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Soft Topology divisor of Zero in Soft Banach Algebra

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ABSTRACT: In this paper the ideas of soft topology of zero, soft spectrum, soft boundary of soft spectrum and soft isomorphic of soft element over soft banach algebras (in short SBA) are discussed. Some basic properties of these ideas in SBA are studied. Finally some new results and theorems about them over SBA are investigated.

KEYWORDS: soft Banach algebras, soft element, soft singular, soft boundary, soft isomorphic.

I.INTRODUCTION

Soft sets was introduced by Molodtsove 1999 [1] as a general mathematical tool for dealing with uncertain objects. Operations on soft set such as soft union , soft intersection , soft equality was introduced by Maji , Biswas and Roy 2003 [2] . Shabir and Naz 2011[3] introduce and study the concept of soft topological spaces over a soft set and introduce some related concepts such as soft subspace, soft closure set and their properties. Cagman [4] introduce the soft closure point (set), soft boundary point(set). The concept of soft element was studied by Das, Samanta [5].Wardowski [6]introduce a new notion of soft element and establish its natural relation with soft sets in soft topological spaces, soft relation and soft map in anew manner. Thakur and Samanta [7] introduced the notion of soft banach algebras and studied some of its preliminary properties.

II.PRELIMINARIES

Definition (2.1) [1]: Let X be a universe set and E be a set of parameters, $P(X)$ the power set of X and $A \subseteq E$. A pair (F, A) is called soft set over X with respect to A and F is a mapping given by $F: A \to P(X)$, $(F, A) = \{F(e) \in P(X): e \in A\}$.

Definition (2.2) [2]: Let (F, A) and (G, B) be two soft sets over X, we say that $(F, A)$is a soft subset of $(G, B)$and denoted by $(F, A) \tilde{\subseteq} (G, B)$, if:

- $A \subseteq B$.
- $F(e) \subseteq G(e), \forall e \in A$.

Also, we say that (F, A) and (G, B) are soft equal is denoted by $(F, A) = (G, B)$, if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$.It is clear that:

$(\boldsymbol{i})\widetilde{\emptyset}_A$is a soft subset of any soft set (F, A).

$(\boldsymbol{ii})$ Any soft set $(F, A)$is a soft subset of $\tilde{X}_A$.

Definition (2.3) [2]:

$(\boldsymbol{i})$ The intersection of two soft sets (F, A) and(G, B) over a universe X is the soft set$(H, C) = (F, A)\widetilde{\cap}(G, B)$, where $C = A\cap B$ and for all $e \in C$, write $(H, C) = (F, A)\widetilde{\cap}(G, B)$such that $H(e) = F(e)\cap G(e)$.

$(\boldsymbol{ii})$ The union of two soft sets (F, A) and(G, B) over X is the soft set (G, B) , where $C = A\cup B$ and $\forall\, e \in A$ we write $(H, C) = (F, A)\widetilde{\cup}(G, B)$, such that

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Vol. 5, Issue 4 , April 2018

$$H(e) = \begin{cases} F(e) & , \ if\ e \in A \backslash B \\ G(e) & , \ if\ e \in B \backslash A \\ F(e) \cup G(e) & , if\ e \in A \cap B \end{cases}.$$

$(\boldsymbol{iii})$ The difference of two soft sets (F, A) and (G, A) over X , denoted by $(H, C) = (F, A)\tilde{\backslash}(G, A)$ is defined as $H(e) = F(e)\backslash G(e)$, for all $e \in A$.

Definition (2.4) [8]: The soft complement of a soft set (F, A) over a universe X is denoted by $(F, A)^c$ and it is defined by $(F, A)^c = (F^c, A)$, where F^c is a mapping given by $F^c: A \to P(X)$, $F^c(e) = X \backslash F(e)$, for all $e \in A$. i.e. $(F, A)^c = \{(e, X\backslash F(e)) : \forall\ e \in A\}$.

