



Assessment of Air Quality in Coimbatore City using TOPSIS Method

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ABSTRACT: The main objective of this paper is to analyse air pollution level of different areas in various climatic conditions (summer, winter, monsoon). The quality of air was determined based on National Ambient Air Quality Standards (NQAAS). This case study shows that the multi criteria decision making method is applied for assessment of air pollution in Coimbatore city.

KEYWORDS: Multi criteria decision making, Entropy, Air pollution, Topsis.

I. INTRODUCTION

Decision making is the method of finding the best choice from all feasible alternatives. Much of the decision making in the real world takes place in an environment in which constraints and the consequences of possible actions are not known accurately and hence fuzzy set theory can be used to deal with imprecision in decision making. In classical multi criteria decision making (MCDM) methods the ratings and the weights of the criteria are known accurately. TOPSIS was one of the classical methods, first developed by Hwang and Yoon (1981) [1] for solving a MCDM problem wherein the process of TOPSIS the performance ratings and the weights of the criteria were given as crisp values. TOPSIS method is called as ideal solution. It is an efficient multiple attribute decision making method. This method is to construct the ideal solutions and minus ideal solutions to the problems of multiple attributes and uses the two standards of being close to the ideal solutions and being far away from the minus ideal solutions as the criteria of evaluating the feasible projects. The rule of ranking projects is to associate each alternative project with x^+ and x^- . If one of the projects is close to x^+ and far away from x^- at the same time, then the project is the best project of the alternative projects[6]. TOPSIS method gives the best alternatives by maximizing the distance to negative-ideal solution and minimizing the distances to the ideal solution [2]. TOPSIS method to evaluate the performance of manufacturing enterprises by Yurdakul & le (2005)[7]. Nan Li, Ying Wang [3] used TOPSIS method with entropy weight to evaluate the development level of Chinese regional service industry. Topsis was used for financial investment in advanced manufacturing system [9]. It was also used for analysing company performances [10]. SahayaSudha A and Rachel InbaJeba J adopted TOPSIS for selection of planting of crops by rotation[4].

II. TOPSIS METHOD FOR DETERMINING AIR QUALITY

In this method the first weight is determined using Entropy method and then TOPSIS method is applied to determine the air quality.

The following are the steps of Entropy method for Weight Determination and TOPSIS method:

Step 1: In matrix M, P_{ij} is of the i^{th} alternatives to the j^{th} factor:

$$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}, (1 \leq i \leq m, 1 \leq j \leq n) \text{---- (1)}$$

Step 2: Determine the entropy value e_j of the j^{th} factor which becomes

$$e_j = -k \sum_{i=1}^m P_{ij} \ln P_{ij}, (k = \frac{1}{\ln m}, 1 \leq j \leq n) \text{---- (2)}$$

Step 3: Variation coefficient of the j^{th} factor d_j can be defined by the following equation:

$$d_j = 1 - e_j, (1 \leq j \leq n) \text{---- (3)}$$

Step 4: Determine the weight of the entropy w_j of index j :

$$w_j = \frac{1-e_j}{n-\sum_{j=1}^n e_j} \text{---- (4)}$$

Step 5: Construct the normalized weighted decision matrix $V = (v_{ij})_{m \times n}$:

$$v_{ij} = w_j \times p_{ij} \text{----- (5)}$$

where w_j is the weight of the i^{th} criterion and $\sum_{j=1}^n w_j = 1$

Step 6: Calculate the positive ideal and negative ideal solutions

$$V^+ = \{v_1^+, v_2^+, \dots, v_n^+\}, \text{ where } v^+ = \{\max(v_{ij}) \text{ if } j \in J; \min(v_{ij}) \text{ if } j \in J\}$$

$$V^- = \{v_1^-, v_2^-, \dots, v_n^-\}, \text{ where } v^- = \{\min(v_{ij}) \text{ if } j \in J; \max(v_{ij}) \text{ if } j \in J\} \text{----- (6)}$$

Step 7: Determine the separation measures, using the m-dimensional Euclidean distance

$$S^+ = \sqrt{\sum_{j=1}^n (V_{ij} - V^+)^2}, \text{ where } (1 \leq i \leq m, 1 \leq j \leq n) \text{----- (7)}$$

$$S^- = \sqrt{\sum_{j=1}^n (V_{ij} - V^-)^2}, \text{ where } (1 \leq i \leq m, 1 \leq j \leq n) \text{----- (8)}$$

Step 8: Evaluate the relative closeness to ideal solution

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}, i = 1, 2, \dots, m \text{----- (9)}$$

In this step, the option with C_i^+ closer to 1 is chosen.

