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Modeling and Forecasting Volatility of Financial Data using Geometric Brownian Motion

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ABSTRACT: This study presents modeling and forecasting volatility of financial data in Bursa Malaysia using Geometric Brownian Motion. Stock market is the main platform for investors to participate and own some of their shares in a certain company. The changes of share prices on daily basis make the stock market more volatile and very difficult to predict because of economic factors of the country. From the Geometric Brownian Motion simulation, most of the graph will heading toward a direction with some deviation. The simulation is including the random walk from Wiener Process or Brownian Motion, which is the stochastic process for random behavior of share prices in stock market. Hundreds of simulation is done for generating the forecast value and it is selected based on the value of drift rate and volatility rate with the statistical test of forecast accuracy. For the prediction value distribution and interval, the Geometric Brownian Motion generating function is defined to produce the final value of commodity.

KEYWORDS*:* Modelling, Forecasting, Random Walk, Wiener Process, Geometric Brownian Motion.

I. INTRODUCTION

Stock market is the main platform for investors to participate and own some of their shares in a certain company. The investors turn out to be a part of the company members, and thus shares not only profits, but also losses of the company. The changes of share prices on daily basis make the stock market more volatile and very difficult to predict because of economic factors of the country. When investors buy stock market, it does not guarantee any returns, but it makes stocks are really risky in investments and can gain high return. Therefore, the model created from Geometric Brownian Motion will help the investors to give a better view and predict share prices in a short period of time.

The Malaysia Stock Market, Kuala Lumpur Stock Exchange (FTSE KLCI), started to have a downward trend in July 1997 as it fell below its psychological level of 1000 points. According to [1], the contagious and outrages effects of speculative activity on the Thai Bath had led to the crisis in 1997. Based on the economic report 1997/98, in September 1997, the KLCI touched its lowest level since 20th April 1993 of 675.15. In September 1998, the KLCI fell sharply to as low to as low as 262.7 points from 1077.3 points in June 1997 follow the implementation of exchange control on 1st September 1998 based on [1].

According to [2], the dot-com bubble, which a historic speculative bubble was covering roughly 1997–2000 with a climax on March 10 of 2000, it was burst in March 2000 and it had rapidly effected the growth of Malaysia's economy. The KLCI which was recovering from 1997 crash took another hit and went down again. From 2003 to 2007, the index point of KLCI started to rise again and eventually surpassed the 1500 points in the early of 2008. But in 2008, the global recession crisis that began from United States housing bubble is transmitted to Malaysia based on [2] and the KLCI fell below 1000 points again markedly in September 2008.

II. SIGNIFICANCE OF THE SYSTEM

This study emphasizes on the model development based on Geometrics Brownian Motion to investigate the trends and forecast of Malaysian Stock Market. The study of literature survey is presented in section III, Methodology is discussed in section IV, section V represent results and discussion and Conclusion of this study is presented in section VI.

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III. LITERATURE SURVEY

According to [3], Malaysia turns the wheel again to get the momentum in terms of share prices but the growth rate is really volatile. Volatility plays crucial roles in various area of finance, especially in risk management, derivatives pricing and portfolio optimization which include all the important aspects in determining the pattern of the share prices in market. From this, many studies have been conducted to perform a better result in forecasting and determining the shape and pattern of stocks, shares, commodities, unit trust and bonds of a country.

Geometric Brownian Motion is an example of stochastic process that satisfies a stochastic differential equation. According to [4], a French mathematician, Louis Bachelier mentioned in his PhD thesis in 1900 that the stock price dynamics follows Brownian Motion. More than half a century later, Samuelson modified Bachelier model in 1965 and discussed that the return rates, instead of the stock prices, follow Geometric Brownian Motion which can avoid negative value from [5].

Time series of stock prices, interest rate and exchange rate fluctuations are some of the common examples of stochastic processes. According to [6], Geometric Brownian Motion is the most used stochastic process as it is always positive because it follows log-normal distribution. In 1993, Steven Heston proposed a model for the stock price dynamics using Geometric Brownian Motion with stochastic volatility. The model is known as Heston model. This model assumes that the volatility of the asset is not constant but follows a random process. The Geometric Brownian Motion can estimate the long time mean of variance, the rate of relaxation to this mean, the variance noise and the drift parameter in modelling the volatility of financial data.

