ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 5, Issue 4 , April 2018

Sustainable Algorithms of Suboptimal Estimation Based on Methods of Condition-Gaussian Filtration

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ABSTRACT: Stable algorithms for suboptimal estimation of unobserved coordinates of joint random processes are presented on the basis of conditional Gaussian filtering methods. To make the procedure of suboptimal estimation more stable, an algorithm was used to find the pseudo inverse matrix using direct numerical analysis of the recursive type. The above algorithms allow to increase the accuracy and stability of the procedure for estimating the unobserved coordinates of random processes.

KEYWORDS: stable algorithms, suboptimal estimation, conditional Gaussian filtration.

I.INTRODUCTION

The problem of estimating the realization of random processes containing some useful information and taking place in the presence of various types of interference arises in many areas of science and technology. Depending on the purpose, a variety of technical transmission or retrieval systems operate under different conditions and different requirements are imposed on them [1-10]. If the information parameters of the useful signal change on the observation segment, the problem of estimating the parameters is also transformed into a filtration problem. In solving this problem, it is necessary to obtain an estimate of the realization of a directly unobserved random process on the basis of observing the realization of a random process and known a priori information. The result of solving filtration problems is the optimal rules for estimating random processes and the corresponding structural diagrams of the optimal estimation devices, as well as the potential performance characteristics of their functioning[1,4,5,8]. We note that in the solution of various filtration problems, the concept of conditional Gaussian filtration turns out to be very effective [3,11-13].

II. FORMULATION OF THE PROBLEM

Consider the joint process $Z^T(k+1)=[X^T(k+1),Y^T(k+1)]$, which is described by the stochastic difference equation[3,11,12]:

$$Z(k+1)=\Phi_z[k+1,k;Z(k)]+\Gamma_z[k+1,k;Z(k)]N_z(k)\ ,\ k=0,1,2,..., \tag{1}$$

where $Z(k+1)$ is the vector column of size $(n+m)\times 1$; $X(k+1)$ $-(n\times 1)$-dimensional unobserved component of the vector $Z(k+1)$; $Y(k+1)$ -part of the components of vector $Z(k+1)$, available for direct observation; $\Phi_z[\cdot]$ and $\Gamma_z[\cdot]$ are known vector functions of their arguments; $N_z(k)$ are the vectors of independent Gaussian random variables with zero mathematical expectations and unit variances.

Suppose that the functions on the right-hand side of equation (1) have the form

$$\Phi_z[k+1,k,Z(k)]=\Phi_{zx}[k+1,k;Y(k)]X(k)+\Phi_{zy}[k+1,k;Y(k)]=$$

$$=\begin{bmatrix}\Phi_{xx}[k+1,k;Y(k)]X(k)+\Phi_{xy}[k+1,k;Y(k)]\\ \Phi_{yx}[k+1,k;Y(k)]X(k)+\Phi_{yy}[k+1,k;Y(k)]\end{bmatrix}, \tag{2}$$

$$\Gamma_z=\Gamma_z[k+1,k;Y(k)]=[\Gamma_x^T[k+1,k;Y(k)]\Gamma_y^T[k+1,k;Y(k)]]^T\ . \tag{3}$$

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

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According to (3), we can write

$$B_{zz} = \Gamma_z \Gamma_z^T = \left[\begin{array}{c|c} B_{xx} & B_{xy} \\ \hline B_{yx} & B_{yy} \end{array}\right]. \tag{4}$$

In optimal or quasi-optimal discrete filtering algorithms, the result of each newly obtained measurement must be processed to determine the state vector estimate. However, in practical implementation of these algorithms, a certain calculation time is required to process the result of each next measurement, which depends on the dimensions of the state and observation vectors, and also on the form of the functional dependences of the matrices Φ and B entering the right-hand parts of the filtering algorithms for conditionally Gaussian processes of the form [12-14]

$$\begin{aligned} \hat{X}(k+1) &= \Phi_{xx}[k+1,\, k; Y(k)]\hat{X}(k) + \Phi_{xy}[k+1,\, k; Y(k)] + K(k+1)\times \\ &\times \{\mathrm{Y}(k+1) - \Phi_{yx}[k+1,\, k; Y(k)]\hat{X}(k) - \Phi_{yy}[k+1,\, k; Y(k)]\}, \end{aligned} \tag{5}$$

$$K(k+1) = [\Phi_{xx} P(k) \Phi_{yx}^T + \mathrm{B}_{xy}][\Phi_{yx} P(k) \Phi_{yx}^T + \mathrm{B}_{yy}]^{-1}, \tag{6}$$

$$P(k+1) = [\Phi_{xx} P(k) \Phi_{xx}^T + B_{xx}] - K(k+1)[B_{xy} \Phi_{xx} P(k) \Phi_{yx}^T]^T. \tag{7}$$