Step 9: Arranging the evaluation objects according to C_i^+ .

According to formula (9), the value of C_i^+ should lies between 0 and 1. The finer value means the evaluation objects is closer to the optimal level.

III. NUMERICAL EXAMPLE

Air is a mixture of gases in Earth's atmosphere. According to the [World Health Organization](#) (WHO), air pollution is one of the biggest killers in the world, it causes around three million people to die prematurely each year[5]. In this work, the air pollutants concentration like $SO_2, NO_2, PM_{10}, PM_{2.5}$ were measured in selected four locations in Coimbatore city.

A. Status of Air Pollution in Coimbatore City:

An attempt has been made to evaluate the concentrations of ambient air pollutants in different type of zones for one year during January to December 2015 from four different areas for different seasons such as summer, monsoon and winter.

Table 1: Name of the ambient air measuring locations

Location	Zone Type	Direction
Kurichi	Industrial	South
RS Puram	Commercial	West
Peelamedu	Residential	East
Ganapathi	Residential	North

The average ambient air quality data for different parameters in three different seasons is given below:

Table 2: Specification of 4 different pollutants in Monsoon

Alternatives	Criteria			
	SO_2 (C1)	NO_2 (C2)	PM_{10} (C3)	$PM_{2.5}$ (C4)
Kurichi	10.5	12.1	90.2	62.2
RS Puram	18.1	11.9	97	62.3
Peelamedu	9.6	11.6	81.3	57.5
Ganapathi	7.6	10.5	69	51.1

Table 3: Specification of 4 different pollutants in Summer

Alternatives	Criteria			
	<i>SO₂</i> (C1)	<i>NO₂</i> (C2)	<i>PM₁₀</i> (C3)	<i>PM_{2.5}</i> (C4)
Kurichi	24.3	15.2	112.2	82.3
RS Puram	20.4	14.2	97.2	78.6
Peelamedu	19.2	12.3	90.9	67.2
Ganapathi	12.9	10.5	75.3	63.6

Table 4: Specification of 4 different pollutants in Winter

Alternatives	Criteria			
	<i>SO₂</i> (C1)	<i>NO₂</i> (C2)	<i>PM₁₀</i> (C3)	<i>PM_{2.5}</i> (C4)
Kurichi	19.2	18.2	102.8	68.3
RS Puram	19.2	18.2	90.8	68.3
Peelamedu	15.6	10.4	84.3	60.9
Ganapathi	13.5	15.5	71.3	58.2

The below figures represents the average ambient air quality data in $\mu\text{g}/\text{m}^3$ of four different zones from three different seasons:

