

Algorithms of Sustainable Identification of Dynamic Objects in the Closed Control Line

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ABSTRACT: Algorithms for the stable identification of dynamic objects based on the results of their normal functioning in a closed control loop are presented. To improve the accuracy of the calculation of the parameters of the object, the regularization principle and pseudo-inversion methods based on the singular expansion are used. The resulted algorithms allow under certain noise signal conditions to carry out a stable identification of dynamic objects in a closed control loop.

KEYWORDS: dynamic object, sustainable identification, closed control line, regularization, pseudoinverse, singular expansion.

I. INTRODUCTION

When constructing adaptive control systems, the task of identifying a dynamic object arises from the results of its normal functioning in a closed control loop. It is known [1-5] that under certain noise-signal conditions, the use of identification methods developed for open systems leads to incorrect results. For this reason, the development of effective procedures for identifying dynamic objects based on the results of their normal functioning in a closed loop of control remains at present an urgent problem.

II. FORMULATION OF THE PROBLEM

Let the control object in the closed system given in the following figure be described by a n -order difference equation [5,6]

$$x[k] = \sum_{i=1}^p a_i x[k-i] + \sum_{j=1}^q b_j \eta[k-j], \tag{1}$$

$$y[k] = x[k] + v[k]; \eta[k] = u[k] + w[k],$$

where $\eta[k]$ and $x[k]$ are the input and output of the object; $w[k]$ and $v[k]$ - the disturbance and the error of the output measurement, respectively; $u[k]$ and $y[k]$ - control action and output measurement result used as input and output data in the identification procedure; a_i, b_j - unknown parameters of the object.

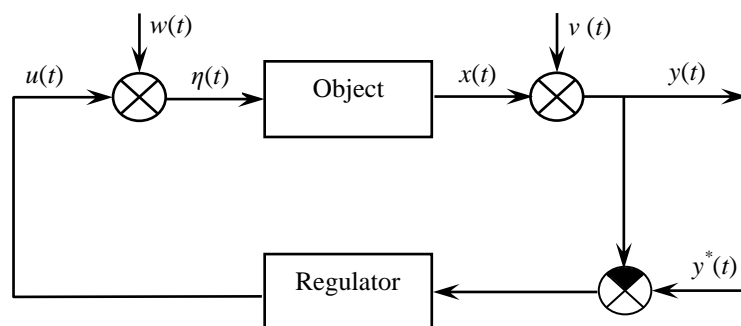


Fig. Block diagram of the control loop

Perturbation $w[k]$ is a stationary random process with a known autocorrelation function $R_{ww}[i]$, for example

$$R_{ww}[i] = D_w \cdot e^{-\alpha|i|},$$

where D_w and α – are known statistical characteristics $w[k]$.

The measurement error $v[k]$ is a sequence of uncorrelated random variables

$$R_{vv}[i] = D_v \cdot \delta[i],$$

where D_v is the known variance of error $v[k]$, $\delta[i]$ is the symbol of Kronecker.

Sequence $v[k]$ is uncorrelated with disturbance $w[k]$, the history of signals $u[k]$ and $y[k]$.

The difference equation of the controller has the form:

$$u[k] = \sum_{i=1}^{l1} c_i u[k-i] + \sum_{j=0}^{l2} d_j \varepsilon[k-j], \tag{2}$$

$$\varepsilon[k] = y^*[k] - y[k],$$

where $y^*[k]$ - the current value of the job; $\varepsilon[k]$ - a signal of a mismatch; c_i, d_j - known regulator parameters.

To reduce further calculations, we assume that in equation (1) $p=q$, and in equation (4) $l1=l2=l$. Equation (1) of the object can be written in the following form

$$Y = H \cdot \theta + E + \Psi \cdot \theta, \tag{3}$$

where $Y^T = [y[p+1] \ y[p+2] \ \dots \ y[p+N]]$;

$$H = \begin{bmatrix} y[p] & \dots & y[1] & u[p] & \dots & u[1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y[p+N-1] & \dots & y[p] & u[p+N-1] & \dots & u[N] \end{bmatrix};$$

$$\theta = [a_1 \ \dots \ a_p \ b_1 \ \dots \ b_p]^T;$$

$$E = [v[p+1] \ v[p+2] \ \dots \ v[p+N]]^T;$$

$$\Psi = \begin{bmatrix} -v[p] & \dots & -v[1] & w[p] & \dots & w[1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -v[p+N-1] & \dots & -v[1] & w[p+N-1] & \dots & w[N] \end{bmatrix}.$$

Multiply the equation of the object (3) on the left by H^T

$$H^T Y = H^T H \theta + H^T E + H^T \Psi \theta. \tag{4}$$

Dropping the term $H^T E$ in (4) does not cause the least square method (LSM) estimator $\hat{\theta}$ to shift. At the same time, $M(H^T \Psi) \neq 0$ for any order of p objects, so the LSM-estimator

$$\hat{\theta} = (H^T H)^{-1} H^T Y, \tag{5}$$

which does not take into account this factor, is biased [4,5,7]. For this reason, we consider the influence of the last term on the bias of the estimate (5).

