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Construction of ε mixed estimation for solving the first edge problem for the equation of non-isotropic diffusion

Tokhir Tozhiev, Kuvvatali Rahimov, Jasurbek Solijonov

PhD, Department of Information Technology, Fergana State University, Fergana, Uzbekistan
PhD student, Department of Information Technology, Tashkent University of Information Technologies, Tashkent, Uzbekistan
Student, Fergana State University, Fergana, Uzbekistan

ABSTRACT: Using method of Monte Carlo is constantly expanding in various fields as well as the development of computing technology. Increasing the speed and storage capacity of modern computing systems will allow us to solve various problems using the statistical modeling method. In this article, using the theory of martingales and Markov moments, an algorithm is constructed for solving initial-boundary value problems for the generalized non-isotropic diffusion equation.

KEY WORDS: Monte Carlo method, estimation, diffusion, martingale, unbiasedness, dispersion.

I. INTRODUCTION

To solve many classical problems of mathematical physics, a number of probabilistic representations are known. However, they do not always directly lead to a simple Monte Carlo algorithm for solving the problem. In a certain sense, it can be considered that linear transport problems are an exception to the general rule that many problems of mathematical physics obey, and in particular, boundary value problems for elliptic parabolic equations. The presence of a deep connection between differential equations and random processes requires a comprehensive study of it and opens up the prospect of creating new efficient numerical methods for solving practical problems. Such a connection has been known for a long time and at first a well-developed theory of differential equations was widely used in probability theory. It should be noted that the Monte Carlo method was used mainly for solving stationary problems of mathematical physics. There is a small number of works devoted to the development of the Monte Carlo method for solving nonstationary problems. Among them are the works [2], [6]. In our article, an algorithm for solving boundary value problems for the generalized nonisotropic diffusion equation is constructed using a sharoid algorithm based on the mean value formulas obtained in [6].

II FORMULATION OF THE PROBLEM

Consider in $(n+1)$ dimensional space R^{n+1} points $(y,z,t), y\in R^k, z\in R^l, t\in R^1, k+l=n$ equation $\frac{\partial u(y,z,t)}{\partial t}-(Lu)(y,z,t)=f(y,z,t)$ with differential operator with constant coefficients $\alpha^{ij},\beta_j^m,(1\le ij\le k, 1\le m\le l)$,

$$(Lu)(y,z,t)\equiv\sum_{ij=1}^{k}\alpha^{ij}\frac{\partial^2 u(y,z,t)}{\partial y^i\partial y^j}+\sum_{j=1}^{k}\sum_{m=1}^{l}\beta_j^m y^j\frac{\partial u(y,z,t)}{\partial z^m}.$$

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Let $\alpha=\left(\alpha^{ij}\right)-k\times k$ be matrix, $\beta=\left(\beta_j^m\right)-l\times k$ - matrix, $D\subset R^n$ is limited domain with border ∂D. Combine spatial variables (y,z) in one spatial variable - column $x=\left(y^T,z^T\right)\in R^n$. Consider in space R^{n+1} variables $(x,t)=(x_1,x_2,...,x_n,t)$ cylinder $\Omega=D*[0,T]$.

We will consider the following initial-boundary problem. For functions $\phi(x,t)\in C\left(\partial D*[0,\mathrm{T}]\right)$ and $\varphi(x)\in C(D)$, find function $u(x,t)\in C\left(\bar{D}*[0,\mathrm{T})\right)\cap C^{(2,1)}\left(\bar{D}*[0,\mathrm{T})\right)$ satisfying in the cylinder Ω to the equation

$$\frac{\partial u(y,z,t)}{\partial t}-(Lu)(y,z,t)=f(y,z,t),(x,t)\in\Omega \tag{1}$$

marginal conditions

$$u(x,t)=\varphi(x,t),\ x\in\partial D,\ t\in[0,T) \tag{2}$$

and initial conditions

$$u(0,t)=\phi(x,t),x\in D \tag{3}$$

Further, for convenience, we introduce matrices of size $n*n$ (in a block record), which have the following form

$$a=\begin{bmatrix} m_1 & m_2\\ m_2^T & m_3\end{bmatrix},\ C=\begin{bmatrix} I_k & 0\\ -\beta & I_l\end{bmatrix}=\exp\begin{bmatrix} 0 & 0\\ -\beta & 0\end{bmatrix},\ d(\rho)=\begin{bmatrix} \rho^{\frac{1}{2}}I_k & 0\\ 0 & \rho^{\frac{3}{2}}I_l\end{bmatrix}$$

block $m_1=\frac{1}{4}\left[\alpha^{-1}+3\beta^T\omega\beta\right]$ - size to $k\times k$, block $m_2=\frac{3}{2}\beta^T\omega$ - size to $k\times l$, m_2^T - size to $l\times k$, $m_3=3\omega$ - size to $l\times l$, $I_k,o,-\beta,I_l$ - similar sizes.

