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Algorithm of Solution of Geometrically Nonlinear Problem of Rods with Arbitrary Mechanical Geometrical Characteristics

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ABSTRACT: In this paper, based on Hamilton-Ostrogradsky's variation principle the equations of motion of geometrically nonlinear problem of rods with natural boundary and initial conditions are derived. Based on Finite difference method of the second order of accuracy, a system of equations of motion of the rods is solved. An algorithm for solving the formulated problem is given.

KEYWORDS: Algorithm, Hamilton-Ostrogradsky's variation principle rods, nonlinear problem, arbitrary, geometrically nonlinear

I.INTRODUCTION.

An intensive development of nonlinear theory of elasticity has begun in the 40s of the 20th century. Since then, the scope of nonlinear theory of elasticity is constantly expanding. Not only rubber-like and polymer materials have become an object of study in modern mechanics of solids, but also the tissues of living organisms. The walls of blood vessels, cell membranes, protein molecules are prone to strong deformation and to describe them the considerably nonlinear models are required.

The problem of strong bending of a prismatic beam by end moments is a nonlinear version of one of Saint-Venant problems [1]. The solution of the other nonlinear problem - the problem of torsion was given by L.M. Zubov [2]. Within the framework of linear theory of elasticity the problem of bending of prismatic body was solved by Saint-Venant about 170 years ago. Since then, Saint Venant problem of bending was generalized in different directions. However, these generalizations do not go beyond the small deformations. An exception is the nonlinear plane problem of pure bending of elastic strip; its solution is given in the work by A.I. Lurie [3,4].

At present stage of development of the mechanics of continuous media the interest to the problems of nonlinear theory of elasticity is explained by several reasons. Firstly, in practice, different bodies experience finite deformation, where materials exhibit substantial elastic properties. Their behavior is very different from the one predicted by linear theory. Proper accounting of non-linearity is particularly important in calculating the products made of elastomeric, plastic and other materials. Secondly, a number of phenomena, experimentally observed at certain strains (e.g., torsion) can not be described theoretically, retaining in the solution only linear terms relative to the gradient of displacement.

Thirdly, new materials, and non-linear behavior of known ones require the development of new mathematical models, which adequately describe their properties. Therefore, the solution (in the framework of nonlinear theory of elasticity) of the problems for certain basic experiments (tension, torsion, bending, etc.) using various determinant relationships allows us to check the suitability of the latter, to experimentally determine their characteristics at high strains, as well as to compare the behavior of various materials.

The building of the adequate mathematical models of these materials with full account of the nonlinearity should be based primarily on modeling of classic experiments, and, consequently, on the solution of fundamental problems of the theory of elasticity, describing a simple deformation of bodies (tension, torsion, bending, etc.). At the same time the solution of boundary value problems of the nonlinear theory of elasticity in most cases is difficult because the elastic potentials used present quite complex expressions, reducing to essentially nonlinear equations; their solution can't be found in analytical form.

At the same time, the majority of the studied nonlinear elastic potentials present a rather bulky expressions, which make analytical derivation of the boundary value problem of equilibrium even in the cases of simple loading an extremely time-consuming one and not always reliable. In addition, the change of the specific potential energy function often leads to the need to derive all the equations afresh. However, the process of derivation of boundary value problems of equilibrium is strictly algorithmic.

The issues of development in the field of algorithmization of the theory of calculation and automation of solutions of problems of nonlinear elastic elements of structures were studied by V.K. Kabulov [5-7], A.V. Tolok [8] T. Buriev [9], K.Sh. Babamuratov [10], F.B. Badalov [11] B. Kurmanbaev [12], T. Yuldashev [13], Sh.A. Nazirov [14-18] and their followers.

As is known, the calculation of thin-walled rods is much more complicated than solid ones. Thin-walled structure best suit to the requirements of economic feasibility, with adequate strength and rigidity. This explains their wide use in various fields of mechanical engineering, construction, aviation, etc.