- It is clear that $\widetilde{\emptyset}_A{}^c = \tilde{X}_A$; $\tilde{X}_A{}^c = \widetilde{\emptyset}_A$.
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Definition (2.5) [7]: Let V be a vector space over a field K and let A be a parameter set. Let G be a soft set over (V, A). Now G is said to be a soft vector space or soft linear space of V over K if $G(\lambda)$ is a vector subspace of $V, \lambda \in A$.

Definition (2.6) [7]: Let G be a vector space of V over K Then a soft element of G is said to be a soft vector of G . In a similar manner a soft element of the soft set (K, A) is said to be a soft scalar, being the scalar field.

Definition (2.7) [9]: Let $\tilde{X}$ be the absolute soft vector space i.e, $\tilde{X}(\lambda) = X, \forall \lambda \in E$. Then a mapping $\|.\| : SE(\tilde{X}) \to R(E)^*$ is said to be soft norm on the soft vector space $\tilde{X}$ if $\|.\|$ satisfies the following conditions:
$(\boldsymbol{i})\|\tilde{x}^e\| \geqq \bar{0}, for\ all\ \tilde{x}^e \tilde{\in} \tilde{X}.$
$(\boldsymbol{ii})\|\tilde{x}^e\| = \bar{0}\ if\ and\ only\ if\ \ \tilde{x}^e = \widetilde{\Theta}^e.$
$(\boldsymbol{iii})\|\tilde{\lambda}\tilde{x}^e\| \geqq |\tilde{\lambda}|\|\tilde{x}^e\|, for\ all\ \tilde{x}^e \tilde{\in} \tilde{X} and for\ every\ soft\ scalar \tilde{\lambda}.$
$(\boldsymbol{iv})\|\tilde{x}^e + \tilde{y}^e\| \leqq \|\tilde{x}^e\| + \|\tilde{y}^e\|, for\ all\ \tilde{x}^e, \tilde{y}^e \tilde{\in} \tilde{X}.$
The soft vector space $\tilde{X}$ with a soft norm $\|.\|$ on $\tilde{X}$ is said to be a soft normed linear space and is denoted by $(\tilde{X}, \|.\|)$.

Definition (2.8) [7]: A sequence of soft elements $\{\tilde{x}^e{}_n\}$ in a soft normed linear space$(\tilde{X}, \|.\|)$. is said to be convergent and converges to a soft element $\tilde{x}^e$ if $\|\tilde{x}^e{}_n - \tilde{x}^e\| \to \bar{0}$ as $n \to \infty$ This means for every $\tilde{\epsilon} \gneqq \bar{0}$, chosen arbitrary, there exists a natural number $= N(\epsilon)$, such that $\bar{0} \leqq \|\tilde{x}^e{}_n - \tilde{x}^e\| \lneqq \tilde{\epsilon}$, whenever $n > N$. We denote this by $\tilde{x}^e{}_n \to \tilde{x}^e as\ n \to \infty$ or $\lim_{n\to\infty} \tilde{x}^e{}_n = \tilde{x}^e$. $\tilde{x}^e$ is said to be the limit of the sequence $\tilde{x}^e{}_n$ as $n \to \infty$.

Definition (2.9) [7]: A sequence of soft elements $\{\tilde{x}^e{}_n\}$ in a soft normed linear space$(\tilde{X}, \|.\|)$. is said to be Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\epsilon} \gneqq \bar{0}$, there exists $m \in N$ such that:
$\|\tilde{x}^e{}_i - \tilde{x}^e{}_j\| \tilde{<} \tilde{\epsilon}, \forall i, j \geq m,$ i.e $\|\tilde{x}^e{}_i - \tilde{x}^e{}_j\| \to \bar{0}\ as\ i, j \to \infty.$