The matrix a is symmetric and positive defined and therefore it can be represented as a product of some matrix on its transposed b^T, videlicet $a=b^Tb$, $d(p)$ - diagonal matrix, $\omega=\left(\beta\alpha\beta^T\right)^{-1}$.

III. REPRESENTATION OF SOLUTIONS

Then $Z(x,t;y,\tau)$ - fundamental solution of equation (1) with point singularity on the point (y,τ) has the form [2]

$$Z(x,t;y,\tau)=\pi^{-\frac{n}{2}}\|a\|^{\frac{1}{2}}(t-\tau)^{-\frac{\gamma}{2}}\exp\left\{-(y-Cx)^T d\left(\frac{1}{t-\tau}\right)ad\left(\frac{1}{t-\tau}\right)(y-cx)\right\}$$

where $\|a\|$ determinant of the matrix a, $\gamma=k+3l$.

We introduce a domain depending on the parameter $r>0$

$$B_r(x,t)=\left\{(y,\tau):Z(x,t;y,\tau)>\pi^{-\frac{n}{2}}\|a\|^{\frac{1}{2}}r^{-\gamma},t>\tau\right\},$$

which will be called the sharoid of radius r with the center at the point (x,t) and its border $\partial B_r(x,t)$ spheroid.

For $r\to 0$ $B_r(x,t)$ and $\partial B_r(x,t)$ monotonously tightened to (x,t). Therefore there exists $r=r(x,t)>0$, which $\bar{B}_r(x,t)\subset\bar{\Omega}$ for $(x,t)\in\Omega$. We present one of the ways to select parameter $r>0$.

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Let $R(x)$ be the distance from x to the bound of *D*, $|x| = \max_{1\le i\le k}|x_i|$, μ - the largest eigenvalue of the matrix *a*, $\rho = \max\left\{t^{\frac{1}{2}}, t^{\frac{3}{2}}\right\}$,

$$\beta' = \max_{1\le i\le l}\left\{\sum_{i,j=1}\left|\beta^{ij}\right|\right\},\quad V_1 = \sqrt{\frac{\gamma}{8\mu\rho^2 e}},$$

$$V_2 = \beta'|x|,\qquad r_1(x) = \left[\frac{-V_2 + \sqrt{V_2^2 + 4R(x)V_1}}{2V_1}\right]^{\frac{1}{2}}.$$

Lemma 1. If $r = r(x,t) = \min\{r_1(x), t^{\frac{1}{2}}\}$ at $r(x,t)\in\Omega$ then $\bar{B}_r(x,t)\in\bar{\Omega}$.

Evidence: From $|y-x| \le R^2(x)$ it follows that $\bar{B}_r(x,t)\subset\bar{\Omega}$.

$$|y-x| \le |y-Cx| + |Cx-x| \le R(x) \tag{4}$$

From the definition of $B_r(x,t)$ and using some mathematical transformations for the first term of (4) we get

$$\left(d(t-\tau)(y-Cx)\right)^{\mathrm{T}} a\left(d(t-\tau)(y-Cx)\right) \le \mu\left|d(t-\tau)(y-Cx)\right|^2 \le$$
$$\le \mu\left\|d(t-\tau)\right\|^2 |y-Cx|^2 \le \mu\rho^2|y-Cx|^2.$$

Since $\left\|d(t-\tau)\right\|^2 \le \rho^2$.

Let $g(t-\tau) = (t-\tau)^4 \ln\frac{r^2}{t-\tau}$, then $\max_{\tau} g(t-\tau) = \frac{e^{-1}r^8}{4}$ for $\tau = t - r^2 e^{-\frac{1}{4}}$.