A unified theory of thin-walled rods was proposed in the works of V.Z.Vlasov [19], G.Yu.Dzhanelidze [20] and V.K. Kabulov [5]. At present practice requirements lead to the need to study the deformation of the elements considering their geometric nonlinearity.

Applied theory of rods vibrations is built on the basis of a number of static and kinematic hypotheses relative to the law of distribution of displacements, strains and stresses in the sections of the bar.

Displacements of the rod points under joint longitudinal, transverse and torsional vibrations are represented in the form [5]:

$$\left.\begin{aligned} &u_1(x,y,z,t) = u - \frac{\partial v}{\partial x} z - \frac{\partial w}{\partial x} y + \varphi(y,z)\frac{\partial \theta}{\partial x} + \psi_1(z)\beta_1 + \psi_2(y)\beta_2, \\ &u_2(x,y,z,t) = v - z\theta, \qquad u_3(x,y,z,t) = w + y\theta \end{aligned}\right\}$$

Assuming that $\partial v/\partial x = \alpha_1 + \beta_1,\ \partial w/\partial x = \alpha_2 + \beta_2,$

where θ–is an angle of torsion, u, v, w–displacements of a middle line of the rod, α_1, α_2– the angles of sections rotation under pure bending, β_1, β_2– the angles of transverse shear, u_1, u_2, u_3–the components of displacement vectors, *x,y,z*–spatial variables, $\varphi(y,z)$– Saint-Venant function of torsion, defined from

$$\nabla^2\varphi = 0, \qquad \partial\varphi/\partial n = ly - mz.$$

The first relation is presented in the form:

$$u_1 = u - z\alpha_1 - y\alpha_2 + \varphi(x,y,z)\vartheta + a_1(x,y,z)\beta_1 + a_2(x,y,z)\beta_2$$

where $a_1=\psi_1(z)-z$, $a_2=\psi_2(y)-y$, $\vartheta \neq \partial\theta/\partial x$–is a linear torsion.

When building an applied theory, the transition from the study of vibrations of three-dimensional body to one-dimensional body has a crucial importance. A complete solution of this problem can be derived from a discrete-continuum method developed V.Z.Vlasov, G.Yu.Dzhanelidze and V.K.Kabulov.

Task Definition. Based on Vlasov-Dzhanelidze-Kabulov refined theory the displacements of the rod are taken in the form [5]:

$$\left.\begin{aligned} &u_1(x,y,z) = u - z\alpha_1 - y\alpha_2 + \varphi(x,y,z)\vartheta + a_1(x,y,z)\beta_1 + a_2(x,y,z)\beta_2 \\ &u_2(x,y,z) = v + z\theta, \qquad u_3(x,y,z) = w - y\theta. \end{aligned}\right\} \tag{1}$$

Here the sought for functions are reduced to twelve ($u, v, w, \varphi, \vartheta, \theta, \alpha_1, \alpha_2, \beta_1, \beta_2, a_1 .u\ a_2$) and an external loading is not restricted; the functions $u, v, w, \vartheta, \theta, \alpha_1, \alpha_2, \beta_1, \beta_2$ are the functions along spatial variables x and t.

The theory of rods can be generalized in two directions. First of all the coordinate functions can be regarded as unknown, and to determine them based on Hamilton-Ostrogradski principle it is necessary to derive the corresponding differential equation. Such a theory is conventionally called a "one-dimensional" one.

Another way of generalizing the vibrations of rods leads to the solution of a mathematical problem of the theory of elasticity with strict regard to the boundary conditions.

In a particular case, consider the vibrations of the rod form:

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$$u_1(x,y,z,t) = u - z\alpha, \quad u_3(x,y,z,t) = w.\} \tag{2}$$

where u, w –are the displacements of a middle line of the rod, α–an angle of section rotation under pure bending, u_1, u_3 –the components of displacements vector. Here the sought for functions u, w, α are the functions along the spatial variables x and t, and an external loading is not restricted.