IV. METHODOLOGY

In this research, the materials will be used is the past data of share prices in FTSE KLCI Bursa Malaysia. The data is taken from Yahoo! Finance website which is from December 1993 until April 2016. The data from January 2016 to April 2016 will be used as validation where it will be compare with the forecast data.

The past data will be used to forecast the data for twelve months which from January 2016 until December 2016. The methods will be used for this research is Geometric Brownian Motion. Geometric Brownian Motion is an example of stochastic process that satisfies a stochastic differential equation. The random behavior of the share prices proved that Geometric Brownian Motion is highly suitable for the short term investment.

A Geometric Brownian Motion model is a continuous-time stochastic process explained by [7], in which the logarithm of the randomly varying quantity follows a Brownian Motion also known as Wiener process. Wiener process or a Brownian Motion process can be defined as the stochastic process $\{X(t), t \geq 0\}$ is called a Wiener process Brownian Motion refers to the limiting process for a random walk as the time steps go to zero indicates as $X(t)$. The properties of Brownian Motion are very important for financial model and it is explained by [7].

The main mathematical model used in this research is Geometric Brownian Motion. Geometric Brownian Motion grows in the stochastic calculus. Stochastic calculus is a branch of mathematic that deals with uncertainty such as in stock market and foreign exchange. According to [7], investor main concern will be the return on investment which is referred to the percentage growth in the value of an asset.

The quantity S_i is the asset value on the i^{th} day and the return from the day to other day $i+1$ which is given by:

$$R_i = \frac{S_{i+1} - S_i}{S_i} \qquad (1)$$

Rate of return can be explained as the rate of profit or loss in investment. The positive value of rate of returns indicates increase of profits, while a negative value means that the investors will face the loss. Higher rate of return value gives higher profit of gaining.

By knowing the rate of return, the mean of distribution of the drift, μ can be calculated as follows, where M is the number of returns in the sample:

$$\mu = \bar{R} = \frac{1}{M} \sum_{i=1}^{M} R_i \qquad (2)$$

For the sample of standard deviation, the volatility, σ, is defined as the fluctuation of the share prices which the price of security moves up and down referring to [8].

Volatility is found by calculating the annualized standard deviation of daily change in price where standard deviation is a statistical measure of dispersion around a central tendency:

$$\sigma = \sqrt{\frac{1}{(M-1)} \sum_{i=1}^{M} (R_i - \bar{R})^2} \qquad (3)$$

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High volatility refers to high share prices rapidly moves up and down over short periods of time. In other words, it refers to the risk of level, since the fluctuation of the prices is unpredictable and uncertain. Investing in stock market is really risky and investor will face either loss or profit after the investment. Therefore, the volatility of the rate of return, or the standard deviation, can be used as the measurement of risk level according to [9]. Higher volatility refers to the higher level of risk.

Integration of derivation from the Geometric Brownian Motion formed a stochastic equation. The integration of the model formed an equation, which is:

$$e^{lnS} = e^{\left(\mu+\frac{1}{2}\right)t+\sigma[X(t)-X(0)]}.e^{c} \tag{4}$$

In the equation above, $e^c = S(0)$, ⍰ is drift, σ is volatility, $X(t)$ is random value and $S(t)$ is the price of stock at time t. The stochastic differential equation for $\log S$ is:

$$S(t) = S(0)e^{\left(\mu+\frac{1}{2}\sigma^2\right)t+\sigma[X(t)-X(0)]} \tag{5}$$

The random value of $X(t) - X(0)$ can be represented as Wiener process, $W(t)$, which is the movement of the random walk for the stochastic process.

$$S(t) = S(0)e^{\left(\mu+\frac{1}{2}\sigma^2\right)t+\sigma[W(t)]} \tag{6}$$

This stochastic differential equation is particularly important in modeling of many asset classes. Equation in above is the asset price model that is able to predict an asset price at specific time t.

The data of the share prices in Bursa Malaysia acquired from 3rd December 1993 to 14th April 2016 will be analyzed daily and weekly. Simulation of random walk, which is Wiener process, is simulated and a range of random number from -1 to 1 is generated to represents the Brownian Motion.