From the inequality $\mu\rho^2|y-Cx| \le \frac{e^{-1}r^8\gamma}{8}$ we get $|y-Cx|^2 \le \frac{e^{-1}r^8\gamma}{8\mu\rho^2}$ or $|y-Cx|^2 \le r^4\sqrt{\frac{e^{-1}\gamma}{8\mu\rho^2}}$.

For the second term (4) we have

$$|x-Cx|^2 \le r^4\|\beta\|^2|x_k|^2 \le r^4\beta'^2|x|_k^2 \text{ or } |x-Cx| \le r^2\beta'|x|_k.$$

Finally for the inequality (4), we get

$$|y-x| \le v_1 r^4 + v_2 r^2 \le R(x).$$

Solving the inequality $v_1 r^4 + v_2 r^2 \le R(x)$, we obtain

$$r < \left(\frac{-v_2 + \sqrt{v_2^2 + 4R(x)v_1}}{2v_1}\right)^{\frac{1}{2}} = r_1(x).$$

From $t-\tau > 0$ and $\tau > t - r^2$ it follows that $r \le \sqrt{t}$. So for

$$r = \min\{r_1(x), \sqrt{t}\}. \tag{5}$$

$B_r(x,t)\subset\bar{\Omega}$. The lemma has been proved.

Let $r = r(x,t) > 0$ be such that $B_r(x,t)\in\bar{\Omega}$.

Then, using the formulas of the parabolic average [1], for solving the problem (1) - (3) we get the following representation

$$u(x,t)=\int_0^1 P_1(\rho)\int_{S_1(0)} P_2(H)u(y(e^{-\frac{\rho}{\gamma}}),\tau(e^{\frac{\rho}{\gamma}}))dsd\rho+\bar{f}(x,t), \quad (6)$$

where

$$\bar{f}(x,t)=\int_{B_r(x,t)}\left[Z(x,t;y,\tau)-\pi^{-\frac{n}{2}}\|a\|^{\frac{1}{2}}r^{-\gamma}\right]f(y,\tau)dyd\tau$$

$S_1(0)-(n-1)$- dimensional unit sphere, $H\in S_1(0)$ - unit n - dimensional vector, $P_1(\rho)$ - is the gamma density of a distributed random variable with the parameter $1+\frac{n}{2}$, $P_2(H)=\frac{H^T b\alpha b H^T}{\gamma\sigma_n}$ - is the density of a random vector, $\tau(\lambda)=t-r^2\lambda^2$, $y(\lambda,H)=e^{-r\lambda\beta}x+\left(\gamma\ln(\frac{1}{\lambda})\right)^{\frac{1}{2}}d\left(r^2\lambda^2\right)b^{-1}H$, σ_n surface of the unit sphere.

IV. CONSTRUCTING MARKOV CHAINS

Let be

$$P_2(H)=P_2(H_1,H_2,...,H_n)=\frac{1}{\gamma\sigma_n}(\sum_{ij=1}^{n}q_{ij}H_iH_j)\chi_{S_1(0)}(H)$$

where $\chi_{S_1(0)}(H)$ is the indicator of the set $S_1(0)$, σ_n is the surface of the unit sphere, $q=4b\alpha b^T$. Since $H_1,H_2,...,H_n$ are the coordinates of the unit vector and $H_1^2+H_2^2+...+H_n^2=1$ we get $H^T qH\le\mu_1$, where μ_1 is the largest eigenvalue of the matrix q. Then one can simulate a random vector with distribution density (6) by method of Neumann. We will present an algorithm of modeling.

Algorithm:

1) Simulated $\omega=(\omega_1,\omega_2,...,\omega_n)$ isotropic vector and γ_1 is a uniformly distributed random variable;

2) $E=\mu_1\gamma_1\int_x f(x)dx$;

3) If $(\sum_{ij=1}^{n}q_{ij}\omega_i\omega_j)/\gamma\ge E$, then ω is accepted, otherwise it is repeated paragraph 1).

Let $\{\xi_j\}_{j=1}^{\infty}$ be a sequence of independent gamma distributed random variables with the parameter $\left(1+\frac{n}{2}\right)$, $\{\omega^j\}_{j=1}^{\infty}$ is a sequence of independent random vectors with distribution density $P_2(H)$.