Definition (2.10) [7]: The operator $T: SE(\tilde{X}) \to SE(\tilde{Y})$ is said to be continuous at $\tilde{x}^e{}_0 \tilde{\in} \tilde{X}$ if for every sequence $\{\tilde{x}^e{}_n\}$ of soft elements of $\tilde{X}$ with $\tilde{x}^e{}_n \to \tilde{x}^e{}_0$ as $n \to \infty$. We have $T(\tilde{x}^e{}_n) \to T(\tilde{x}^e{}_0)$ as $n \to \infty$ i.e. , $\|\tilde{x}^e{}_n - \tilde{x}^e{}_0\| \to \bar{0}$ as $n \to \infty$ implies
$\|T(\tilde{x}^e{}_n) - T(\tilde{x}^e{}_0)\| \to \bar{0}$as $n \to \infty$. If T is continuous at each soft element of $\tilde{X}$, then T is said to be continuous operator.

Definition (2.11) [7]: Let $T: SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft linear operator, where $\tilde{X}$,$\tilde{Y}$ are soft normed linear space. The operator T is called bounded if there exists some positive soft real number $\widetilde{M}$ such that for all $\tilde{x}^e \tilde{\in} \tilde{X}$, $\|T(\tilde{x}^e)\| \leqq \widetilde{M}\|\tilde{x}^e\|$.

Definition (2.12) [7]: Let T be a bounded soft linear operator from $SE(\tilde{X})$ into $SE(\tilde{Y})$. Then the norm of the operator T denoted by $\|T\|$, is a soft real number defined as the following:
For each $\lambda \in E$, $\|T\|(\lambda) = \inf\{t \in \mathbb{R}; \|T(\tilde{x}^e)\|(\lambda) \leq t. \|\tilde{x}^e\|(\lambda), for\ each\ \tilde{x}^e \tilde{\in} \tilde{X}\}$.

Definition (2.13) [7]: An operator $T: SE(\tilde{X}) \to SE(\tilde{Y})$ is called injective or one-to-one if $T(\tilde{x}^e{}_1) = T(\tilde{x}^e{}_2)$ implies $\tilde{x}^e{}_1 = \tilde{x}^e{}_2$. It is called surjective or onto if $R(T) = SE(\tilde{Y})$. The operator T is bijective if T is both injective and surjective.

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Definition (2.14) [7]: Let V be an algebra over a scalar field $\mathbb{R}$ and let E be the parameter set and F_E be a soft set over V. Now, $F_E$is called soft algebra (for short SA) of V over $\mathbb{R}(E)$ if $F(e)$ is a sub algebra of V for all $e \in E$.It is very easy to see that in a SA the soft elements satisfy the properties:
$(\boldsymbol{i})(\tilde{x}^e\tilde{y}^e)\tilde{z}^e = \tilde{x}^e(\tilde{y}^e\tilde{z}^e)$.
$(\boldsymbol{ii})\tilde{x}^e(\tilde{y}^e \tilde{+} \tilde{z}^e) = \tilde{x}^e\tilde{y}^e \tilde{+} \tilde{x}^e\tilde{z}^e$; $(\tilde{y}^e \tilde{+} \tilde{z}^e)\tilde{x}^e = \tilde{y}^e\tilde{x}^e \tilde{+} \tilde{z}^e\tilde{x}^e$.
$(\boldsymbol{iii})\tilde{\alpha}(\tilde{x}^e\tilde{y}^e) = (\tilde{\alpha}\tilde{x}^e)\tilde{y}^e = \tilde{x}^e(\tilde{\alpha}\tilde{y}^e)$, where for all $\tilde{x}^e$,$\tilde{y}^e, \tilde{z}^e \tilde{\in} F_E$ and for any soft scalar $\tilde{\alpha}$. If $F_E$is also soft Banach space w.r.t. a soft norm that satisfies the inequality $\|\tilde{x}^e\tilde{y}^e\| \leq \|\tilde{x}^e\|\|\tilde{y}^e\|$ and $F_E$is contains the unitary element $\tilde{u}^e$such that $\tilde{x}^e\tilde{u}^e = \tilde{u}^e\tilde{x}^e = \tilde{x}^e$ with $\|\tilde{u}^e\| = \bar{1}$, then is called soft Banach algebra (for short $\boldsymbol{SBA}$).