For the overwhelming majority of practical problems, it is usually required to provide the highest possible accuracy of the state vector estimation, including in the presence of certain limitations, in any case, for the given models of state and observation vectors it is first expedient to synthesize optimal or quasi-optimal algorithms for processing information[2,4,12,15].

In the derivation of quasi-optimal algorithms for estimation, these simplifications are directed primarily at the analytical record of the corresponding recurrence relations, and in deriving suboptimal algorithms they must also ensure the fulfilment of the specified constraints [2,11,16].
Therefore, it is possible to significantly reduce the total computational costs if, instead of (5), we use a suboptimal estimation algorithm

$$\begin{aligned} X^*(k+1) &= \Phi_{xx}[k+1, k; Y(k)]X^*(k) + \Phi_{xy}[k+1, k; Y(k)] + \\ \overline{K}(k+1)&\{Y(k+1) - \Phi_{yx}[k+1, k; Y(k)]X^*(k) - \Phi_{yy}[k+1, k; Y(k)]\}, \end{aligned} \tag{8}$$

in which the sub-optimal values of the transmission coefficient matrix $\overline{K}(k+1)$ are determined in some manner in advance. The quality of estimating the vector of unobserved components using the suboptimal algorithm (8) will depend substantially on a more or less successful selection of these transmission factors.

Let us consider the choice of the transmission coefficients of the suboptimal filter (8). To significantly reduce the computational costs for each evaluation cycle, when passing from the optimal algorithms (5) - (7) to the suboptimal algorithm (8), these coefficients can be calculated in advance, i.e. before processing a specific implementation of the observation vector. Thus, we arrive at the problem of parametric optimization [3,5], a fairly general method of solving which is as follows [12-14]. On the basis of (5), (6), taking into account (8), the equation for estimation errors can be written in the form

$$\begin{aligned} E(k+1) &= \{\Phi_{xx}[k+1, k; Y(k)] - \overline{K}(k+1)\Phi_{yx}[k+1, k; Y(k)]\}E(k) + \\ &\{\Gamma_x[k+1, k; Y(k)] - \overline{K}(k+1)\Gamma_y[k+1, k; Y(k)]\}N_z(k). \end{aligned} \tag{9}$$

The problem of parametric optimization can be formulated as follows. Find a sequence of matrices $\overline{K}(k+1)$ for which the average risk is minimal at each step

$$\begin{aligned} R_G[\overline{K}(k+1)] &= \int\int\int E^T(k+1) B E(k+1) \times \\ &\times p[E(k+1), X(k+1), Y(k+1)] dE(k+1) dX(k+1) dY(k+1). \end{aligned} \tag{10}$$

On the basis of the necessary condition for the extremum, the length of the determination of the unknown matrix $\overline{K}(k+1) = [\overline{K}_{ij}]$, $i = \overline{1,n}$, $j = \overline{1,m}$, can be arrived at to the system of equations

$$\frac{\partial}{\partial \overline{K}_{ij}} R_G[\overline{K}(k+1)] = 0. \tag{11}$$

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

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If the solution of the system of equations (11) exists, then the found sequence of matrices $\overline{K}(k+1)$ will provide, within the framework of the suboptimal filter structure given by expression (8), the best estimate quality for all realizations of state and observation vectors, in the sense of minimum mean risk (10).

