-PFD from *gos* has been obtained. Further, the particular cases of recurrence relation for single moments of order statistics and record values are deduced from *gos*.

Theorem 2.1: Let X be a non-negative continuous random variable and follows Kumaraswami power function distribution in (1.4). Suppose that $j>0$ and $1\le r\le n$

$$E\left[X^{j}(r,n,m,k)\right]-E\left[X^{j}(r-1,n,m,k)\right]$$
$$=\frac{j\lambda^{a\theta}}{\gamma_r ab\theta}E\left[X^{j-a\theta}(r,n,m,k)\right]-\frac{j}{\gamma_r ab\theta}\left[X^{j}(r,n,m,k)\right] \tag{2.1}$$

Proof: From (1.2)**,** we have

$$E\left[X^{j}(r,n,m,k)\right]=\frac{C_{r-1}}{(r-1)!}\int_0^{\lambda}x^{j}\left[\bar{F}(x)\right]^{\gamma_r-1}g_m^{r-1}\left[F(x)\right]f(x)dx$$

Integrating above integral treating $\left[\bar{F}(x)\right]^{\gamma_r-1}f(x)$ as integrand and rest part for differentiation, we have

$$E\left[X^{j}(r,n,m,k)\right]=\frac{jC_{r-1}}{\gamma_r(r-1)!}\int_0^{\lambda}x^{j-1}\left[\bar{F}(x)\right]^{\gamma_r}g_m^{r-1}\left[F(x)\right]dx+\frac{C_{r-1}}{\gamma_r(r-2)!}\int_0^{\lambda}x^{j}\left[\bar{F}(x)\right]^{\gamma_{r-1}-1}g_m^{r-2}\left[F(x)\right]f(x)dx$$

which implies that

$$E\left[X^{j}(r,n,m,k)\right]-E\left[X^{j}(r-1,n,m,k)\right]=\frac{jC_{r-1}}{\gamma_r(r-1)!}\int_0^{\lambda}x^{j-1}\left[\bar{F}(x)\right]^{\gamma_r}g_m^{r-1}\left[F(x)\right]dx$$

Now in view of (1.7), we get

$$E\left[X^{j}(r,n,m,k)\right]-E\left[X^{j}(r-1,n,m,k)\right]$$

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$$=\frac{jC_{r-1}\lambda^{a\theta}}{\gamma_r(r-1)!ab\theta}\int_0^{\lambda} x^{j-a\theta}\left[\overline{F}(x)\right]^{\gamma_r-1} g_m^{r-1}\left[F(x)\right] f(x)dx$$

$$-\frac{jC_{r-1}}{\gamma_r(r-1)!ab\theta}\int_0^{\lambda} x^{j}\left[\overline{F}(x)\right]^{\gamma_r-1} g_m^{r-1}\left[F(x)\right] f(x)dx$$

and hence the result.

Remark 2.1: Setting $m=0$ and $k=1$ in Theorem (2.1), we get the recurrence relation for single moments of order statistics of Kw-PFD as

$$E\left[X_{r:n}^{j}\right]-E\left[X_{r-1:n}^{j}\right]=\frac{j\lambda^{a\theta}}{ab\theta(n-r+1)}E\left[X_{r:n}^{j}\right]-\frac{j}{ab\theta(n-r+1)}\left[X_{r:n}^{j}\right]$$

Remark 2.2: Setting $m=-1$ and $k\geq 1$ in Theorem (2.1), we get the recurrence relation for single moments of upper k^{th} record of Kw-PFD as

$$E\left[X^{j}(r,n,-1,k)\right]-E\left[X^{j}(r-1,n,-1,k)\right]$$
$$=\frac{j\lambda^{a\theta}}{ab\theta k}E\left[X^{j-a\theta}(r,n,-1,k)\right]-\frac{j}{ab\theta k}E\left[X^{j}(r,n,-1,k)\right]$$

Remark 2.3: Setting $\lambda=1$ and $\theta=1$ in (2.1) we get recurrence relation for single moments of *gos* from Kumaraswami distribution.

III. Recurrence relation for product moment of *gos* from Kw-PFD

In this section, the recurrence relation for product moment of *gos* from Kw-PFD has been obtained. Further, the recurrence relations for product moments of order statistics and record values are obtained as particular case of *gos*.

