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Expected Number of Level Crossings of Legendre Polynomials

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ABSTRACT:-The aim of this paper is to estimate the number of real zeros of a orthogonal random polynomial under different condition when the coefficients belong to the domain of attraction of orthogonal properties. Let $y=\sum_{k=0}^{n} y_k(w)\Psi_k(t)$ be random polynomial such that [*y_0 (w), y_1 (w),……y_n(w)*] is a sequence of mutually independent, normally distributed random variables with mean zero and variance unity and [Ψ_0 (t),……. Ψn (t)] be a sequence of normalized orthogonal Legendre polynomials, defined by $\Psi_n(t)=\sqrt{n+1/2}\,P_n(t),$ where $P_n(t)$ is the classical Legendre polynomial. Then, for any constant K such that $(K^2/n)\to 0$ as $n\to\infty$, the mathematical expectation of number of real zeros of the equation $y=\sum_{k=0}^{n} y_k(w)\Psi_k(t)$ =k is asymptotic to $\sqrt{n/3}$.

I. Basic idea about no of zeros or no of level crossing

A. Level crossing: in general a level crossing means crossing of a railway or a railroad crossing is a place where a line and a road intersect each other on the same level.In mathematics no of level crossing means no of zeros or point of intersection between a curve and x-axis.For example: the trigonometric functions sin x and cos x the point of intersection of the curve with x axis are shown in the following figures called number of zeros or number of level

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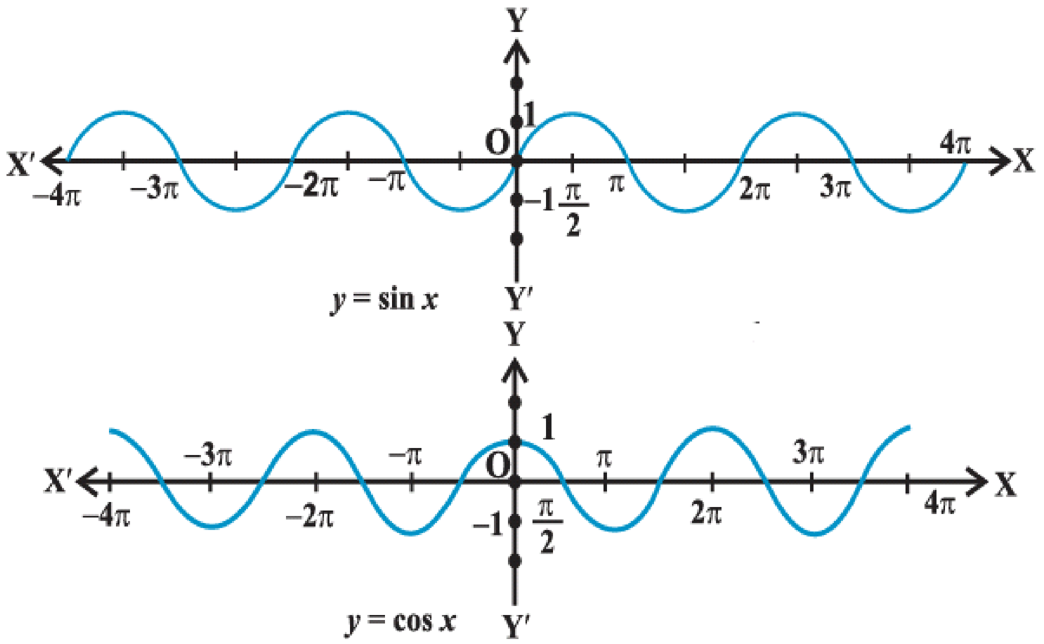
crossings of the above two curves with x-axis.



. **Figure 1:-** Here number of zeros for sin x is nπ and no of zeros for cos x is (2n+1)π/2 for n is any integer.