In Ω we define a Markov chain $\{x^j,t^j\}_{j=0}^{\infty}$ using the following recurrence relations:

$$x^0=x, t^0=t, t^j=t^{j-1}-r_{j-1}^2\exp\left(-\frac{2\xi_j}{\gamma}\right),$$

$$x_i^j=x_i^{j-1}-r_{j-1}^2\exp\left(-\frac{\xi_j}{\gamma}\right)^{\xi^{\frac{1}{2}}}\sum_{m=1}^{n}b_{im}\omega_i^j,$$

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$$x_{k+p}^{j}=x_{k+p}^{j-1}-r_{j-1}^{2}\exp(\frac{-2\xi}{\gamma})\sum_{c=1}^{k}\beta_{pc}x_{c}^{j}+r_{j-1}^{3}\exp(\frac{-3\xi}{\gamma})\xi^{\frac{1}{2}}\sum_{\nu=1}^{n}b_{k+p},\nu^{\omega_{i}^{j}}$$

We define a sequence of random variables $\{\eta_l\}_{l=0}^{\infty}$ by the following equality

$$\eta_l=\sum_{j=1}^{l-1}h(x^j,t^j)f(y^j,\tau^j)+u(x^l,\tau^l) \quad (7)$$

where (y^j,τ^j) is a random point of the $B_r(x,t)$ which has distribution density for the fixed (x^j,t^j).

Let $\{\Im_l\}_{l=0}$ be a sequence of σ -algebras generated by random variables $\xi_1,\xi_2,...,\xi_l$, sequence of vectors $\omega^0,\omega^1,...,\omega^l$ and random points $(y^0,\tau^0),(y^1,\tau^1),...,(y^{l-1},\tau^{l-1})$, $u_{f,\psi,\varphi}(x,t)$ - solution of problem (3)-(5), corresponding to the given f,ψ,φ,

$$\frac{[Z(x^j,t^j;y,\tau)-\| a\|^{\frac{1}{2}}\pi^{-\frac{n}{2}}r^{-\gamma}]}{h(x^j,t^j)},$$

$$h(x,t)=\iint_{B_r(x,t)}[Z(x,t;y,\tau)-\| a\|^{\frac{1}{2}}\pi^{-\frac{n}{2}}r^{-\gamma}]dyd\tau$$

Theorem 1. a). Sequence $\{\eta_l\}_{l=0}^{\infty}$ forms a martingale with respect to sequences of σ -algebra $\{\Im_l\}_{l=0}^{\infty}$.

b) If $u_{f^2,0,0}(x,t)<+\infty$ and $u_{|f|,0,0}(x,t)<+\infty$, then η_l will be quadratic integrable.

Proof. Firstly, we prove that $\{\eta_l\}_{l=0}^{\infty}$ forms a martingale. From the definition of $\Im_l$ it is obvious that η_l is $\Im_l$ measurable.

$$E_{(x,t)}(\frac{\eta_{l+1}}{\Im_l})=E_{(x,t)}\left((\sum_{j=0}^{l}h(x^j,t^j)f(t^j,\tau^j)+u(x^{l+1},t^{l+1}))\frac{1}{\Im_l}\right)=$$

$$=E_{(x,t)}\left((\sum_{j=0}^{l-1}h(x^j,t^j)f(t^j,\tau^j)+h(x^l,t^l)f(t^l,\tau^l)+u(x^{l+1},t^{l+1}))\frac{1}{\Im_l}\right)=$$

$$=E_{(x,t)}\left(\sum_{j=0}^{l-1}h(x^j,t^j)f(t^j,\tau^j)\frac{1}{\Im_l}\right)+E_{(x,t)}\left(h(x^l,t^l)f(t^l,\tau^l)\frac{1}{\Im_l}\right)+E_{(x,t)}(u(x^{l+1},t^{l+1})\frac{1}{\Im_l}).$$

Since $\sum_{j=0}^{l-1}h(x^j,t^j)f(t^j,\tau^j)$ is $\Im_l$ is measurable, then from the properties of the conditional mathematical expectation it follows that

$$E_{(x,t)}\left((\sum_{j=0}^{l-1}h(x^j,t^j)f(y^j,\tau^j))\frac{1}{\Im_l}\right)=\sum_{j=0}^{l-1}h(x^j,t^j)f(y^j,\tau^j)$$