Share prices in the stock market are really volatile and randomly change. The behaviors of this change are simulated through the random walk process. The process will be moving randomly in 10 different simulations.

Hundreds of simulations are done and random walk with the closest behavior to the stock prices are chooses to proceed in the Geometric Brownian Motion model. From the model acquired in (6), $S(t) = S(0)e^{\left(\mu+\frac{1}{2}\sigma^2\right)t+\sigma[W(t)]}$, each of the value is inserted to complete the model for the stock market, which $S(0)$ is the initial stock price, μ is the drift of the share prices, σ is the volatility of the share prices, t is time and $X(t)$ is the random walk that have been simulated.

Sets of forecast data is then produced with the model acquired from Geometric Brownian Motion and hundreds of forecast data is simulated to select the most appropriate simulation from random walk in getting close the behavior of the share prices of Bursa Malaysia. In this simulation, 10 sets of forecast data are generated from hundreds of simulation and are compared with the actual data.

The results gained from the simulation shows a random behavior of share prices in Brownian Motion. It is the closest that can be acquired from the hundreds simulation based on the drift and volatility of each of the sets of forecast simulation compared with the actual drift and volatility. The closest forecast simulation with actual share prices is selected with the most precise value of drift and volatility. The selecting process between all the simulations is to ensure the forecast data are closest in the pattern and trend according to upward and downward movement of the share prices in Bursa Malaysia.

According to [10], there are three measurement of forecasting model which involve time period, t. The measurements are number of period forecast, n, actual value in time period at time, t, Yt and forecast value at time period t, Ft. The widely used to evaluate the forecasting method that considers the effect of the magnitude of the actual values, is the mean absolute percentage error (MAPE).

It can be calculated as follows:

$$MAPE = \frac{\sum\left|\frac{Y_t - F_t}{Y_t}\right|}{n} \times 100 \tag{7}$$

Besides that, the accuracy of financial data can be acquired with the value of mean absolute deviation (MAD). The value calculated from MAD indicates that the forecast is more accurate with the lowest value of MAD. It can be calculated as follows:

$$MAD = \frac{\sum|Y_t - F_t|}{n} \tag{8}$$

Furthermore, the accuracy of financial data also can be determined through the value of root mean square error (RMSE). The value calculated from RMSE indicates that the smaller the value, the better. It can be calculated as follows:

$$RMSE = \sqrt{\frac{\sum(Y_t - F_t)^2}{n}} \tag{9}$$

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V. RESULTS AND DISCUSSIONS

The data of the share prices in Bursa Malaysia acquired from 3rd December 1993 to 14th April 2016 will be analyzed daily, weekly and for the first quarter of 2016. The data is being analyzed by a simulation form the Geometric Brownian Motion model using Microsoft Excel**.** The steps and procedures to carry out the Geometric Brownian Motion method are applied to the data series to fulfill the objective of the research.

Simulation of Random Walk, which is Wiener Process, is simulated and a range of random number from -1 to 1 is generated to represents the Brownian Motion. Share prices in the stock market are really volatile and randomly change. The behaviors of this change are simulated through the Random Walk process.

Hundreds of simulations are done and random walk with the closest behavior to the stock prices are chooses to proceed in the Geometric Brownian Motion model. From the model acquired in (6), $S(t) = S(0)e^{\left(\mu+\frac{1}{2}\sigma^2\right)t+\sigma[X(t)-X(0)]}$, each of the value is inserted to complete the model for the stock market, which $S(0)$ is the initial stock price, μ is the drift of the share prices, σ is the volatility of the share prices, t is the time increase by day and $X(t)$ is the Random Walk that have been simulated.

Sets of forecast data is then produced with the model acquired from Geometric Brownian Motion and hundreds of forecast data is simulated to select the most appropriate simulation from Random Walk in getting close the behavior of the share prices of Bursa Malaysia. In this simulation, 10 sets of forecast data are generated from hundreds of simulation and are compared with the actual data.

The results gained from the simulation shows a random behavior of share prices in Brownian Motion. It is the closest that can be acquired from the hundreds simulation based on the drift and volatility of each of the sets of forecast simulation compared with the actual drift and volatility.

The closest forecast simulation with actual share prices is selected with the most precise value of drift and volatility. The selecting process between all the simulations is to ensure the forecast data are closest in the pattern and trend according to upward and downward movement in the share prices of Bursa Malaysia.