Orthogonal polynomial: An orthogonal polynomial is a family of polynomial such that any two different polynomial in the sequence are orthogonal to each other under some inner product.
The most widely used orthogonal polynomials are classical orthogonal polynomial consisting of**a**. Legendre polynomial, **b.** Hermite polynomial , **c**. Jacobi polynomial
d. Bessel polynomial

Legendre polynomial: The solution Legendre differential equation are a set of functions known as the Legendre polynomials. The polynomials are defined on[-1,1].
We can call Legendre polynomials in Mathematics using: LegendreP[n,x].
Where n represent the polynomial, and x is the variable.

Solution to differential equation:

Legendre polynomial are one of the solutions to the Legendre differential equation ;

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0 \quad \_\_\_\_\_\_\_\_\_\_(1)$$

Here n is a constant is known as Legendre differential equation
To obtain the solution of (1)we shall use series solution method.
An expression for the Legendre polynomial Pn(x) is given by the formula known as Rodrigues's formula of degree n

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

Recursive formula for Legendre polynomial:

(a).$\frac{d}{dx}P_n(x) - x\frac{d}{dx}P_{n-1}(x) = nP_{n-1}(x)$

(b)$x\frac{d}{dx}P_n(x) - \frac{d}{dx}P_{n-1}(x) = nP_n(x)$

(c) $(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$

(d) $\frac{d}{dx}P_n(x) - \frac{d}{dx}P_{n-1}(x) = (2n+1)P_n(x)$

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Legendre polynomials are defined to be orthonormal, meaning the integral of a product of Legendre polynomials is either zero or one. In other words there is either an orthonormal constant N s.t

$$N\int_{-1}^{1} P_n(x)P_n(x)\,dx = 1$$

They are orthogonal in[-1,1]

1. $\int_{-1}^{1} P_n(x)P_m(x)dx = 0\ for\ m\ not\ eqaul\ to\ n$

2. $\int_{-1}^{1} P_n(x)^2 dx = \frac{2}{2n+1}$

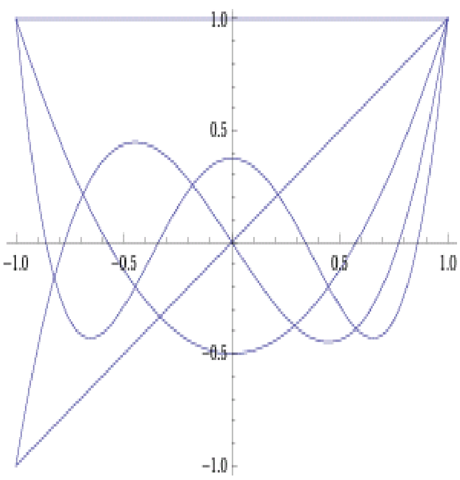
Zeros of Legendre polynomial: The Legendre polynomials are

For n=1 the Legendre polynomial is $P_0(x) = 1, P_1(x) = x\ , P_2(x)=\frac{1}{2}(-1+3x^2), P_3(x) = \frac{1}{2}(-3x+5x^3)$and so on.

We can plot the first several polynomials :

In[135]:= Plot[LegendreP[n, x] /. n → {0, 1, 2, 3, 4}, {x, -1, 1}]

Out[135]=



How many roots does the n^{th} Legendre polynomial have.

we know that the even order Legendrepolynomial are even and odd orders are odd function .According to the general result about the zeros of solutions, thek^{th} polynomial should have k zeros in the interval[-1,1].

II.Theorem 1.1

Let $y = \sum_{k=0}^{n} y_k(w)\Psi_k(t)$ be random polynomial such that [y_0 (w), y_1 (w),……y_n(w)] is a sequence of mutually independent, normally distributed random variables with mean zero and variance unity and [Ψ_0 (t),……. Ψn (t)] be a sequence of normalized orthogonal Legendre polynomials, defined by $\Psi_n(t) = \sqrt{n + 1/2}\,P_n(t),$ where $P_n(t)$ is the

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classical Legendre polynomial. Then, for any constant K such that $(K^2/n)\to 0$ as $n\to\infty$, the mathematical expectation of number of real zeros of the equation $y=\sum_{k=0}^{n} y_k(w)\Psi_k(t)$=k is asymptotic to $\sqrt{n/3}$.