$h(x^l,t^l)$ - $\Im_l$ is measured and $f(x^l,t^l)$ does not depend on $\Im_l$ and

$$E_{(x,t)}\left(h(x^l,t^l)f(y^l,\tau^l)\frac{1}{\Im_l}\right)=h(x^l,t^l)E_{(x,t)}f(y^l,\tau^l),$$

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$u(x^{l+1},t^{l+1})$ does not depend on $\Im_l$, it means $E_{(x,t)}(u(x^{l+1},t^{l+1})\frac{1}{\Im_l})=E_{(x,t)}u(x^{l+1},t^{l+1})$ and finally we get

$$E_{(x,t)}(\frac{\eta_{l+1}}{\Im_l})=\sum_{j=0}^{l-1}h(x^j,t^j)f(y^j,\tau^j)+h(x^l,t^l)E_{(x,t)}f(y^l,\tau^l)++E_{(x,t)}u(x^{l+1},t^{l+1}).$$

Using the formulas (6) and (7) we get

$$E_{(x,t)}(\frac{\eta_{l+1}}{\Im_l})=\sum_{j=0}^{l-1}h(x^j,t^j)f(y^j,\tau^j)+u(x^l,t^l)=\eta_l.$$

It follows that $\{\eta_l\}_{l=0}^{\infty}$ forms a martingale with respect to $\{\Im_l\}_l^{\infty}$.

We will prove that $E_{(x,t)}\eta_l^2\le+\infty.$

To do this it is enough to show that

$$I=E_{(x,t)}\left(\sum_{j=0}^{l-1}h(x^j,t^j)f(y^j,\tau^j)\right)^2<+\infty.$$

Separating I into two terms I_1,I_2, we will show the finiteness of I:

$$I=E_{(x,t)}\left(\sum_{j=0}^{l-1}h^2(x^j,t^j)f^2(y^j,\tau^j)\right)+2E_{(x,t)}\sum_{j=0}^{l-2}\sum_{k=j+1}^{l-1}h(x^j,t^j)h(x^k,t^k)f(y^j,\tau^j)f(y^k,\tau^k)$$

From (7) and the condition *r(x, t)* $\le t$ we obtain that $h(x,t)\le(\frac{\gamma}{\gamma+2})^{1+\frac{n}{2}}t$

$$I_1=E_{(x,t)}\sum_{j=0}^{l-1}h^2(x^j,t^j)f^2(y^j,\tau^j)\le(\frac{\gamma}{\gamma+2})^{1+\frac{n}{2}}tE_{(x,t)}\sum_{j=0}^{l-1}h(x^j,t^j)f(y^j,\tau^j)\le$$

$$\le(\frac{\gamma}{\gamma+2})^{1+\frac{n}{2}}tu_{f^2,0,0}(x,t)<+\infty.$$

$$I_2=2E_{(x,t)}\sum_{j=0}^{l-2}\sum_{k=j+1}^{l-1}h(x^j,t^j)h(x^k,t^k)f(y^j,\tau^j)f(y^k,\tau^k)\le2(\max_{x\in\bar{D},t>\tau}u_{|f|,0,0}(x,t))u_{|f|,0,0}(x,t)<+\infty.$$

The theorem has been proved.

Lemma 2. For the function $\bar{f}(x,t)$ the following relation is valid

$$\bar{f}(x,t)-\left(\frac{\gamma}{\gamma+2}\right)^{1+\frac{n}{2}}r^2\mathrm{E}f\left(y(\xi,\zeta,\omega),\tau(\xi,\zeta)\right),$$

where

$$y(\xi,\zeta,\omega)=e^{-r^2}\exp\left(-\frac{2\xi}{\gamma+2}\right)\zeta^{\frac{2}{\gamma}}\beta_x+\left(\frac{\gamma}{\gamma+2}\xi\right)^{\frac{1}{2}}d\left(r^2\exp\left(-\frac{2\xi}{\gamma+2}\right)\right)\zeta^{\frac{2}{\gamma}}b^{-1}\omega, \qquad (8)$$

$$\tau(\xi,\zeta)=t-\exp\left(-\frac{2\xi}{\gamma+2}\right)\zeta^{\frac{2}{\gamma}}.$$

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Here ξ is gamma distributed random variable with the parameter $\frac{n}{2}$, ζ is beta distributed random variable with the parameter $(\frac{2}{\gamma},2)$ and ω is random unit vector.