Theorem (2.15) [7]: $\mathfrak{A}$ is a (SBA) iff $\mathfrak{A}(\lambda)$ is a Banach algebra $\lambda \in E$.

Theorem (2.16) [7]: In (SBA) if $\tilde{x}^e{}_n \to \tilde{x}^e$ and $\tilde{y}^e{}_n \to \tilde{y}^e$ then $\tilde{x}^e{}_n\tilde{y}^e{}_n \to \tilde{x}^e\tilde{y}^e$. i.e., multiplication in a (SBA) is continuous.

Theorem (2.17) [7]: Every parameterized family of crisp Banach algebras on a crisp space V can be considered as a (SBA) on the soft vector space $\tilde{V}$.

Definition (2.18) [7]: Let $\mathfrak{A}$ is a $(\boldsymbol{SBA})$ with $\tilde{u}^e$. Then $\tilde{x}^e \tilde{\in} \mathfrak{A}$ is said to be soft regular, if $\tilde{x}^e$ is invertible (i.e. there a soft element $\tilde{x}^{e^{-1}}$called the inverse of $\tilde{x}^e$, such that $\tilde{x}^e(\tilde{x}^e)^{-1} = (\tilde{x}^e)^{-1}\tilde{x}^e = \tilde{u}^e$). A non-soft regular element is called soft singular (it's not invertible).

III.MAIN RESULTS

Definition (3.1) [7]: Let $\mathfrak{A}$ be soft Banach algebra. A soft element $\tilde{z}^e \tilde{\in} \mathfrak{A}$ is called soft topology divisor of zero (shortly STDZ) if there exists a sequence $\{\tilde{z}^e{}_n\}$in $\mathfrak{A}$ with $\|\tilde{z}^e{}_n\| = \bar{1}$ such that either $\tilde{z}^e \tilde{.} \tilde{z}^e{}_n \to \tilde{\Theta}^e$ or$\tilde{z}^e{}_n \tilde{.} \tilde{z}^e \to \tilde{\Theta}^e$. i.e.$\widetilde{Z_E} = \{\tilde{z}^e \tilde{\in} \mathfrak{A}: \tilde{z}^e \text{ is STDZ}\}$.

Theorem (3.2) [7]: Every (STDZ) is soft singular, i.e. $\widetilde{Z_E} \tilde{\subseteq} S_E$.

Definition (3.3)[7]: Let$(\tilde{X}, \|\tilde{.}\|)$ be soft normed space and $\tilde{Y} \tilde{\in} S(X)$.Asoft element $\tilde{a} \tilde{\in} \tilde{X}$is called soft boundary element (Bd) of $\tilde{Y}$ if there exist two soft sequence $\tilde{x}^e{}_n$ and $\tilde{y}^e{}_n$ of soft element in $\tilde{Y}$and Y^crespectively such that $\tilde{x}^e{}_n \to \tilde{a}$ and$\tilde{y}^e{}_n \to \tilde{a}$.

Theorem (3.4) [7]: $Bd(S_E) = \overline{SE} \tilde{\cap} \overline{(\mathfrak{A} - S_E)} = \overline{SE} \tilde{\cap} \overline{G_E}$.

Theorem (3.5) [7]: $Bd(S_E) \tilde{\subseteq} \widetilde{Z_E}$.

Definition (3.6): An soft algebra $\mathfrak{A}$ is said to be soft division algebra if every $\tilde{x}^e \tilde{\in} \mathfrak{A}$,
$\tilde{x}^e \neq \tilde{\Theta}^e$ is soft regular.