III.INTRODUCTION

Let $f(t)=f(t,w)=\sum_{k=0}^{n} y_k(w)\Psi_k(t)$ Where $\{y_0, (w), y_1 (w),\ldots\ldots y_n(w)\}$ is a sequence of independent random variables on a probability space (Ω, A,P), each normally distributed with mathematical expectation zero and variance one. Let $\{\Psi_0 (t),\ldots\ldots \Psi n (t)\}$ be a sequence of normalized orthogonal Legendre polynomials. Let ENn(f: α,β) be the expected number of real zeros of the equation f(t)=k in the interval α≤t≤β, where multiple zeros are counted only once. We know, from the work of ***Das [2],*** that in the interval –1st≤t≤1, all save a certain exceptional set, the functions f(t) have $\frac{\sqrt{n}}{\sqrt{3}}+0(n^{1/3})$ zeros, on the average, when n is large and k=0. The measure of the exceptional set does not exceed exp (-n/3).

Farahmand [3] has shown that when $\Psi_k (t)=t^k$, has excepted number of k-level crossings of the algebraic polynomial $Q(t)=\sum_{k=0}^{n} y_k(w)t^k$ satisfies

$EN_n(Q: -1,1)= (1/\pi) \log (n/K^2)$,

$EN_n(Q: -\infty,-1)= EN_n(Q:1, \infty)\sim(2 \pi)^{-1}\log n$.

A corresponding estimate, when Ψ_k (t)=coskt, is also due to ***Farahamand [4]*** who showed that the random trigonometric polynomial $T(t)=\sum_{k=0}^{n} y_k(w)\cos kt$ has $(\frac{2}{\sqrt{3}})n+0(n^{1/3})$ expected number of real zeros as long as $(K^2/n)\to 0$ as $n\to\infty$.

Comparison of these results with our theorem shows the difference and similarity of behaviour of algebraic and trigonometric polynomial with the orthogonal polynomial considered by us. Thus, the number of crossings of the algebraic polynomial with the level K decreases as K increases, while for trigonometric polynomial and orthogonal polynomial f(t) the average number of level crossings remains fixed with probability one, as long as $(K^2/n)\to 0$ as $n\to\infty$.

In order to estimate EN_n (f: -1,1) for the polynomial

$$f(t)=\sum_{k=0}^{n} y_k(w)\Psi_k(t)=K, \qquad (2)$$

To find number of level crossings of the above polynomial in the interval (-1,1) we divide the interval (-1,1) into subintervals (-1, -1+€), (-1+€, 1-€) and (1-€,1) as in the, where $\in= n^{-\frac{4}{7+\delta}}, \delta>0$. First, we derive number of level crossings of the above polynomial in the interval (-1+€, 1-€) using a well known formula to find the number of level crossing of any polynomial called Kac-Rice formula, we use its extended formula for our use.

IV. Extended Kac-Rice Formula for EN_n (f: α,β)

From ***Das***[2] and ***Crammer***we find that the expected number of real zeros of the equation f(t)=0 in the interval (α,β) satisfies:

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$$EN_n(f:\alpha,\beta)=\int_{\alpha}^{\beta}dt\int_{-\infty}^{\infty}|y|\Phi(x.y)dy, \qquad (3)$$

where Φ(x,y) is the density function of the distribution of f(t) and its derivative f'(t).