Proof. We introduce a domain

$$B_r=\{(y,t): y^T d(\frac{1}{\tau})ad(\frac{1}{\tau})y<\frac{\gamma}{2}\ln\frac{r^2}{\tau},\tau>0\}$$

the resulting mirror image of the $B_r(0,0)$ sharoid with respect to the plane $\tau=0$. These regions will also be called sharoids(with radius r). Then we have

$$f(x,t)=\frac{\|a\|^{\frac{1}{2}}}{\pi^{\frac{n}{2}}r^{\gamma}}\int_{B_r}[r^{\gamma}\tau^{-\frac{\gamma}{2}}\exp(-y^T d(\frac{1}{\tau})ad(\frac{1}{\tau})y)-1)f(e^{-\tau\beta}x+y,t-\tau)dyd\tau .$$

In this integral we will change integration variables (y,τ) by (ρ,λ,θ) using the formula

$$y=(\gamma\ln(\frac{1}{\lambda}))^{\frac{1}{2}}d(\tau)d^{-1}H(\theta),\tau=\lambda^2\rho^2.$$

By simple computations, it can be shown that

$$dyd\tau\|a\|^{-\frac{1}{2}}\gamma^{\frac{n}{2}}\lambda^{\gamma+1}(\ln(\frac{1}{\lambda}))^{\frac{n}{2}-1}\rho^{\gamma+1}d\rho d\lambda ds$$

$$r^{\gamma}\tau^{-\frac{\gamma}{2}}\exp(-y^T d(\frac{1}{\tau})ad(\frac{1}{\tau})y)=r^{\gamma}\rho^{-\gamma}$$

The domain of integration is transformed into the cylinder $(0\le\rho\le r)\times(0\le\lambda\le 1)\times S_1(0)$. From here we get

$$\overline{f}(x,t)=\frac{\gamma^{\frac{n}{2}}}{\pi^{\frac{n}{2}}r^{\gamma}}\int_0^r\rho(r^{\gamma}-\rho^{\gamma})d\rho\int_0^1\lambda^{\gamma+1}(\ln\frac{1}{\lambda})^{\frac{n}{2}-1}d\lambda\times$$

$$\times\iint_{S_1(0)}f(e^{-\lambda^2\rho^2\beta}x+(\gamma\ln\frac{1}{\lambda})^{\frac{1}{2}}d(\rho^2\lambda^2)b^{-1}H(\theta),t-\lambda^2\rho^2)ds$$

using changing variables we get:

$$\overline{f}(x,t)=\frac{r^2}{\gamma\pi^{\frac{n}{2}}}(\frac{\gamma}{\gamma+2})^{\frac{n}{2}}\int_0^1 v^{\frac{2}{\gamma}-1}(1-v)dv\int_0^{\infty}e^{-z}z^{\frac{n}{2}-1}dz\int_{S(0)}f(y(z,v,H(\theta)),\tau(z,v))ds=$$

$$=r^2(\frac{\gamma}{\gamma+2})^{1+\frac{n}{2}}\int_0^1 P_1(v)dv\int_0^{\infty}P_2(z)dz\int_{S_1(0)}P_3(H)f(y(z,v,H(\theta)),\tau(z,v))ds=$$

$$=r^2(\frac{\gamma}{\gamma+2})^{1+\frac{n}{2}}Ef(y(\xi,\zeta,\omega),\tau(\xi,\zeta)).$$

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Where ξ is a random variable with distribution density $P_2(z)=\frac{e^{-z}z^{\frac{n}{2}-1}}{\Gamma(\frac{n}{2})}$ (gamma - distributions),

ζ is a random variable with a distribution density $P_1(v)=\frac{v^{\frac{2}{\gamma}-1}(1-v)}{B(\frac{2}{\gamma},2)}$, ω - random vector with distribution density

$P_3(H(\theta))=\frac{1}{\sigma_n}$ $(\sigma_n=\frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$ - is the unit surface spheres). The lemma is proved.

Let $N_\varepsilon=\inf\left\{l:(x^l,t^l)\in(\partial\Omega)_\varepsilon\right\}$- the moment of the first hit of the process (x^l,t^l). At $\left(\partial\Omega\right)_\varepsilon$,i.e. N_ε the moment of stopping the process (Markov moment).