Theorem 3.1: Let X be a non-negative continuous random variable and follows Kw-PFD in (1.4). Suppose that $i,j>0$ and $1\leq r<s\leq n$ the following recurrence relation is satisfied.

$$E\left[X^{i}(r,n,m,k)X^{j}(s,n,m,k)\right]-E\left[X^{i}(r,n,m,k)X^{j-1}(s-1,n,m,k)\right]$$

$$=\frac{j\lambda^{a\theta}}{ab\theta\gamma_s}E\left[X^{i}(r,n,m,k)X^{j-a\theta}(s,n,m,k)\right]-\frac{j}{ab\theta\gamma_s}E\left[X^{i}(r,n,m,k)X^{j}(s,n,m,k)\right] \quad (3.1)$$

Proof: From (1.3), we have

$$E\left[X^{i}(r,n,m,k)X^{j}(s,n,m,k)\right]$$

$$=\frac{C_{s-1}}{(r-1)!(s-r-1)!}\int_0^{\lambda} x^{j}\left[\overline{F}(x)\right]^{m} f(x)g_m^{r-1}\left[F(x)\right]I(x)dx \quad (3.2)$$

where

$$I(x)=\int_x^{\lambda}\left[h_m(F(y))-h_m(F(x))\right]^{s-r-1}\left[\overline{F}(y)\right]^{\gamma_s-1} f(y)dy \quad (3.3)$$

Solving the integral *I(x)* by parts and substituting the resulting expression in (3.2), we get

$$E\left[X^{i}(r,n,m,k)X^{j}(s,n,m,k)\right]-E\left[X^{i}(r,n,m,k)X^{j-1}(s-1,n,m,k)\right]$$

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$$=\frac{jc_{s-1}}{\gamma_s(r-1)!(s-r-1)!}\int_0^{\lambda}\int_x^{\lambda} x^i y^{j-1}\left[\bar{F}(x)\right]^m f(x) g_m^{r-1}\left[F(x)\right]\left[\bar{F}(y)\right]^{\gamma_s}$$

$$\times\left[h_m(F(y))-h_m(F(x))\right]^{s-r-1} dydx$$

In view of (1.7), we obtain

$$=\frac{jc_{s-1}}{\gamma_s(r-1)!(s-r-1)!}\int_0^{\lambda}\int_x^{\lambda} x^i y^{j-1}\left[\bar{F}(x)\right]^m f(x) g_m^{r-1}\left[F(x)\right]\left[\bar{F}(y)\right]^{\gamma_s-1}$$

$$\times\left[h_m(F(y))-h_m(F(x))\right]^{s-r-1}\left\{\frac{1}{ab\theta}\left[\lambda^{a\theta}x^{1-a\theta}-x\right]f(x)\right\}dydx$$

and hence the theorem.

Remark 3.1: Setting $m=0$ and $k=1$ in Theorem (3.1), we get the recurrence relation for product moments of order statistics of Kw-PFD as

$$E\left[X_{r:n}^{i}X_{s:n}^{j}\right]-E\left[X_{r:n}^{i}X_{s-1:n}^{j-1}\right]$$

$$=\frac{j\lambda^{a\theta}}{ab\theta(n-s+1)}E\left[X_{r:n}^{i}X_{s:n}^{j-a\theta}\right]-\frac{j}{ab\theta(n-s+1)}E\left[X_{r:n}^{i}X_{s:n}^{j}\right]$$

Remark 3.2: Setting $m=-1$ and $k\geq 1$ in Theorem (3.1), we get the recurrence relation for product moments of upper k^{th} record of Kw-PFD as

$$E\left[X^i(r,n,-1,k)X^j(s,n,-1,k)\right]-E\left[X^i(r,n,-1,k)X^j(s-1,n,-1,k)\right]$$

$$=\frac{j\lambda^{a\theta}}{ab\theta k}E\left[X^i(r,n,-1,k)X^{j-a\theta}(s,n,-1,k)\right]-\frac{j}{ab\theta k}E\left[X^i(r,n,-1,k)X^j(s,n,-1,k)\right]$$

Remark 3.3: Setting $\lambda=1$ and $\theta=1$ in (3.1) we get recurrence relation for product moments of *gos* from Kumaraswami distribution.

IV. Characterization of Kw-PFD

In this section we have discussed the characterization of Kw-PFD. Characterization of probability distribution plays an important role in probability and statistical science. Before applied a particular model on real world data it is necessary to check that given continuous probability distribution satisfied the underlying assumptions. In this section, we have obtained a characterization result of Kw-PFD based on recurrence relation of single moments from *gos*.