$$X=X_n(t)=\sum_{k=0}^{n}\left[\Psi_k(t)\right]^2$$

Let $$Y=Y_n(t)=\sum_{k=0}^{n}\left[\Psi_k(t)\right]\left[\Psi'_k(t)\right],$$

$$Z=Z_n(t)=\sum_{k=0}^{n}\left[\Psi'_k(t)\right]^2$$

$$and\ \Delta^2=X_n(t)Z_n(t)-Y_n^{\,2}(t).$$

Then, the joint density of (f,f) is

$$\Phi(x,y)=(2\pi\Delta)^{-1}\exp\left\{-(Zx^2-2Yxy+Xy^2)/(2\Delta^2)\right\}. \qquad (4)$$

Let $S=\left(X^{1/2}y\right)/\sqrt{2\Delta}.$

From (3), we have

$$\int_{-\infty}^{\infty}|y|\Phi(x,y)dy=(\Delta/\pi x)\exp(-Zk^2/2\Delta^2)\int_{-\infty}^{\infty}|s|\exp\left(\frac{\sqrt{2YKs}}{\Delta X^{1/2}}\right)ds. \qquad (5).$$

Putting p= $\dfrac{\sqrt{2YKs}}{\Delta X^{1/2}}$ in (5), we have

$$\int_{-\infty}^{\infty}|y|\Phi(x,y)dy=\int_{-\infty}^{\infty}|s|\{\exp(ps)+(-ps)\}\exp(-s^2)\,ds.$$

$$=\theta\{p\}+\theta(-p),$$

Where $\theta(p)=\int_0^{\infty}|s|\{\exp(ps-s^2)\,ds$

$$=\tfrac{1}{2}+(p/2)\exp(p^2/4)\int_0^{\infty}\exp\{-(s-p/2)^2\,ds \qquad (6)$$

$$\tfrac{1}{2}+(p/2)\exp(p^2/4)\{\sqrt{\pi/2}+\operatorname{erf}(p/2)$$

Where f (x)= $\int_0^{\infty}\exp(-t^2)\,dt.$

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Hence from (3), (4), (5) and (6)

$$EN_n(f:\alpha,\beta)=\int_\alpha^\beta \frac{\Delta}{\pi X}\exp\{-Zk^2/(2\Delta^2\}dt$$

$$+\int_\alpha^\beta(\sqrt{2/\pi})\frac{|KY|}{X^{3/2}}\exp\{-k^2/2X\}erf(|KY|/X^{1/2}\Delta\sqrt{2})dt$$

$=I_1(\alpha,\beta)+I_2(\alpha,\beta)$ (7)

we divide the interval (-1,1) into subintervals (-1, -1+€), (-1+€, 1-€) and (1-€,1) Combining we prove theorem.

V. Average Number of Level Crossings in the Interval (-1+ε, 1- ε)

The ***ChristoffelDarboux formula, Sansone [5]*** for Legendre polynomials

$P_n(t)$ reads as follows:

$$\sum_{k=0}^{n}(2k+1)P_k(u)P_k(t)=(n+1)\left\{\frac{P_{n+1}(u)P_n(t)-P_n(u)P_{n+1}(t)}{u-t}\right\} \quad (8)$$

Putting $h_k=(2k+1)^{-1}$ and $\mu_n=(n+1)$ and following the procedure described in last section of the paper, we have

$$X=\sum_{k=0}^{n}[\Psi_k(t)]^2=(n+1)[P_{n+1}'(t)]P_n(t)-P_n'(t)P_{n+1}(t) \quad (9)$$

$$Y=\sum_{k=0}^{n}[\Psi_k(t)]\Psi'_k(t)=\frac{n+1}{2}[P''_{n+1}'(t)]P_n(t)-P_{n-1}'(t)P'''_n(t) \quad (10)$$

$$+\frac{n+1}{2}[P''_{n+1}'(t)]P'_n(t)-P'_{n+1}'(t)P''_n(t)$$

For Legendre polynomial $P_n(t)$, we have the relations

$$(1-t^2)P_{n+1}''(t)=2tP'_{n+1}(t)-(n+1)(n+2)P_{n+1}(t) \quad (11)$$

And $$(1-t^2)P''_n(t)=2tP'_n(t)-n(n+1)P_n(t) \quad (12)$$

From (11) and (12),

we obtain $(1-t^2)(P_{n+1}''(t)P'_n(t)-P''_n(t)P'_{n+1}(t))$

$$=n(n+1)[P_{n+1}''(t)P_n(t)-P'_n(t)P_{n+1}(t)]-2(n+1)P_{n+1}(t)P'_n(t) \quad (13)$$