In general form Hamilton- Ostrogradsky's variation principle is written [5,14-18,21-25]:

$$\int_t (\delta K - \delta \Pi + \delta A) dt = 0; \tag{3}$$

where K, П–are kinetic and potential energy; A–a work of internal volume and surface forces.

Determination of the variation of kinetic energy. In calculation of the variation of kinetic energy the following relation is used

$$\int_t \delta K dt = \int_t \int_v \left(\rho \frac{\partial u_1}{\partial t} \delta \frac{\partial u_1}{\partial t} + \rho \frac{\partial u_3}{\partial t} \delta \frac{\partial u_3}{\partial t} \right) dv dt;$$

here ρ–is a specific mass density of the material of a body (assumed to be constant).

The operation of integration is conducted by parts

$$\int_t \delta K dt = \int_v \left(\rho \frac{\partial u_1}{\partial t} \delta u_1 + \rho \frac{\partial u_3}{\partial t} \delta u_3 \right) dv \bigg|_t - \int_t \int_v \left[\frac{\partial}{\partial t} \left(\rho \frac{\partial u_1}{\partial t} \right) \delta u_1 + \frac{\partial}{\partial t} \left(\rho \frac{\partial u_3}{\partial t} \right) \delta u_3 \right] dv dt. \tag{4}$$

Substituting the expressions u_1, u_3 from (2) in the variation of kinetic energy (4) and opening brackets under the sign of the variation after the integration operation on the cross sections of the rod, and introducing the designations, one obtains [14–18,21-25]:

$$\int_t \delta K dt = \int_x \rho \left\{ \left[F \frac{\partial u}{\partial t} - S_y \frac{\partial \alpha}{\partial t} \right] \delta u - \left[S_y \frac{\partial u}{\partial t} - J_y \frac{\partial \alpha}{\partial t} \right] \delta \alpha_1 + F \frac{\partial w}{\partial t} \delta w \right\} dx \bigg|_t +$$

$$+ \int_t \int_x \rho \left\{ \left[- F \frac{\partial^2 u}{\partial t^2} + S_y \frac{\partial^2 \alpha}{\partial t^2} \right] \delta u + \left[S_y \frac{\partial^2 u}{\partial t^2} - J_y \frac{\partial^2 \alpha}{\partial t^2} \right] \delta \alpha_1 - \left[F \frac{\partial^2 w}{\partial t^2} \right] \delta w \right\} dx dt, \tag{5}$$

where $F = \int_y \int_z dzdy; \; S_y = \int_y \int_z zdzdy; \; J_y = \int_y \int_z z^2 dzdy.$

A. Determination of the variation of potential energy.

For the variation of potential energy one obtains:

$$\int_t \delta \Pi dt = \int_t \int_v (\sigma_{11} \delta \varepsilon_{11} + \sigma_{13} \delta \varepsilon_{13}) dv dt. \tag{6}$$

Form the Cauchy relations [5,21-22,26]:

$$\varepsilon_{11} = \gamma_{11} = \frac{\partial u_1}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u_1}{\partial x} \right)^2 + \left(\frac{\partial u_3}{\partial x} \right)^2 \right]; \; \varepsilon_{13} = \varepsilon_{31} = 2\gamma_{13} = \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} + \frac{\partial u_1}{\partial x} \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} \frac{\partial u_3}{\partial z}. \tag{7}$$

Stress components are taken as [5, 21-22, 26]:

$$\sigma_{11} = \frac{E}{1+\mu} \left[\varepsilon_{11} + \frac{\mu}{1-2\mu} e \right]; \sigma_{13} = \frac{E}{2(1+\mu)} \varepsilon_{13}; \; e = \varepsilon_{11} + \varepsilon_{33}; \sigma_{11} = \frac{E(1-\mu)}{(1-2\mu)(1+\mu)} \varepsilon_{11}, \tag{8}$$

where E - is elasticity modulus, G - shear modulus, μ - Poisson's ratio.