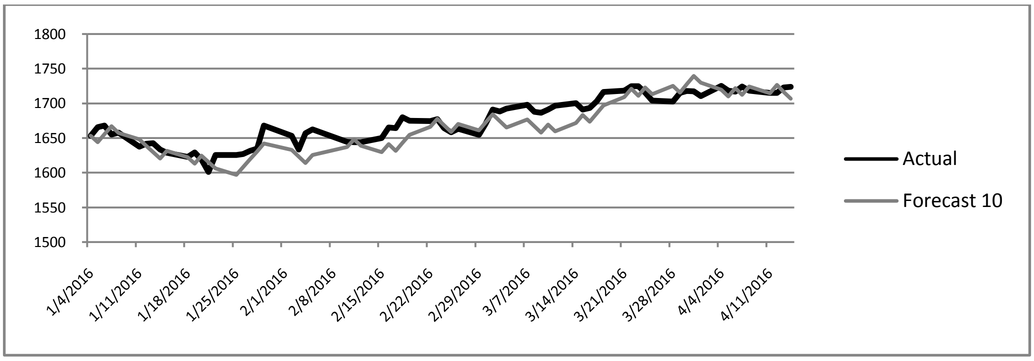

Figure 1: Comparison between the actual share prices and tenth forecast simulation trend for the first quarter of 2016.

Table 1: Comparison between the actual rate and forecast rate for first quarter of 2016.

Rate	μ (Drift Rate)	σ (Volatility Rate)
Actual	0.000616	0.006323
Forecast	0.000475	0.006370

Table 2: Statistical test value of the forecast simulation for first quarter of 2016.

Accuracy Measurement	MAPE	MAD	RMSE
Value	0.83635	14.0137	17.3017

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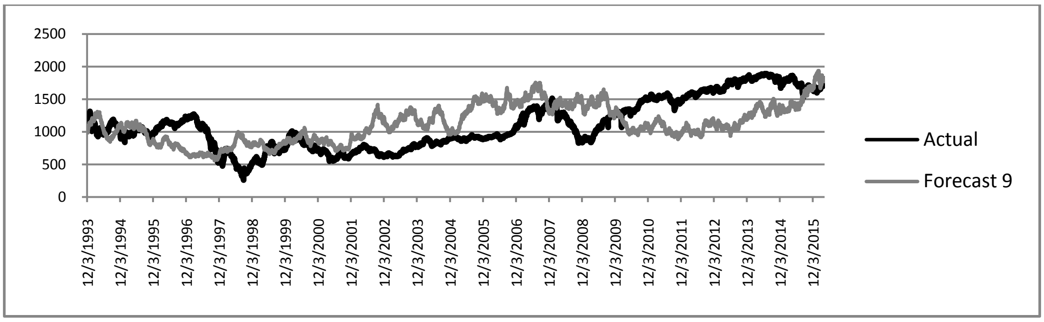

Figure 2: Comparison between the actual share prices and ninth forecast simulation trend for daily data from December 1993 to April 2016.

Table 3: Comparison between the actual and forecast rate for daily data.

Rate	μ (Drift Rate)	σ (Volatility Rate)
Actual	0.000192	0.014042
Forecast	0.000201	0.014047

Table 4: Statistical test value of the forecast simulation for daily data.

Accuracy Measurement	MAPE	MAD	RMSE
Value	13.22685	326.8053	381.1768

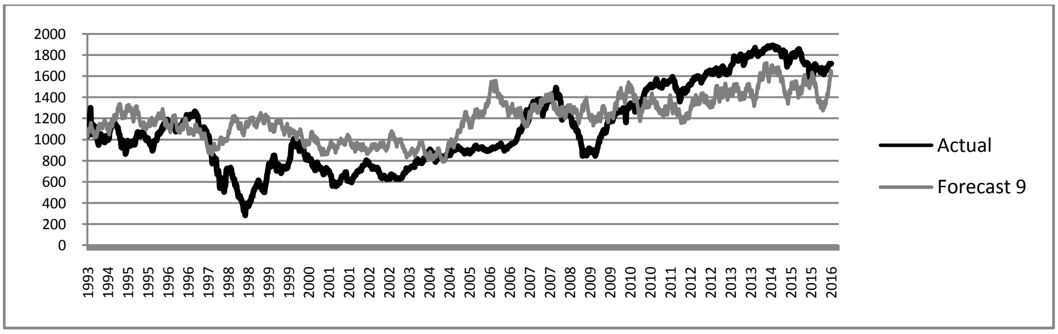

Figure 3: Comparison between the actual share prices and ninth forecast simulation trend for weekly data from December 1993 to April 2016.