An analytical solution of the problem of parametric optimization, even for conditionally Gaussian processes, encounters considerable difficulties [3,6,11]. The exact solution of the problem of parametric optimization of the suboptimal estimation algorithm (8) can be found in the practically important special case, when the matrices Φ_{xx}, Φ_{yx}, Γ_x and Γ_y do not depend on the observation vector $Y(k)$. In this case, the process $E(k+1)$, defined by equation (9), itself will be Markovian for Gaussian the initial distribution of its distribution will be Gaussian at any other time. The change in the parameters of this distribution in time is described by the difference equations [12,13,16]:

$$M\{E(k+1)\} = [\Phi_{xx}[k+1,k] - \overline{K}(k+1)\Phi_{yx}[k+1,k]]M\{E(k)\}, \tag{12}$$

$$\begin{aligned} P(k+1) &= M\{[E(k+1) - M\{E(k+1)\}][E(k+1) - M\{E(k+1)\}]^T\} = \\ &= \{\Phi_{xx}[k+1,k] - \overline{K}(k+1)\Phi_{yx}[k+1,k]\}P(k)\{\Phi_{xx}[k+1,k] - \\ &- \overline{K}(k+1)\Phi_{yx}[k+1,k]\}^T + \Gamma_x[k+1,k]\Gamma_x^T[k+1,k] + \overline{K}(k+1)\times \\ &\times \Gamma_y[k+1,k]\Gamma_y^T[k+1,k]\overline{K}^T(k+1) - \overline{K}(k+1)\Gamma_y[k+1,k]\Gamma_x^T[k+1,k] - \\ &- \Gamma_x[k+1,k]\Gamma_y^T[k+1,k]\overline{K}^T(k+1). \end{aligned} \tag{13}$$

Since at the initial moment of time an a priori mathematical expectation of the state vector is usually taken for an estimate, the initial value of the mathematical expectation of the estimation error is zero. In this case, according to (12), it will be zero for any other instants of time, i.e. the estimate at the output of the filter (8) will be unbiased regardless of the choice of the matrix $\overline{K}(k+1)$.

The right-hand side of equation (13) is a quadratic function of $\overline{K}(k+1)$. Adding it to a complete square, we rewrite (13) in the form

$$\begin{aligned} P(k+1) &= \{\Phi_{xx}P(k)\Phi_{xx}^T + \Gamma_x\Gamma_x^T - [\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T][\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T]^{-1}\times \\ &\times [\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T]^T\} + \{[\overline{K}(k+1) - (\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T)(\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T)^{-1}]\times \\ &\times [\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T][\overline{K}(k+1) - (\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T)(\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T)^{-1}]^T\}. \end{aligned}$$

Since the mathematical expectation of the estimation error is zero, in this case the average risk can be described by the equation

$$R_G[\overline{K}(k+1)] = tr[BP(k+1)]. \tag{14}$$

Thus, the average risk (14) will be minimal if $\overline{K}(k+1)$ is chosen so that the second term of the resulting equality is zero [4,12,14]. Hence we have

$$\overline{K}(k+1) = (\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T)(\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T)^{-1}, \tag{15}$$

$$\begin{aligned} P(k+1) &= \Phi_{xx}P(k)\Phi_{xx}^T + \Gamma_x\Gamma_x^T + [\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T]\times \\ &\times [\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T]^{-1}[\Phi_{xx}P(k)\Phi_{yx}^T + \Gamma_x\Gamma_y^T]^T\}. \end{aligned} \tag{16}$$

We note that the result obtained does not depend on the matrix B in (14). If the state vector is completely observable, then at least for linear systems it is always possible to estimate the components of the state vector with a finite mean square error and, consequently, the elements of the matrix $P(k+1)$ will remain finite for the coefficients of the state and observation state models that are limited for all time instants [5,15]. This circumstance can be used to select the transmission coefficients of the suboptimal filter (8) without solving the parametric optimization problem.

Thus, in calculating the matrices $\overline{K}(k+1)$ and $P(k+1)$ in expressions (15) and (16), it is necessary to reverse the matrix $A(k) = [\Phi_{yx}P(k)\Phi_{yx}^T + \Gamma_y\Gamma_y^T]$ at each step. The efficiency of the suboptimal estimation algorithm (8) essentially depends on the accuracy of the inversion of this matrix.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

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III. SOLUTION OF THE TASK

In a number of computational problems of linear algebra, it is required to find the pseudoinverse matrix G^+ in terms of the Moore-Penrose relation to the matrix G. An extensive literature is devoted to a discussion of the problems of numerical determination of pseudoinverse matrices [17-20]. One of the most popular and widely used methods for finding a pseudo-inverse matrix is to use recursive algorithms that generalize the recursive method for determining pseudo-solutions of linear algebraic systems [21].