According to Cauchy relation (7) and considering that $\partial u_3 / \partial z = 0$, the variations of potential energy (6) is presented in the form

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$$\int_t \delta\Pi dt = \int_t \int_v \left[\sigma_{11}\left(\delta\frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial x}\delta\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x}\delta\frac{\partial u_3}{\partial x} \right) + + \sigma_{13}\left(\delta\frac{\partial u_1}{\partial z} + \delta\frac{\partial u_3}{\partial x} + \frac{\partial u_1}{\partial x}\delta\frac{\partial u_1}{\partial z} + \frac{\partial u_1}{\partial z}\delta\frac{\partial u_1}{\partial x} \right) \right] dv dt. \quad (9)$$

In (9) similar items are given:

$$\int_t \delta\Pi dt = \int_t \int_v \left[\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right)\delta\frac{\partial u_1}{\partial x} + \left(\sigma_{13} + \frac{\partial u_1}{\partial x}\sigma_{13} \right)\delta\frac{\partial u_1}{\partial z} + \left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right)\delta\frac{\partial u_3}{\partial x} \right] dv dt \;.(10)$$

Substituting the expressions of displacements u_1 and u_3 from (2) under the sign of the variation in the variation of potential energy (10), one obtains

$$\int_t \delta\Pi dt = \int_t \int_v \left[\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right)\delta\frac{\partial}{\partial x}(u - z\alpha) - \left(\sigma_{13} + \frac{\partial u_1}{\partial x}\sigma_{13} \right)\delta\alpha + \left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right)\delta\frac{\partial w}{\partial x} \right] dv\, dt.$$

Here similar items are given:

$$\int_t \delta\Pi dt = \int_t \int_v \left[\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right)\delta\frac{\partial u}{\partial x} - \left(\sigma_{11}z + \frac{\partial u_1}{\partial x}\sigma_{11}z + \frac{\partial u_1}{\partial z}\sigma_{13}z \right)\delta\frac{\partial \alpha}{\partial x} + + \left(-\sigma_{13} - \frac{\partial u_1}{\partial x}\sigma_{13} \right)\delta\alpha + \left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right)\delta\frac{\partial w}{\partial x} \right] dv dt \quad (11)$$

The operation of integration is conducted by parts. Then the variation of potential energy (11) has the form

$$\int_t \delta\Pi dt = \left\{ \int_y \int_z \left[\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right)\delta u - \left(z\sigma_{11} + \frac{\partial u_1}{\partial x}z\sigma_{11} + \frac{\partial u_1}{\partial z}z\sigma_{13} \right)\delta\alpha + + \left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right)\delta w \right] dzdy \Big|_x - \int_v \left[\frac{\partial}{\partial x}\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right)\delta u - \left[\frac{\partial}{\partial x}\left(z\sigma_{11} + \frac{\partial u_1}{\partial x}z\sigma_{11} + + \frac{\partial u_1}{\partial z}z\sigma_{13} \right) + \left(-\sigma_{13} - \frac{\partial u_1}{\partial x}\sigma_{13} \right) \right]\delta\alpha + \frac{\partial}{\partial x}\left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right)\delta w \right] dv \right\} dt. \quad (12)$$

In (12) the integral in the section of the rod is marked out and one obtains

$$\int_t \delta\Pi dt = \int_t \left\{ \int_y \int_z \left[\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right)\delta u - \left(z\sigma_{11} + \frac{\partial u_1}{\partial x}z\sigma_{11} + \frac{\partial u_1}{\partial z}z\sigma_{13} \right)\delta\alpha + + \left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right)\delta w \right] dzdy \Big|_x - \left\{ \int_x \left[\int_y \int_z \left[\frac{\partial}{\partial x}\left(\sigma_{11} + \frac{\partial u_1}{\partial x}\sigma_{11} + \frac{\partial u_1}{\partial z}\sigma_{13} \right) \right] dzdy \right]\delta u - - \left[\int_y \int_z \left[\frac{\partial}{\partial x}\left(\sigma_{11}z + \frac{\partial u_1}{\partial x}\sigma_{11}z + \frac{\partial u_1}{\partial z}\sigma_{13}z \right) - \left(\sigma_{13} + \frac{\partial u_1}{\partial x}\sigma_{13} \right) \right] dzdy \right]\delta\alpha + + \left[\int_y \int_z \left[\frac{\partial}{\partial x}\left(\sigma_{13} + \frac{\partial u_3}{\partial x}\sigma_{11} \right) \right] dzdy \right]\delta w \right\} dx \right\} dt. \quad (13)$$