Lemma 3. The following inequality holds: $E_{(x,t)}N_\omega\le\left(\frac{\gamma+2}{\gamma}\right)^{1+\frac{n}{2}}\frac{t}{\theta(\varepsilon)}$.

Proof. Taking $u(x,t)=t$ and applying the formulas of relations (6) and (7) we get

$$t=u_{1,0,0}(x,t)\ge E_{(x,t)}\sum_{j=1}^{N_\varepsilon-1}h(x^j,t^j)=(\frac{\gamma}{\gamma+2})E_{(x,t)}\sum_{j=1}^{N_\varepsilon-1}r^2(x^j,t^j).$$

From the definition of $r(x,t)$ it follows that

$$r^2(x^j,t^j)=\min\{r_1^2(x^j),t^j\}\ge\min\{\frac{-v_2+(v_2^2+4v_1R(x_j))^{\frac{1}{2}}}{2v_1}\}\ge\min\{\frac{-v_2+(v_2^2+4v_1\varepsilon)^{\frac{1}{2}}}{2v_1},\varepsilon\}=\theta(\varepsilon).$$

Therefore $E_{(x,t)}N_\varepsilon\le\left(\frac{\gamma+2}{\gamma}\right)^{1+\frac{n}{2}}\frac{t}{\theta(\varepsilon)}$. The lemma has been completed.

Theorem 2. Let the conditions of Theorem 1 are fulfilled. Then η_{N_ω} will be an unbiased estimate for $u\left(x,t\right)$. The variance is finite.

Proof. From the Theorem 1 it follows that η_l is square integrable and hence η_l is uniformly integrable and $N_\varepsilon<+\infty$, and the moment stopping the process is Markov moments. Therefore, according to Doob's theorem "On free choice transformations" [5] and the formula relations

$$u(x^{j-1},t^{j-1})=E_{x^{j-1},t^{j-1}}u(x^j,t^j)+\overline{f}(x^{j-1},t^{j-1})$$

and $D\eta_{N_\varepsilon}=E_{\eta_1}=u(x,t)$ i.e η_{N_ε} is an unbiased estimate for $u(x,t)$. From the definition of random variables η_{N_ε} and η_∞ you can see that $D\eta_{N_\varepsilon}\le D\eta_\infty$.

$$\eta_\infty^2=(\sum_{i=0}^{\infty}h(x^i,t^i)f(y^i,\tau^i)+u(x^\infty,t^\infty))^2=(\sum_{i=0}^{\infty}h(x^i,t^i)f(y^i,\tau^i))^2+$$

$$+2u(x^\infty,t^\infty)\sum_{i=0}^{\infty}h(x^i,t^i)f(y^i,\tau^i)+u^2(x^\infty,t^\infty)=\sum_{i=0}^{\infty}h^2(x^i,t^i)f^2(y^i,\tau^i)+$$

$$+2\sum_{i=0}^{\infty}h(x^i,t^i)f(y^i,\tau^i)\sum_{j=i+1}^{\infty}h(x^j,t^j)f(y^j,\tau^j)\ +2u(x^\infty,t^\infty)\sum_{i=0}^{\infty}h(x^i,t^i)f(y^i,\tau^i)++u^2(x^\infty,t^\infty).$$

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$$E_{(x,t)}\eta_\infty^2 \le (\frac{\gamma+2}{\gamma})^{1+\frac{n}{2}} tu_{f^2,0,0}(x,t) + 2(\max_{x\in\bar{D},\tau<t} u_{|f|,0,0}(x,\tau)u_{|f|,0,0}(x,t) +$$
$$+2u_{0,\varphi,\psi}(x,t)u_{|f|,0,0}(x,t) + u_{0,\varphi^2,\psi^2}(x,t).$$

$D\eta_\infty = E\eta_\infty^2 - (E\eta_\infty)^2 = E\eta_\infty^2 - u^2(x,t) < +\infty$ i.e. $D\eta_{N_\varepsilon} < +\infty$.