Table 5: Comparison between the actual and forecast rate weekly data.

Rate	μ (Drift Rate)	σ (Volatility Rate)
Actual	0.000562	0.022446
Forecast	0.000529	0.022466

Table 6: Statistical test value of the forecast simulation weekly data.

Accuracy Measurement	MAPE	MAD	RMSE
Value	17.1173	231.0986	280.5932

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From the statistical test value of mean absolute percentage error (MAPE), mean absolute deviation (MAD) and root mean square error (RMSE) of the first quarter of 2016, the forecast accuracy of the simulation can be clearly stated that the simulation from Geometric Brownian Motion model is highly accurate forecast compare with the daily and weekly data. Due to the random behavior of the share prices, Geometric Brownian Motion is highly suitable for the short term investment.

The Geometric Brownian Motion model gives a good opportunity to decide new and gain profit after two weeks of investment. Moreover, this model is an accurate model, the calculation of Geometric Brownian Motion is much easier and less data is needed to forecast the future closing prices. Based on the three set of forecast data, the drift rate and volatility rate from the data of first quarter of 2016 is selected based on the value of forecast accuracy MAPE, MAD and RMSE. Hence, the forecast for year is 2016 is done by using the simulation from Geometric Brownian Motion model.

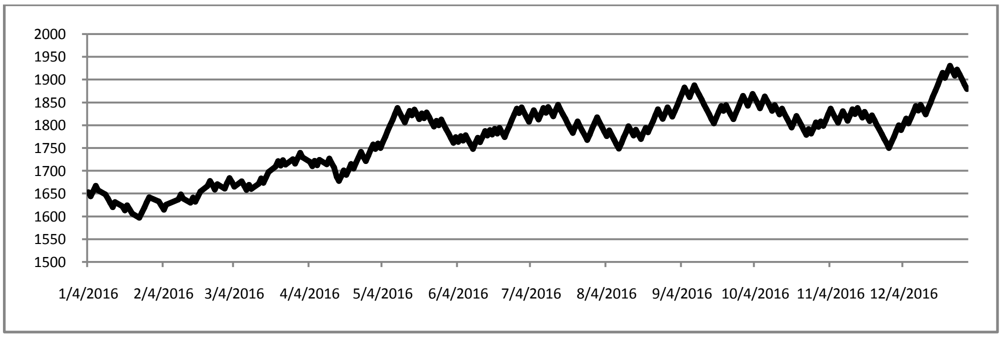

Figure 4: Forecast data from January to December daily for the year 2016.

Due to the random behavior of the share prices, Geometric Brownian Motion is highly suitable for the short term investment. The Geometric Brownian Motion model gives a good opportunity to decide new and gain profit after two weeks of investment. Moreover, this model is an accurate model, the calculation of Geometric Brownian Motion is much easier and less data is needed to forecast the future closing prices.

VI. CONCLUSION

The research conducted is fulfilled the objectives of the research. the two objectives of this research are achieved. The first objective is to acquire a model in forecasting index prices in FTSE KLCI Bursa Malaysia by using Geometric Brownian Motion. This objective had been achieved by getting the model from the derivation of Brownian Motion, Wiener process and random walk.

The second objective is to forecast the financial data using Geometric Brownian Motion to get the most accurate forecast data for index prices in FTSE KLCI Bursa Malaysia. The model acquired from Geometric Brownian Motion is used to forecast the share prices of Bursa Malaysia in year 2016. The result showed that the Geometric Brownian Motion is highly accurate to forecast index prices in short period.

Based on the three set of forecast data, the drift rate and volatility rate from the data of first quarter of 2016 is selected based on the value of forecast accuracy MAPE, MAD and RMSE. Hence, the forecast for year is 2016 is done by using the simulation from Geometric Brownian Motion model.

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