In (13) the expressions are calculated and introducing the designations one obtains

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$$\int_y\int_z \sigma_{11}\frac{\partial u_1}{\partial x}dzdy=\int_y\int_z \sigma_{11}\left(\frac{\partial u}{\partial x}-z\frac{\partial \alpha}{\partial x}\right)dzdy=\int_y\int_z \left(\sigma_{11}\frac{\partial u}{\partial x}-z\sigma_{11}\frac{\partial \alpha}{\partial x}\right)dzdy=N_x\frac{\partial u}{\partial x}-M_y\frac{\partial \alpha}{\partial x};$$

$$\int_y\int_z \sigma_{13}\frac{\partial u_1}{\partial z}dzdy=\int_y\int_z \sigma_{13}(-\alpha)dzdy=\int_y\int_z (-\sigma_{13}\alpha)dzdy=-Q_3\alpha;$$

$$\int_y\int_z \sigma_{13}\frac{\partial u_1}{\partial x}dzdy=\int_y\int_z \sigma_{13}\left(\frac{\partial u}{\partial x}-z\frac{\partial \alpha}{\partial x}\right)dzdy=\int_y\int_z \left(\sigma_{13}\frac{\partial u}{\partial x}-z\sigma_{13}\frac{\partial \alpha}{\partial x}\right)dzdy=Q_3\frac{\partial u}{\partial x}-M_{13}(\sigma_{13}z)\frac{\partial \alpha}{\partial x};$$

$$\int_y\int_z \sigma_{11}z\frac{\partial u_1}{\partial x}dzdy=\int_y\int_z \left(\sigma_{11}z\frac{\partial u}{\partial x}-z^2\sigma_{11}\frac{\partial \alpha}{\partial x}\right)dzdy=M_y\frac{\partial u}{\partial x}-M_{11}\left(z^2\sigma_{11}\right)\frac{\partial \alpha}{\partial x};$$

$$\int_y\int_z z\sigma_{13}\frac{\partial u_1}{\partial z}dzdy=\int_y\int_z z\sigma_{13}(-\alpha)dzdy=\int_y\int_z (-z\sigma_{13}\alpha)dzdy=-M_{13}(z\sigma_{13})\alpha;$$

$$\int_y\int_z \sigma_{13}z\frac{\partial u_1}{\partial x}dzdy=\int_y\int_z \left(\sigma_{13}z\frac{\partial u}{\partial x}-z^2\sigma_{13}\frac{\partial \alpha}{\partial x}\right)dzdy=M_{13}(z\sigma_{13})\frac{\partial u}{\partial x}-M_{13}(\sigma_{13}z^2)\frac{\partial \alpha}{\partial x};$$

$$\int_y\int_z \sigma_{11}\frac{\partial u_3}{\partial x}dzdy=\int_y\int_z \sigma_{11}\left(\frac{\partial w}{\partial x}\right)dzdy=\int_y\int_z \left(\sigma_{11}\frac{\partial w}{\partial x}\right)dzdy=N_x\frac{\partial w}{\partial x};$$

$$\int_y\int_z \sigma_{13}\frac{\partial u_3}{\partial x}dzdy=\int_y\int_z \sigma_{13}\left(\frac{\partial w}{\partial x}\right)dzdy=\int_y\int_z \sigma_{13}\frac{\partial w}{\partial x}dzdy=Q_3\frac{\partial w}{\partial x}, \qquad (14)$$