And $$(1-t^2)(P_{n+1}''(t)P_n(t)-P_{n+1}(t)P''_n(t))$$

$$=2t[P_{n+1}'(t)P_n(t)-P_{n+1}(t)P'_n(t)]-2(n+1)P_n(t)P_{n+1}(t) \quad (14)$$

Differentiating (13) and (14), and multiplying them respectively by $P_n(t)$ and $P_{n+1}(t)$ we obtain, after simplifications

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$$(1-t^2)(P_{n+1}'''(t)P_n(t)-P'''_n(t)P_{n+1}(t))$$
$$=\left[\frac{16t}{1-t^2}-(n^2+3n+2)\right]\left[P_{n+1}'(t)P_n(t)-P'_n(t)P_{n+1}(t)P\right] \quad (15)$$
$$+2n(n+1)P_{n+1}(t)P'_n(t)-\frac{16(n+1)}{1-t^2}P_n(t)P_{n+1}(t)$$

From *Sansone [5]* we have

$$(1-t^2)(P_n'(t)=nP_{n-1}(t)-ntP_n(t) \quad (16)$$

and $$(1-t^2)(P_{n+1}'(t)=(n+1)P_n(t)-(n+1)tP_{n+1}(t). \quad (17)$$

Hence, using (16) and (17), we have

$$(1-t^2)(P_{n+1}'(t)P_n(t)-P_{n+1}(t)P'_n(t))$$
$$=(n+1)\left[P^2{}_n(t)+P^2{}_{n+1}(t)-2tP_n(t)P_{n+1}(t)\right] \quad (18)$$

In the range $\varepsilon\leq\gamma\leq\pi-\varepsilon$ and $0<\varepsilon<\pi/2$, the asymptotic estimate of $P_n(t)$, for $t=\cos\gamma$, is given by ***Sansone [5]***

$$\left(\frac{2}{\pi n\sin\gamma}\right)^{1/2}\cos\left[(n+1/2)\gamma-\frac{\pi}{4}\right]+O(n(\sin\gamma)^{-3/2}). \quad (19)$$

Hence

$$P_n{}^2(t)+P^2{}_{n+1}(t)-2tP_n(t)P_{n+1}(t)$$
$$\frac{2}{\pi n\sin\gamma}\left[\cos^2\left\{(n+1/2)\gamma-\frac{\pi}{4}\right\}\cos^2\left\{n+3/2)\gamma-\frac{\pi}{4}\right\}\right]$$
$$-2\cos\gamma\cos\left\{(n+1/2)\gamma-\frac{\pi}{4}\right\}\cos\left\{n+3/2)\gamma-\frac{\pi}{4}\right\}+O(n^{-2}\operatorname{cosec}^2\gamma) \quad (20)$$
$$=\frac{2}{n\pi}(1-t^2)^{1/2}+O(n^{-2}(1-t^2)^{-1}.$$

From (17) and (19), we obtain

$$P_{n+1}'(t)+P_n(t)-P'_n(t)P_{n+1}(t)$$
$$=\frac{2}{\pi}(1-t^2)^{-1/2}+O(n^{-1}(1-t^2)^{-1/2}. \quad (21)$$

By the first theorem of ***Sansone [5]*** we have

$$\left|P_n'(t)\right|\leq 8n^{1/2}(1-t^2)^{-5/4}.$$

Hence

$$P_n(t)P_n'(t)=0(1-t^2)^{-3/2}. \quad (22)$$

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With the help of above estimates we are in a position to calculate asymptotic estimates of X, Y and Z.