Theorem (3.7): If $\mathfrak{A}$ is soft Banach algebra with$\tilde{u}^e$,then for every $\tilde{x}^e \tilde{\in} \mathfrak{A}$ can be expressed as $\tilde{\alpha} \tilde{.} \tilde{u}^e$,for all $\tilde{\alpha} \tilde{\in} \mathbb{C}(E)$.
Proof:
Suppose that $\exists\, \tilde{x}^e \tilde{\in} \mathfrak{A}$ can't be expressed as$\tilde{\alpha} \tilde{.} \tilde{u}^e$
i.e. $\tilde{x}^e \neq \tilde{\alpha} \tilde{.} \tilde{u}^e$ for all $\tilde{\alpha} \tilde{\in} \mathbb{C}(E)$
So$\tilde{x}^e - \tilde{\alpha} \tilde{.} \tilde{u}^e \neq \tilde{\Theta}^e$, for $\tilde{\alpha} \tilde{\in} \mathbb{C}(E)$
Then$(\tilde{x}^e - \tilde{\alpha} \tilde{.} \tilde{u}^e)^{-1}$ is exist, $\tilde{\alpha} \tilde{\in} \mathbb{C}(E)$
hence ($\tilde{x}^e - \tilde{\alpha} \tilde{.} \tilde{u}^e$) is soft regular , $\tilde{\alpha} \tilde{\in} \mathbb{C}(E)$ Consequently $\delta(\tilde{x}^e) = \tilde{\emptyset}$.
This is contradiction with $\delta(\tilde{x}^e) \neq \tilde{\emptyset}$.

Definition (3.8) [10]: Let (F, A) and (H, B) be two soft groups over G and K respectively, and let $f: G \to K$ and $g: A \to B$ be two functions. Then we say that (f, g) is a soft homomorphism, and that (F, A) is soft homomorphic to

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Vol. 5, Issue 4 , April 2018

(H,B). The latter is written as $(F,A)\sim(H,B)$, if the following conditions are satisfied:

(1) f is a homomorphism from G onto K,

(2) g is a mapping from A onto B, and

(3) $f\big(F(x)\big) = H\big(g(x)\big)\ for\ all\ x \in A$.

In this definition, if f is an isomorphism from G to K and g is a one-to-one mapping from A onto B, then we say that $(f,g)$is a soft isomorphism and that (F,A) is soft isomorphic to (H,B). The latter is denoted by $(F,A) \cong (H,B)$.

Theorem (3.9): Every SBA with $\tilde{u}^e$over $\mathbb{C}(\mathrm{E})$ which is soft division algebra is soft isomorphic with $\mathbb{C}(\mathrm{E})$.

Proof:

Let $\mathfrak{A}$ be soft division algebra, then by [theorem (3.7)] implies

$\mathfrak{A} = \{\tilde{x}^e = \tilde{\alpha}\tilde{.}\tilde{u}^e\ , \tilde{\alpha} \tilde{\in} \mathbb{C}(\mathrm{E})\ \}$

Define$\vartheta: \mathfrak{A} \to \mathbb{C}(\mathrm{E}\ )$ by

$\vartheta(\tilde{x}^e) = \ \vartheta(\tilde{\alpha}\tilde{.}\tilde{u}^e) = \ \tilde{\alpha}$

Let $\tilde{x}^e\ = \tilde{y}^e, \tilde{x}^e, \tilde{y}^e\ \tilde{\in}\ \mathfrak{A}$,then

$\tilde{\alpha}\tilde{.}\tilde{u}^e = \tilde{\beta}\tilde{.}\tilde{u}^e$, for some $\tilde{\alpha}, \tilde{\beta}\ \tilde{\in}\ \mathbb{C}(\mathrm{E})$hence $\tilde{\alpha}\ = \tilde{\beta}$, So $\vartheta(\tilde{\alpha}\tilde{.}\tilde{u}^e) = \vartheta\big(\tilde{\beta}\tilde{.}\tilde{u}^e\big)$

Consequnetly $\vartheta(\tilde{x}^e) = \vartheta(\tilde{y}^e)$

Therefore ϑ is well define. Now to prove that ϑ is soft injective.