Figure 1

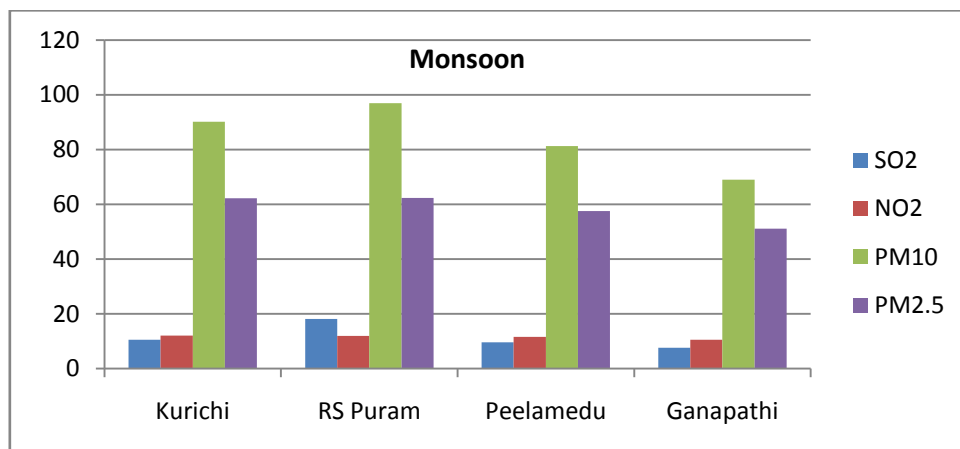


Figure 2

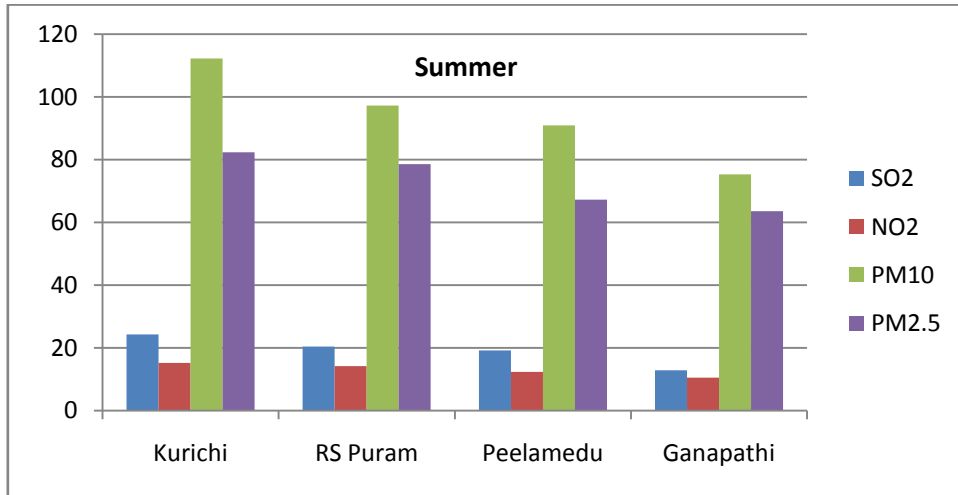
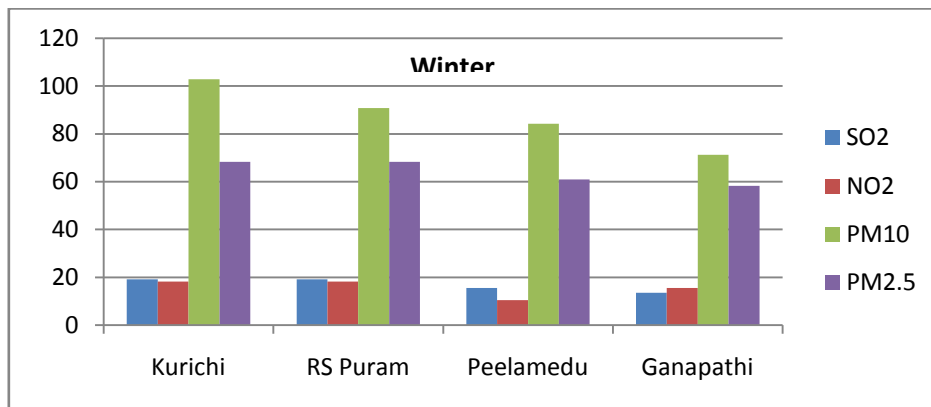


Figure 3



Using Topsis method with Entropy weight, we assessed air quality of Monsoon season.

Step: 1

For the weight using entropy analysis, the procedure is as follows, the decision matrix is shown in Table 2.2.

$$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad (1 \leq i \leq m, 1 \leq j \leq n) ; \sum_{i=1}^4 x_{i1} = 45.8$$

$$P_{11} = \frac{10.5}{45.8} = 0.2293,$$

Similarly, $P_{21} = 0.3952, P_{31} = 0.2096, P_{41} = 0.1659$ etc

Table 5: Entropy Normalization Matrix

Alternatives	Criteria			
	SO ₂	NO ₂	PM ₁₀	PM _{2.5}
Kurichi	0.2293	0.2625	0.2673	0.2668
RS Puram	0.3952	0.2581	0.2874	0.2673