Using (20) and (22) in (12) we have

$$(1-t^2)(P_{n+1}{}''(t)P'_n(t)P''_n(t)P'_{n+1}(t)$$
$$=\frac{2n^2}{\pi}(1-t^2)^{1/2}+O(n^{-2}(1-t^2)^{-3/2}.$$

Hence

$$P_{n+1}{}''(t)-P''_n(t)P'_{n+1}(t)$$
$$=\frac{2n^2}{\pi}(1-t^2)^{3/2}+O(n(1-t^2)^{-5/2}. \qquad (23)$$

From (16), (22) and (23), we have

$$(1-t^2)(P_{n+1}{}'''(t)P_n(t)-P'''_{n+1}(t)$$
$$=\frac{2}{\pi}\left(\frac{16t}{1-t^2}-n-n^2\right)(1-t^2)^{-1/2}+O(n(1-t^2)^{-3/2}.$$

So that

$$(P_{n+1}{}'''(t)P_n(t)-P'''_{n+1}(t)P_{n+1}(t)$$
$$=\frac{2}{\pi}n^2(1-t^2)^{-3/2}+O(n(1-t^2)^{-5/2}. \qquad (24)$$

Using the above estimates in (10), (11) and (12), we obtain

$$X=\frac{2}{\pi}(1-t^2)^{-1/2}(1+O(n^{-1}) \qquad (25)$$

$$Y=O(n(1-t^2)^{-3/2}) \qquad (26)$$

$$Z=\frac{2n^3}{3\pi}(1-t^2)^{-3/2}(1+O(n^{-1})(1-t^2)^{-1}) \qquad (27)$$

From the definition of Δ, we have

$$\Delta^2=XZ-Y^2 \qquad (28)$$

$$=\frac{4n^2}{3\pi^2}(1-t^2)^{-2}(1+O(n^{-2})(1-t^2)^{-3}) \qquad (29)$$

From (28) and the fact that $K^2/n\to 0$ as $n\to\infty$, we have

$$=\frac{K^2}{X^{3/2}}=O(n^{-\frac{2}{7+\delta}})$$

From (28) and (29), we have

$$=\frac{KY}{X^{3/2}}=O(n^{-\frac{2}{7+\delta}})$$

From (25) and (29), we have

$$=\frac{ZK^2}{2\Delta^2}=O(n^{-\frac{2}{7+\delta}}) \qquad (30)$$

From (30) it is clear that

$$\exp(n^{-\frac{2}{7+\delta}})\le(\frac{ZK^2}{2\Delta^2})\le 1 \text{ and}$$

Since $n^{-\frac{2}{7+\delta}}$ tends to zero for large n,

$$\exp(-\frac{ZK^2}{2\Delta^2})-1. \qquad (31)$$

Also from (28) and (31), we have

$$\frac{\Delta}{X}=\sqrt{\frac{n}{3}}(1-t^2)^{-1/2}(1+0\left(\frac{1}{n}\right) \qquad (32)$$

Consequently, we have

$$I_1(-1+\in,1-\in)=\int_{-1+\in}^{1-\in}\frac{\Delta}{\pi X}\exp\{-ZK^2/(2\Delta^2)\}dt$$
$$=\int_{-1+\in}^{1-\in}\frac{n}{\pi\sqrt{3}}(1-t^2)^{-1/2}\left(1+O\left(\frac{1}{n}\right)\right)dt \qquad (33)$$
$$\sim\sqrt{\frac{n}{3}}$$

Since erf (X)≤1

$$I_2(-1+\in,1-\in)=\int_{-1+\in}^{1-\in}\left(\frac{\sqrt{2}}{\pi}\right)\left|\frac{KY}{X^{3/2}}\right|\exp\left\{-\frac{K^2}{2X}\right\}erf\left(\frac{|KZ^{1/2}|}{X^{1/2}\Delta\sqrt{2}}\right)dt$$
$$=O\left(n^{\frac{7}{7+\delta}}\right) \qquad (34)$$