Let $\vartheta(\tilde{x}^e) \neq\ \vartheta(\tilde{y}^e), \tilde{x}^e, \tilde{y}^e\ \tilde{\in}\ \mathfrak{A}$

then $\vartheta(\tilde{\alpha}\tilde{.}\tilde{u}^e\ ) \neq\ \vartheta(\tilde{\beta}\tilde{.}\tilde{u}^e\ )$, for some $\tilde{\alpha}, \tilde{\beta}\ \tilde{\in}\ \mathbb{C}(\mathrm{E})$

So $\tilde{\alpha}\ \neq \tilde{\beta}$and $\tilde{\alpha}\tilde{.}\tilde{u}^e \neq \tilde{\beta}\tilde{.}\tilde{u}^e$,hence $\tilde{x}^e\ \neq\ \tilde{y}^e$

Consequnetly ϑ is soft injective.

Now to show that ϑ is soft Surjective, for all $\tilde{\alpha}\ \tilde{\in}\ \mathbb{C}(\mathrm{E})$then $\tilde{\alpha}\tilde{.}\tilde{u}^e\ = \tilde{x}^e\ \tilde{\in}\ \mathfrak{A}$

So ϑ is soft Surjective.

To show that ϑ is soft homomorphism.

$(\boldsymbol{i})\vartheta(\tilde{x}^e \tilde{+} \tilde{y}^e) = \vartheta(\tilde{\alpha}\tilde{.}\tilde{u}^e \tilde{+} \tilde{\beta}\tilde{.}\tilde{u}^e)$, for some $\tilde{\alpha}, \tilde{\beta}\ \tilde{\in}\ \ \mathbb{C}(\mathrm{E})$

$= \vartheta((\tilde{\alpha}\tilde{+}\tilde{\beta}\ )\tilde{.}\tilde{u}^e) = \tilde{\alpha}\tilde{+}\tilde{\beta} = \vartheta(\tilde{x}^e)\tilde{+}\ \vartheta(\tilde{y}^e\ ) = \vartheta(\tilde{x}^e)\tilde{+}\ \vartheta(\tilde{y}^e)$.

$(\boldsymbol{ii})\ \vartheta(\tilde{\alpha}.\tilde{x}^e) = \vartheta\big(\tilde{\alpha}.\tilde{\beta}\tilde{.}\tilde{u}^e\big) = \ \tilde{\alpha}.\tilde{\beta} = \tilde{\alpha}\,.\vartheta(\tilde{x}^e)$.

$(\boldsymbol{iii})\vartheta(\tilde{x}^e\tilde{.}\tilde{y}^e) = \vartheta(\tilde{\alpha}.\tilde{u}^e\tilde{.}\tilde{\beta}\tilde{.}\tilde{u}^e)$

$= \vartheta\big(\big(\widetilde{\tilde{\alpha}\tilde{.}\tilde{\beta}}\big)\tilde{.}\tilde{u}^e\big) = \tilde{\alpha}\tilde{.}\tilde{\beta} = \vartheta(\tilde{x}^e)\tilde{.}\ \vartheta(\tilde{y}^e\ )$

Since$\vartheta(\tilde{x}^e) = \vartheta(\tilde{\alpha}.\tilde{u}^e) = \tilde{\alpha}$, then $|\vartheta(\tilde{x}^e)| = |\tilde{\alpha}|$

Also $||\tilde{x}^e|| = ||\tilde{\alpha}.\tilde{u}^e|| = |\tilde{\alpha}|.||\tilde{u}^e|| = |\tilde{\alpha}| = |\vartheta(\tilde{x}^e)|$.

Hence $|\vartheta(\tilde{x}^e)| = ||\tilde{x}^e||$. So ϑ is soft isomorphic with $\mathbb{C}(\mathrm{E})$.

Theorem (3.10): If $\widetilde{\Theta}^e$is the only (STDZ) in soft banach algebra $\mathfrak{A}$ over $\mathbb{C}(E)$with $\tilde{u}^e$, then soft $\mathfrak{A} \cong \mathbb{C}(E)$.