The matrix $A(k)$ in the expressions (15) and (16) is square and positive definite. Taking this into account, we introduce the following notation. Let a_t, $t=1,2,...,n$ – be the rows of the matrix $A(k)$, and let γ_{t+1}, $t=1,2,...,n$ – be a sequence of matrices of dimension $(n\times n)$ satisfying the recurrence equation

$$\gamma_{t+1}=\gamma_t-\gamma_t a_{t+1}^T(a_{t+1}\gamma_t a_{t+1}^T)^+ a_{t+1}\gamma_t,\quad \gamma_0=I, \tag{17}$$

$I=I_{(n\times n)}$ – unit matrix.

Denote by c_t, $t=1,2,...,n$, the rows of the matrix $I-\gamma_n$ and define sequences of matrices X_t, $t=1,2,...,n$, of dimension $(n\times n)$ by means of recurrence equations

$$X_{t+1}=X_t+\gamma_t a_{t+1}^T(a_{t+1}\gamma_t a_{t+1}^T)^+(c_{t+1}-a_{t+1}X_t),\ X_0=0, \tag{18}$$

$0=0_{(n\times n)}$ - null matrix.

The systems of equations (17), (18) have the following solutions:

$$\gamma_n=I-A(k)^+A(k),\quad X_n=A(k)^+(I-\gamma_n).$$

By the property of pseudoinverse matrices [17]

$$I-\gamma_n=(A(k)^T)^+A(k)^T=(A(k)A(k)^+)^T=A(k)A(k)^+.$$

Consequently,

$$X_n=A(k)^+A(k)A(k)^+=A(k)^+.$$

When solving systems of recurrence equations (17), (18), it is necessary to calculate the discontinuous function

$$(a_{t+1}\gamma_t a_{t+1}^T)^+=\begin{cases}1/a_{t+1}\gamma_t a_{t+1}^T, & a_{t+1}\gamma_t a_{t+1}^T\neq 0,\\ 0, & a_{t+1}\gamma_t a_{t+1}^T=0.\end{cases} \tag{19}$$

an error in the calculation of which at a value of $a_{t+1}\gamma_t a_{t+1}^T$ close to zero can greatly distort the results obtained. It was shown in [13,22] that the accuracy of the computations can be monitored from the tracks of the matrix γ_t, $t=1,2,...,n$ and can not be computed if their accuracy does not correspond to the conditionality of the matrix $A(k)$.

The system of recurrent equations (19) is successively solved and the values

$$S_p\frac{\gamma_t a_{t+1}^T a_{t+1}\gamma_t}{a_{t+1}\gamma_t a_{t+1}^T},\quad \sum_{i,j=1}^{n}[\gamma_{ij}^2(t)-\gamma_{ij}^2(t+1)]=\|\gamma_t\|^2-\|\gamma_{t+1}\|^2,$$

which must be equal to one, and also the values of $\rho_t=\left\|\gamma_t-\gamma_t^2\right\|/\left\|\gamma_t^2\right\|$, which must be zero. The difference between these parameters and the indicated ones testifies to the accumulation of errors in the calculation of x_t and γ_t. If it turns out that the line with the number τ leads to large errors in $Sp(\gamma_{\tau-1}a_\tau^T a_\tau\gamma_{\tau-1}/a_{\tau-1}\gamma_\tau a_{\tau-1}^*)$, $\|\gamma_{\tau-1}\|^2-\|\gamma_\tau\|^2$, ρ_τ, then it is natural to declare this line as a linear combination of lines $a_1,...,a_{\tau-1}$, putting $a_\tau\gamma_{\tau-1}a_\tau^T=0$, since in this case $a_\tau\gamma_{\tau-1}a_\tau^T$ is always sufficient small value.

IV. CONCLUSION

The above algorithms make it possible to increase the accuracy and stability of the procedure for suboptimal estimation of the unobserved coordinates of random processes.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

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