Thus from (33) and (34), we obtain $EN(f:-1+\in,1-\in)=\sqrt{\frac{n}{3}}+O\left(n^{\frac{7}{7+\delta}}\right)$ (35)

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V. AVERAGE NUMBER OF LEVEL CROSSING IN THE INTERVALS (-1,-1+ε) AND (1- ε ,1)

We show that in the ranges 1- ε≤t≤1 and -1≤t≤-1+ ε, the number of zeros of $\sum_{k=0}^{n} y_k(\omega)\Psi_k(t) = K$ is small, in comparison to those in the interval already considered.

We consider the interval (1- ε,1), to begin with.

Let $F(z)=f(y(\omega), z)$

$$=\sum_{k=0}^{n} y_k(\omega)\Psi_k(z) - K \qquad (36)$$

Where $y(\omega)$, is the random vector $[y_0(\omega), y_1(\omega), \ldots\ldots y_n(\omega)]$

Now $F(y)=f(y\,\omega)$ $=\sum_{k=0}^{n} y_k(\omega)\Psi_k(z) - K$

is a random variable with mean –K and variance $\xi^2 = \sum_{k=0}^{n} \Psi^2{}_k(1) \geq 0,$

and so has the distribution function,

$$\frac{1}{\sqrt{2\pi\xi^2}} \int_{-\infty}^{t} \exp\left(-\frac{(v+k)^2}{2\xi^2}\right) dv.$$

$$P\left(|f(1)| \leq e^{-2n\in}\right)$$

Hence
$$\left(\frac{2}{\pi\xi^2}\right)^{1/2} \int_{-\infty}^{t} \exp\left(-\frac{(v+k)^2}{2\xi^2}\right) dv. \qquad (37)$$

$$\leq \left(\frac{2}{\pi\xi^2}\right)^{1/2} e^{-2n\in} < e^{-n\in}$$

Let $I_n = \max_{0\leq k\leq n}\left(|y_k(\omega)|\right)$ we have, $P(I_n \leq n) > 1 - e^{-n/2}$

Let $T_n = \max_{0\leq k\leq n}\left(|\Psi_k(1+2\in e^{i\theta})|\right)$ Then

$$\left|f(1+2\in e^{i\theta})\right| = \left|\sum_{k=0}^{n} Y_k(\omega)\middle|\Psi_k(1+2\in e^{i\theta})\right|$$
$$\leq \left|\sum_{k=0}^{n} Y_k(\omega)\middle|\Psi_k(1+2\in e^{i\theta})\right| + K$$
$$\leq I_n \sum_{k=0}^{n} T_n + K$$
$$< n^2 T_n + K.$$

Hence we have $P\left(\left|f(1+2\in e^{i\theta}\right| \leq n^2 T_n + K\right) \geq 1 - e^{\frac{-n^2}{2}}$ (38)

1.6 Conclusion:-By considering $y = \sum_{k=0}^{n} y_k(w)\Psi_k(t)$ be random polynomial such that [$y_0(w), y_1(w), \ldots\ldots y_n(w)$] is a sequence of mutually independent, normally distributed random variables with mean zero and variance unity and [$\Psi_0(t), \Psi_1(t), \ldots\ldots$] be a sequence of normalized orthogonal Legendre polynomials, defined by $\Psi_n(t) = \sqrt{n + 1/2}\, P_n(t)$, where $P_n(t)$ is the classical Legendre polynomial. Then, for any constant K such that $(K^2/n) \to 0$ as $n \to \infty$, we found the mathematical expectation of number of real zeros of the equation $y = \sum_{k=0}^{n} y_k(w)\Psi_k(t)$ =k is asymptotic to $\sqrt{n/3}$ .Hence our theorem is proved

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