From η_{N_ω}, the biased but actually realizable estimate is constructed in the standard way $\eta^*_{N_\omega}$.
Let $\psi_1(x,t) = \psi(x,l)$ partial $\partial D\times[0,T]$, $\psi_1(x,0) = \varphi(x)$, $x\in\bar{D}$ and (x^*,t^*) is the closest to (x,t) boundary points of $\partial\Omega$. In the assessment

$$\eta_{N_\varepsilon} = \sum_{j=0}^{N_\varepsilon-1} h(x^j,t^j) f(x^j,\tau^j) + u(x^{N_\varepsilon}, t^{N_\varepsilon}),$$

replace $u(x^{N_\varepsilon}, t^{N_\varepsilon})$ with $\psi_1(x^{*N_\varepsilon}, t^{*N_\varepsilon})$ and we get

$$\eta^*_{N_\varepsilon} = \sum_{j=0}^{N_\varepsilon-1} h(x^j,t^j) f(y^j,\tau^j) + \psi_1(x^{*N_\varepsilon}, t^{*N_\varepsilon}).$$

Theorem 3. Let $u(x,y)$ satisfies the condition Lipschitz and $A(\varepsilon)$ is modulus of continuity of $u(x,t)$. Then the random variable $\eta^*_{N_\varepsilon}$ is a biased estimate for $u(x,t)$. $D\eta^*_{N_\varepsilon}$ bounded function of parameter ε.

Proof. Since $E(x,t)\eta_{N_e} = u(x,t)$,

$$\left|u(x,t) - E_{(x,t)}\eta^*_{N_\varepsilon}\right| = \left|E_{(x,t)}\eta_{N_e} - E_{(x,t)}\eta^*_{N_\varepsilon}\right| = \left|E_{(x,t)}u(x^{N_\varepsilon}, t^{N_\varepsilon}) - E_{(x,t)}\psi_1(x^{*N_\varepsilon}, t^{*N_\varepsilon})\right| \le$$
$$\le E_{(x,t)}\left|u(x^{N_\varepsilon}, t^{N_\varepsilon}) - u(x^{*N_\varepsilon}, t^{*N_\varepsilon})\right| = A(\varepsilon),$$

that is, $\eta^*_{N_\varepsilon}$ is mixed estimate (mixing of which does not exceed $A(\varepsilon)$).

$$D\eta^*_{N_\varepsilon} = E_{(x,t)}(\eta^*_{N_\varepsilon} - E_{(x,t)}\eta^*_{N_\varepsilon})^2 = E_{(x,t)}(\eta^*_{N_\varepsilon} - \eta_{N_e} + \eta_{N_e} - u(x,t) + u(x,t) - E_{(x,t)}\eta^*_{N_\varepsilon})^2 \le$$
$$\le 4E_{(x,t)}(\eta_{N_e} - u(x,t))^2 + 4E_{(x,t)}(\eta^*_{N_\varepsilon} - \eta_{N_e})^2 + 4E_{(x,t)}(u(x,t) - E_{(x,t)}\eta^*_{N_\varepsilon})^2 \le$$
$$\le 4E_{(x,t)}(\psi_1(x^{*N_\varepsilon}, t^{*N_\varepsilon}) - u(x^{N_\varepsilon}, t^{N_\varepsilon}))^2 + 4A^2(\varepsilon) \le 8A^2(\varepsilon) + 4D\eta_{N_e} < +\infty.$$

The theorem is proved.

V. CONCLUSION

In the present work, an unbiased and ε biased estimate for initial and boundary-value problems for the generalized non-isotropic diffusion equation with using the theory of martingales and Markov moments.

REFERENCES

[1] Kolmogoroff A.N., “Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung”, Mathematische Annalen, 1931, December, Volume 104, Issue 1, pp 415-458.
[2] Yermakov S.M., Netrutkin V.V., Sipin A.A., “Random processes for solving classical equations of mathematical physics”, 1984, Moscow: Science, 206 p.
[3] Kuptsov L. P., “On the property of the mean for a generalized equation A. N. Kolmogorova I”, Differential equations, 1983, Vol. XIX, N'2. with. 285-304.
[4] Sobol I.M., “Monte-Carlo Numerical Methods”, 1973, Moscow: Main editors of physical and mathematical literature publishing house, "Science".
[5] Doob J.L., “Classical Potential Theory and its”, 1984, Probabilistic Counterpart. Springer - Varlag. 846 p.

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Vol. 5, Issue 12, December 2018

[6] Khadzhi-Sheikh A., Sparrow EM., "Problem Solving thermal conductivity probabilistic methods", 1962, American works societies of mechanical engineers, v.2, p. 1-18.