Theorem 4.1: The necessary and sufficient condition for a random variable X to be satisfied with *pdf* given in (1.4) for $m\geq -1$ is that

$$\frac{j\lambda^{a\theta}}{ab\theta\gamma_r}E\left[X^{j-a\theta}(r,n,m,k)\right]-\frac{j}{ab\theta\gamma_r}E\left[X^j(r,n,m,k)\right]$$

$$=E\left[X^j(r,n,m,k)\right]-E\left[X^j(r-1,n,m,k)\right] \quad (4.1)$$

If and only if

$$\overline{F}(x)=\frac{1}{ab\theta}\left[\lambda^{a\theta}x^{1-a\theta}-x\right]f(x)$$

Proof: The necessary part follows immediately from (4.1) on the other hand if recurrence relation (4.1) is satisfied, then

$$j\lambda^{a\theta}E\left[X^{j-a\theta}(r,n,m,k)\right]-j\lambda^{a\theta}E\left[X^{j}(r,n,m,k)\right]$$
$$=\gamma_r ab\theta E\left[X^{j}(r,n,m,k)\right]-\gamma_r ab\theta E\left[X^{j}(r-1,n,m,k)\right]$$

$$\gamma_r ab\theta\left[\frac{c_{r-1}}{(r-1)!}\int_0^{\lambda}x^j\left(\overline{F}(x)\right)^{\gamma_r-1}g_m^{r-1}(F(x))f(x)dx-\frac{(r-1)c_{r-1}}{\gamma_r(r-1)!}\int_0^{\lambda}x^j\left(\overline{F}(x)\right)^{\gamma_r+m}g_m^{r-2}(F(x))f(x)dx\right]$$

$$=\gamma_r ab\theta\frac{c_{r-1}}{(r-1)!}\int_0^{\lambda}x^j\left(\overline{F}(x)\right)^{\gamma_r}g_m^{r-2}(F(x))f(x)\left[\frac{g_m(F(x))}{\left(\overline{F}(x)\right)}-\frac{(r-1)\left(\overline{F}(x)\right)^{m}}{\gamma_r}\right]dx$$

Let

$$h(x)=-\frac{\left(\overline{F}(x)\right)^{\gamma_r}g_m^{r-1}(F(x))}{\gamma_r}\tag{4.2}$$

Differentiating both sides of (4.2), we get

$$h'(x)=\left(\overline{F}(x)\right)^{\gamma_r}g_m^{r-2}(F(x))f(x)\left[\frac{g_m(F(x))}{\overline{F}(x)}-\frac{(r-1)\left(\overline{F}(x)\right)^{\gamma_r}}{\gamma_r}\right]$$

Now

$$j\lambda^{a\theta}E\left[X^{j-a\theta}(r,n,m,k)\right]-jE\left[X^{j-a\theta}(r-1,n,m,k)\right]=\gamma_r ab\theta\frac{c_{r-1}}{(r-1)!}\int_0^{\lambda}x^j h'(x)dx\tag{4.3}$$

Integrating RHS in (4.3) by parts and using the value of $h(x)$ from (4.2), we have

which reduces to

$$\frac{c_{r-1}}{(r-1)!}\int_0^{\lambda}x^{j-1}\left(\overline{F}(x)\right)^{\gamma_r-1}g_m^{r-1}(F(x))f(x)\left[\lambda^{a\theta}x^{1-a\theta}-x-ab\theta\frac{\overline{F}(x)}{f(x)}\right]dx=0\tag{4.4}$$

Now applying generalization of *Müntz-Szász* Theorem Hwang and Lin (1984) to (4.4), which state that on a space $L(a,b)$ of summable function defined on (a,b), a sequence of functions $f_n(x)$ is complete on (a,b) if for any $g\in L(a,b)$ the equalities

$$\int_a^b f_n(x)g(x)dx=0,\ \ n=1,2,3,\ldots$$

implies that $g(x)=0$ on (a,b), then we get

$$\lambda^{a\theta}x^{1-a\theta}-x-ab\theta\frac{\overline{F}(x)}{f(x)}=0$$

$$ab\theta\frac{\overline{F}(x)}{f(x)}=\lambda^{a\theta}x^{1-a\theta}-x$$

$$ab\theta\left(\overline{F}(x)\right)=\left[\lambda^{a\theta}x^{1-a\theta}-x\right]f(x)$$

$$\overline{F}(x)=\frac{1}{ab\theta}\left[\lambda^{a\theta}x^{1-a\theta}-x\right]f(x)$$

This proves that $f(x)$ has the form as in (1.4).

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