Peelamedu	0.2096	0.2516	0.2409	0.2467
Ganapathi	0.1659	0.2278	0.2044	0.2192

Step: 2

To find the value of $P_{ij} \ln(P_{ij})$

$$P_{11} \ln(P_{11}) = 0.2293 \ln(0.2293) = -0.3377$$

Similarly, $P_{21} \ln(P_{21}) = -0.3669$, $P_{31} \ln(P_{31}) = -0.3275$, $P_{41} \ln(P_{41}) = -0.2980$

Table 6: Matrix for weight calculations

Alternatives	Criteria			
	SO_2	NO_2	PM_{10}	$PM_{2.5}$
Kurichi	-0.3377	-0.3511	-0.3527	-0.3525
RS Puram	-0.3669	-0.3496	-0.3584	-0.3527
Peelamedu	-0.3275	-0.3472	-0.3429	-0.3453
Ganapathi	-0.2980	-0.3370	-0.3245	-0.3327

Step: 3

$$e_j = -k \sum_{i=1}^m P_{ij} \ln P_{ij}, \left(k = \frac{1}{\ln m}, 1 \leq j \leq n\right) k = \frac{1}{\ln 4} = 0.7213$$

$$e_1 = -0.7213(-1.3301) = 0.9594$$

Similarly, $e_2 = -0.7213(-1.3849) = 0.9989$, $e_3 = 0.9943$, $e_4 = 0.9977$

$$d_1 = 1 - e_1 = 1 - 0.9594 = 0.0406$$

Similarly, $d_2 = 1 - e_2 = 1 - 0.9989 = 0.0011$, $d_3 = 0.0057$, $d_4 = 0.0023$

$$w_j = \frac{1 - e_j}{n - \sum_{j=1}^n e_j}$$

$$n - \sum_{j=1}^4 e_j = 4 - [0.9594 + 0.9989 + 0.9943 + 0.9977] = 0.0497$$

$$w_1 = \frac{d_1}{0.0497} = 0.8169$$

Similarly, $w_2 = \frac{0.0011}{0.0497} = 0.0221$, $w_3 = 0.1147$, $w_4 = 0.0463$

Table 7: Entropy Weight calculations

Entropy Weight	SO_2	NO_2	PM_{10}	$PM_{2.5}$
w_j	0.8169	0.0221	0.1147	0.0463

Now applying different Multi Criteria Decision Making Methods for obtaining ranking solution of the product using Entropy Normalization Matrix.

Step: 4

Elements of matrix of V gain their values from multiplying each column of the entropy normalised decision matrix by the associated entropy weight using $V = (p_{ij} * w_j)$.

$$v_{11} = w_1 * p_{11} = 0.8169 * 0.2293 = 0.1873,$$

Similarly, $v_{21} = w_1 * p_{21} = 0.8169 * 0.3952 = 0.3228, v_{31} = 0.1712, v_{41} = 0.1355$ etc.

Table 8: Weighted Calculation

Alternatives	Criteria			
	SO ₂	NO ₂	PM ₁₀	PM _{2.5}
Kurichi	0.1873	0.0058	0.0307	0.0124
RS Puram	0.3228	0.0057	0.0330	0.0124
Peelamedu	0.1712	0.0056	0.0276	0.0114
Ganapathi	0.1355	0.0050	0.0234	0.0101

Step: 5

The maximum and minimum values in Table 2.6 represent the positive and negative ideal solutions for each decision making as shown below.

$$V^+ = \{v_1^+, v_2^+, \dots, v_n^+\} \text{ where } v^+ = \{\max(v_{ij}) \text{ if } j \in J; \min(v_{ij}) \text{ if } j \in J\}$$

$$V^- = \{v_1^-, v_2^-, \dots, v_n^-\} \text{ where } v^- = \{\min(v_{ij}) \text{ if } j \in J; \max(v_{ij}) \text{ if } j \in J\}$$

$$V^+ = \{0.3228, 0.0058, 0.0330, 0.0124\}$$

$$V^- = \{0.1355, 0.0050, 0.0234, 0.0101\}$$

Step: 6

Table 2.7 presents the separation measure of positive and negative ideal solution for each alternative, which are computed by the using of step 7 & 8.