where

$$N_x=\int_y\int_z\sigma_{11}dzdy;\; M_y=\int_y\int_z\sigma_{11}\cdot zdzdy;\quad Q_3=\int_y\int_z\sigma_{13}dzdy;\quad M_{11}\left(z^2\sigma_{11}\right)= \;=\int_y\int_z\sigma_{11}z^2dzdy;$$

$$M_{13}(z\sigma_{13})=\int_y\int_z(z\sigma_{13})dzdy;\; M_{13}(\sigma_{13}z^2)=\int_y\int_z(\sigma_{13}z^2)dzdy.$$

From (13) an expression (14) is introduced:

$$\int_t\delta\Pi dt=\int_t\left\{\left[N_x+\frac{\partial u}{\partial x}N_x-\frac{\partial \alpha}{\partial x}M_y-Q_3\alpha\right]\delta u-\left[M_y+\frac{\partial u}{\partial x}M_y-\frac{\partial \alpha}{\partial x}M_{11}\left(z^2\sigma_{11}\right)-\alpha M_{13}(z\sigma_{13})\right]\delta\alpha+\right.$$
$$+\left[Q_3+\frac{\partial w}{\partial x}N_x\right]\delta w\Big\}\Big|_x-\left\{\int_x\left[\frac{\partial N_x}{\partial x}+\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}N_x-\frac{\partial \alpha}{\partial x}M_y-\alpha Q_3\right)\right]\delta u+\right.$$
$$+\left[\frac{\partial M_y}{\partial x}+\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}M_y-\frac{\partial \alpha}{\partial x}M_{11}\left(z^2\sigma_{11}\right)-\alpha M_{13}(z\sigma_{13})\right)+\right.$$
$$\left.-\left(Q_3-\frac{\partial u}{\partial x}Q_3-\frac{\partial \alpha}{\partial x}M_{13}(\sigma_{13}z)\right)\right]\delta\alpha+\left[\frac{\partial Q_3}{\partial x}+\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}N_x\right)\right]\delta w\Big\}dx\Big\}dt. \qquad (15)$$

For nonlinear parts of the variation of potential energy the following designations are introduced:

$$R_1=\int_F\left(\frac{\partial u_1}{\partial x}\sigma_{11}-\alpha\sigma_{13}\right)dF;\; R_2=\int_F\frac{\partial w}{\partial x}\sigma_{11}dF;\; R_3=\int_F\left(\frac{\partial u_1}{\partial x}z\sigma_{11}-\alpha z\sigma_{13}\right)dF;\; R_4=\int_F\frac{\partial u_1}{\partial x}\sigma_{13}dF.$$

The variations of potential energy with introduced designations have the form:

$$\int_t\delta\Pi dt=\int_t\left\{[N_x+R_1]\delta u+[Q_3+R_2]\delta w+[M_y+R_3]\delta\alpha-\right.$$

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$$-\int_x \left\{\left[\frac{\partial N_x}{\partial x}+\frac{\partial R_1}{\partial x}\right]\delta u+\left[\frac{\partial Q_3}{\partial x}+\frac{\partial R_2}{\partial x}\right]\delta w-\left[\frac{\partial M_y}{\partial x}+\frac{\partial R_3}{\partial x}+R_4\right]\delta w\right\}dx\Big\}dt \tag{16}$$

Determination of the variation of work of external forces. Consider the variations of the work of external forces:

$$\delta\int_t Adt=\int_v (F_1\delta u_1+F_3\delta u_3)dv+\int_s (q_1\delta u_1+q_3\delta u_3)ds+\int_{s_1}(\varphi_1\delta u_1+\varphi_3\delta u_3)ds_1\big|_x; \tag{17}$$

here F_1, F_3 - are the components of volume forces, per a volume unit, q_1, q_3 – surface forces, respectively, per an area unit of a rod surface, φ_1,φ_3 – boundary stresses.