We have

$$\frac{1}{N} M(H^T \cdot \Psi) = K = \begin{bmatrix} -R_{vy}[0] & \dots & -R_{vy}[p-1] & R_{wy}[0] & \dots & R_{wy}[p-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -R_{vy}[1-p] & \dots & -R_{vy}[0] & R_{wy}[1-p] & \dots & R_{wy}[0] \\ -R_{v\mu}[0] & \dots & -R_{v\mu}[p-1] & R_{w\mu}[0] & \dots & R_{w\mu}[p-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -R_{v\mu}[1-p] & \dots & -R_{v\mu}[0] & R_{w\mu}[1-p] & \dots & R_{w\mu}[0] \end{bmatrix}.$$

Correlation moments appearing in this matrix, we express, using the known [7,8] ratio

$$R_{xz}[i] = \sum_{j=0}^{\infty} \omega_{xz}[j] \cdot R_{xx}[i-j],$$

where x - random impact (w or v); z - the response of a linear impulse system (y or u); ω_{xz} is the weight characteristic of the closed system (1), (2) along the corresponding channel. In turn, the values of the weighted characteristic $\omega_{xz}[j]$ can be expressed in terms of parameters a_i, b_i, c_i, d_i of the closed system.

On the basis of equation (4) we can write

$$\frac{1}{N} H^T Y = \frac{1}{N} H^T H \cdot \theta + K \cdot \theta. \tag{6}$$

Thus, to obtain an unbiased estimate of the vector $\hat{\theta}$, it is necessary to determine the OLS estimate (5) of the vector $\hat{\theta}$ of the corrected estimate of the object parameters [5,7] vector $\hat{\theta}$ using (6)

$$\hat{\theta} = \left(\frac{1}{N} H^T H + K \right)^{-1} \frac{1}{N} H^T Y.$$

A special feature of this problem is its mathematical incorrectness according to Hadamard [9,10], which is associated with the poor conditionality of the matrix H . The methods of regular estimation [9-12] can significantly increase the accuracy of the estimation of parameter $\hat{\theta}$. Among them we should mention a group of methods based on the general regularization principle. A.N.Tikhonov [9], and the effective pseudoinversion method, based on the singular expansion of the matrix [13].

III. SOLUTION OF THE TASK

The regularization principle leads to an estimate of the desired solution $\hat{\theta}$ on the basis of equation

$$\hat{\theta}_\alpha = (H^T H + \alpha I)^{-1} H^T Y, \tag{7}$$

where $\alpha > 0$ is the regularization parameter, I - is the unit matrix.

If the error levels of the initial data are known,

$$h = \|H - \bar{H}\|, \quad \delta = \|Y - \bar{Y}\|,$$

then the regularization parameter α can be determined on the basis of the generalized discrepancy principle [10,12], where \bar{H} and \bar{Y} are the exact values of the matrix operator H and the vector of the right-hand side Y .

If the numbers h and δ are unknown or their computation is associated with significant difficulties, then the regularization parameter α can be determined either on the basis of the quasioptimality method:

$$\|\hat{\theta}_{\alpha_{i+1}} - \hat{\theta}_{\alpha_i}\| = \min, \quad \alpha_{i+1} = \xi \alpha_i, \quad i = 0, 1, 2, \dots, \quad 0 < \xi < 1,$$

or the ratio [9,14]:

$$r_{rel}(\alpha) = r_1(\alpha) / r(\alpha),$$

where

$$r(\alpha) = \|H \hat{\theta}_\alpha - Y\|^2, \quad r_1(\alpha) = \|H \eta_\alpha - (H \hat{\theta}_\alpha - Y)\|_F^2, \\ \eta_\alpha = \alpha (d \hat{\theta}_\alpha / d \alpha).$$

The method of effective pseudoinversion is known [13,15] to be based on the singular expansion of the matrix H , i.e. on its representation in the form

$$H = USV^T,$$

where U - orthogonal ($N \times 2p$) -matrix; V - orthogonal ($2p \times N$) - matrix; S - diagonal ($2p \times 2p$) - matrix.

Columns u_i and v_i of matrices U and V are eigenvectors of matrices HH^T and $H^T H$, and the diagonal elements μ_i of matrix S are positive roots of eigenvalues λ_i of matrix $H^T H$ (or HH^T).



The pseudoinverse Moore-Penrose matrix H^+ makes it possible to obtain the estimate [15]

$$\hat{\theta} = VS^+U^TY = \sum_{i=1}^r \frac{1}{\mu_i} v_i u_i^T, \quad (8)$$

where $S^+ = \text{diag}(s_1^+, \dots, s_r^+)$ – pseudo-inverse matrix for the matrix S ; r – rank of the matrix H , i.e. the number of nonzero singular numbers $\mu_i (i = \overline{1, p})$; $s_i^+ = 1/\mu_i$, if $\mu_i \neq 0$, and $s_i^+ = 0$, if $\mu_i = 0$.

In the case where the rank of the matrix H $r = p$, the pseudoinverse estimate (8) coincides with the estimate (7) for the least squares and, accordingly, is characterized by low accuracy. In connection with this, the so-called effective pseudoinverse matrices [13,15] and estimates

$$\hat{\theta}_\tau = VS_\tau^+U^TY = \sum_{i=1}^{r'} \frac{1}{\mu_i} v_i \cdot u_i^T,$$

where S_τ^+ – effective pseudo-matrix $S = \text{diag}(s_{1\tau}^+, \dots, s_{n\tau}^+)$; $r' < r$, $s_{i\tau}^+ = 1/\mu_i$, if $\mu_i > \tau$, and $s_{i\tau}^+ = 0$, if $\mu_i = 0$.

To obtain estimates of $\hat{\theta}_\alpha$ and $\hat{\theta}_\tau$, having a minimum mean error rate, the regularization parameter α and parameters τ or r' must be consistent with the variance of the measurement errors of vector Y , in practice they are often determined by the residual method.

IV. CONCLUSION

Thus, the above algorithms allow under certain noise signal conditions to perform a stable identification of dynamic objects based on the results of their normal functioning in a closed control loop. The obtained algorithms were used to solve the task of identifying a specific technological object and showed their effectiveness.

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