Proof:

Suppose that$\widetilde{\Theta}^e$is the only (STDZ) in $\mathfrak{A}$

Let $\tilde{x}^e\ \tilde{\in}\ \mathfrak{A}$, then $\delta(\tilde{x}^e) \neq \tilde{\emptyset}$, such that $\delta(\tilde{x}^e) = \{\tilde{\alpha}\ \tilde{\in}\ \mathbb{C}(\mathrm{E}\ )\ :\ \tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e\ \tilde{\in}\ S_E\}$

Also $\mathrm{Bd}((\delta(\tilde{x}^e)) \neq \tilde{\emptyset}$, So$\tilde{\alpha}\ \tilde{\in}\ \mathrm{Bd}(\delta(\tilde{x}^e))$, then$\tilde{\alpha}\ \tilde{\in}\ \ \delta(\tilde{x}^e\ )$ and $\tilde{\alpha}\ \tilde{\in}\ \overline{(\delta(\tilde{x}^e\ )\ )^c}$

Since $\tilde{\alpha}\ \tilde{\in}\ \ \delta(\tilde{x}^e\ )$, then$\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e\ \tilde{\in}\ S_E$. Since $\tilde{\alpha}\ \tilde{\in}\ \overline{(\delta(\tilde{x}^e\ )\ )^c}$. Then $\exists\ \{\tilde{\alpha}_n\}$ in $(\delta(\tilde{x}^e\ )\ )^c$ such that $\tilde{\alpha}_n\ \tilde{\to}\ \tilde{\alpha}$. So $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}_n\tilde{.}\tilde{u}^e\ \tilde{\to}\ \tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e$.

Now $\mathrm{Bd}(S_E) = S_E \cap \overline{G_E}$. Hence $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e\ \tilde{\in}\ \overline{G_E}$ and $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e\ \tilde{\in}\ S_E$

Then by [theorem (3.5)] $\mathrm{Bd}(S_E)\ \tilde{\subseteq}\ \widetilde{Z_E}$

So $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e\ \tilde{\in}\ \widetilde{Z_E}$. Then $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e\ = \widetilde{\Theta}^e$.

Therefore$\tilde{x}^e\ = \tilde{\alpha}.\tilde{u}^e$, consequently $\mathfrak{A} \cong\ \mathbb{C}(E)$.

Theorem (3.11): Let $\mathfrak{A}$ be (SBA) with $\tilde{u}^e$suppose that, $\tilde{\alpha}\ \tilde{>}\ \bar{0}and||\tilde{x}^e.\tilde{y}^e|| \geq \tilde{\alpha}||\tilde{x}^e||\tilde{.}\,||\tilde{y}^e||, \forall\tilde{x}^e, \tilde{y}^e\ \tilde{\in}\ \mathfrak{A}$and $\tilde{\alpha}$ is soft real number, then $\mathfrak{A} \cong \mathbb{C}(\mathrm{E})$

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Proof:
Let $\tilde{z}^e \tilde{\in} \mathfrak{A}$ such that $\tilde{z}^e \tilde{\in} \widetilde{Z_E}$. There is $\{\tilde{z}^e{}_n\} \tilde{\in} \mathfrak{A}$ such that $\tilde{z}^e \tilde{.} \tilde{z}_n \tilde{\to} \tilde{\Theta}^e$
or $\tilde{z}_n \tilde{.} \tilde{z}^e \tilde{\to} \tilde{\Theta}^e$ as $n \to \infty$, $\|\tilde{z}^e{}_n\| = \bar{1}$ and by given $\|\tilde{z}_n \tilde{.} \tilde{z}^e\| \geq \tilde{\alpha}\|\tilde{z}_n\| \tilde{.} \|\tilde{z}^e\|$.
Then $\tilde{\alpha} \tilde{.} \|\tilde{z}^e\| \tilde{\to} \tilde{\Theta}^e$ as $n \to \infty$, implies $\|\tilde{z}^e\| \tilde{\to} \tilde{\Theta}^e$ as $n \to \infty$
Hence $\tilde{z}^e = \tilde{\Theta}^e$. Since $\tilde{\Theta}^e$ is the only(STDZ) in $\mathfrak{A}$. Therefore $\mathfrak{A} \cong \mathbb{C}(E)$.