$$S^+ = \sqrt{\sum_{j=1}^n (V_{ij} - V^+)^2}$$

$$(v_1^+ - v_{11})^2 = (0.3228 - 0.1873)^2 = 0.0184$$

$$(v_2^+ - v_{12})^2 = (0.0058 - 0.0058)^2 = 0$$

$$(v_3^+ - v_{13})^2 = (0.0330 - 0.0307)^2 = 0.000005$$

$$(v_4^+ - v_{14})^2 = (0.0124 - 0.0124)^2 = 0$$

$$S^- = \sqrt{\sum_{j=1}^n (V_{ij} - V^-)^2}$$

$$(v_{11} - v_1^-)^2 = (0.1873 - 0.1355)^2 = 0.0027$$

$$(v_{12} - v_2^-)^2 = (0.0058 - 0.0050)^2 = 0.0000006$$

$$(v_{13} - v_3^-)^2 = (0.0307 - 0.0234)^2 = 0.0001$$

$$(v_{14} - v_4^-)^2 = (0.0124 - 0.0101)^2 = 0.000005$$

$$S_1^+ = \sqrt{0.0184 + 0 + 0.000005 + 0} = 0.1357$$

$$S_1^- = \sqrt{0.0027 + 0.0000006 + 0.0001 + 0.000005} = 0.0530$$

Similarly, $S_2^+ = 0.0001, S_2^- = 0.1876; S_3^+ = 0.1518, S_3^- = 0.0364; S_4^+ = 0.1876, S_4^- = 0$

Table 9: Rank of alternatives

Location	S_i^+	S_i^-	C_i	Ranking
Kurichi	0.1357	0.0530	0.7191	3
RS Puram	0.0001	0.1876	0.0005	4
Peelamedu	0.1518	0.0364	0.8066	2
Ganapathi	0.1876	0	1	1

$$C_1^+ = \frac{S_1^+}{S_1^+ + S_1^-} = \frac{0.1357}{0.1357 + 0.0530} = 0.7191,$$

Similarly, $C_2^+ = 0.0005$, $C_3^+ = 0.8066$, $C_4^+ = 1$

As shown in Table 2.7, the final ranking is based on the highest obtained value of C_i^+ .

The ranking of the alternatives in order are $A_4 > A_3 > A_1 > A_2$

Therefore, Ganapathi (Residential) is low polluted during monsoon season based on the data of Jan 2015- Dec 2015.

Similarly, we get the result for Summer and Winter season, that Ganapathi (Residential) and Peelamedu (Residential) area is low polluted during summer and winter season based on the data of Jan 2015- Dec 2015.

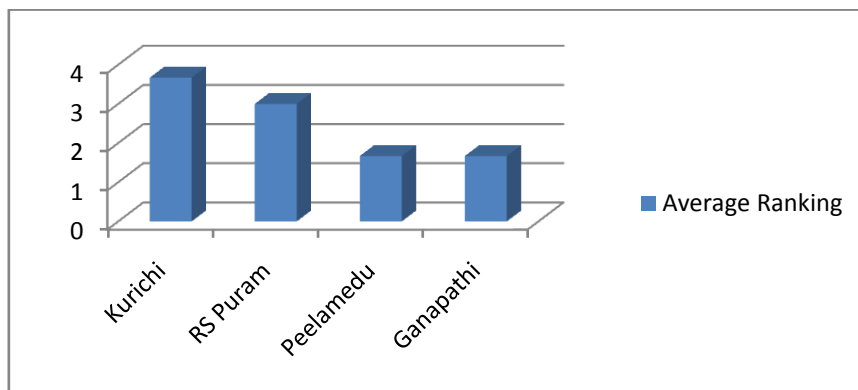
IV CONCLUSION:

From all the three seasons, we can conclude by the average of ranking.

Table 10: The average of ranking

Location	Seasons			Average Ranking
	Monsoon	Summer	Winter	
Kurichi	3	4	4	3.67
RS Puram	4	3	2	3
Peelamedu	2	2	1	1.67
Ganapathi	1	1	3	1.67

Figure 4





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From this analysis, Ganapathi (Residential) is low polluted during summer and monsoon seasons, Peelamedu (Residential) is low polluted during winter season. By the average ranking table, we can conclude that the Kurichi (Industrial area) is highly polluted. Hence, the result reveals that, the air outdoor is often more polluted than inside of many buildings and homes.

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