Theorem (3.12): Let $\grave{\mathfrak{A}}$ be soft Banach sub algebra of soft Banach algebra $\mathfrak{A}$, then for all $\tilde{x}^e \tilde{\in} \grave{\mathfrak{A}}$,we have
$(\boldsymbol{i}) \delta\, \grave{\mathfrak{A}}(\tilde{x}^e) = \delta\mathfrak{A}(\tilde{x}^e)$.
$(\boldsymbol{ii})\ \mathrm{Bd}(\delta\grave{\mathfrak{A}}(\tilde{x}^e)) \tilde{\subseteq} \mathrm{Bd}((\delta\mathfrak{A}(\tilde{x}^e))$.
Proof:
$(\boldsymbol{i})$
Let $\tilde{\alpha} \tilde{\in} \delta\mathfrak{A}(\tilde{x}^e)$, then $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e$ is soft singular of $\mathfrak{A}$.
Hence $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e$ is soft singular of $\grave{\mathfrak{A}}$.
So $\delta\mathfrak{A}(\tilde{x}^e) \tilde{\subseteq} \delta\grave{\mathfrak{A}}(\tilde{x}^e) \ldots\ldots\ldots (1)$
Since $\delta \tilde{\in} \grave{\mathfrak{A}}$, clearly

$$\delta\, \grave{\mathfrak{A}}(\tilde{x}^e) \tilde{\subseteq} \delta\mathfrak{A}(\tilde{x}^e) \quad \ldots\ldots\ldots. (2)$$

from (1) and (2), we get

$$\delta\grave{\mathfrak{A}}(\tilde{x}^e) = \delta\mathfrak{A}(\tilde{x}^e).$$

$(\boldsymbol{ii})$
Let $\tilde{\alpha} \tilde{\in} \mathrm{Bd}(\delta\, \grave{\mathfrak{A}}(\tilde{x}^e))$.
Then $\tilde{\alpha} \tilde{\in} \delta\, \grave{\mathfrak{A}}(\tilde{x}^e) \tilde{\cap} \overline{\left(\delta\grave{\mathfrak{A}}(\tilde{x}^e)\right)^c}$, as $\tilde{\alpha} \tilde{\in} \overline{(\delta\grave{\mathfrak{A}}(\tilde{x}^e\ )\ )^c}$, then there is $\{\tilde{\alpha}_n\}$ in $(\delta\grave{\mathfrak{A}}(\tilde{x}^e))^c$ such that $\tilde{\alpha}_n \tilde{\to} \tilde{\alpha}$ as $n \to \infty$.
Hence $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}_n.\tilde{u}^e \tilde{\to} \tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e$. So $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e \tilde{\in} G_E$, hence $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e \tilde{\in} \overline{G_E}$ also $\tilde{\alpha} \tilde{\in} \delta\grave{\mathfrak{A}}(\tilde{x}^e)$, implies $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e \tilde{\in} S_E$.
Then $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e \tilde{\in} S_E \tilde{\cap} \overline{G_E} = \mathrm{Bd}(S_E)$ by [theorem (3.5)], $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e \tilde{\in} \widetilde{Z_E}$ in $\grave{\mathfrak{A}}$.
Hence $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e \tilde{\in} \widetilde{Z_E}$ in $\mathfrak{A}$ therefore $\tilde{x}^e \tilde{\simeq} \tilde{\alpha}.\tilde{u}^e$ is soft singular in $\mathfrak{A}$.
Then $\tilde{\alpha} \tilde{\in} \delta\mathfrak{A}(\tilde{x}^e)$, also $\tilde{\alpha} \tilde{\in} \mathrm{Bd}\left(\delta\grave{\mathfrak{A}}(\tilde{x}^e)\right)$. consequently $\tilde{\alpha} \tilde{\in} \mathrm{Bd}\big(\delta\mathfrak{A}(\tilde{x}^e)\big)$.

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