In the variations of the work of external forces (16) the expressions of displacements u_1 and u_3 are introduced from (2):

$$\int_t \delta Adt=\int_t\int_v \{F_1\delta(u-z\alpha)+F_3\delta w\}dvdt+\int_t\int_s \{q_1\delta(u-z\alpha)+q_3\delta w\}dsdt+ \\ +\int_t\int_{s_1}(\varphi_1\delta(u-z\alpha)+\varphi_3\delta w)ds_1dt\big|_x. \tag{18}$$

Opening brackets and marking out the integral in the section of the rod, the variation and the work of external forces (18) have the form:

$$\delta\int_t Adt=\int_t\left\{\int_x\left[\int_y\int_z (F_1\delta(u-z\alpha)+F_3\delta w)dzdy+\int_l (q_1\delta(u-z\alpha)+q_3\delta w)dl\right]dx+\right. \\ \left.+\int_{s_1}(\varphi_1\delta(u-z\alpha)+\varphi_3\delta w)ds_1\big|_x\right\}dt=\int_t\left\{\int_x[(f_1+\overline{q}_1)\delta u-(M(f_1)+M(q_1))\delta\alpha+\right. \\ \left.+(f_3+\overline{q}_3)\delta w]dx+[\overline{\varphi}_1\delta u+M(\varphi_1)\delta\alpha+\overline{\varphi}_3\delta w]_x\right\}dt. \tag{19}$$

where

$$\overline{\varphi}_1=\int_y\int_z \varphi_1 dzdy;\quad \overline{\varphi}_3=\int_y\int_z \varphi_3 dzdy;\quad M(\varphi_1)=\int_y\int_z \varphi_1\cdot zdzdy;$$
$$f_1=\int_y\int_z f_1 dzdy;\quad f_3=\int_y\int_z f_3 dzdy;\quad M(f_1)=\int_y\int_z F_1\cdot zdzdy;$$
$$\overline{q}_1=\int_y\int_z \varphi_1 dzdy;\quad \overline{q}_3=\int_y\int_z \varphi_3 dzdy;\quad M(q_1)=\int_y\int_z q_1\cdot zdzdy.$$

B. Derivation of determinant equations of spatially loaded rods.

Results obtained for the variation of kinetic energy (5), potential energy (16) and the work of external forces (19) are introduced into Hamilton-Ostrogradsky's variation principle (3) and derive the system of differential equations with corresponding initial and boundary value conditions.

$$-\rho F\frac{\partial^2 u}{\partial t^2}+\rho S_y\frac{\partial^2\alpha}{\partial t^2}+\frac{\partial N_x}{\partial x}+\frac{\partial R_1}{\partial x}+\left(f_1+\overline{q}_1\right)=0; \\ -\rho F\frac{\partial^2 w}{\partial t^2}+\frac{\partial Q_3}{\partial x}+\frac{\partial R_2}{\partial x}+\left(f_3+\overline{q}_3\right)=0; \\ \rho S_y\frac{\partial^2 u}{\partial t^2}-\rho I_y\frac{\partial^2\alpha}{\partial t^2}+\frac{\partial M_y}{\partial x}+\frac{\partial R_3}{\partial x}+R_4-\left(M(f_1)+M(\overline{q}_1)\right)=0. \tag{20}$$

Initial conditions:

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$$\left[\rho F\frac{\partial u}{\partial t}-\rho S_y\frac{\partial \alpha}{\partial t}\right]\delta u\Big|_t=0;\ \left[\rho F\frac{\partial w}{\partial t}\right]\delta w\Big|_t=0;\ \left[-\rho S_y\frac{\partial u}{\partial t}+\rho I_y\frac{\partial \alpha}{\partial t}\right]\delta\alpha\Big|_t=0. \qquad (21)$$

Boundary conditions:

$$\left[-N_x-R_1+\overline{\varphi}_1\right]\delta u\big|_x=0;\ \left[-Q_3-R_2+\overline{\varphi}_3\right]\delta w\big|_x=0;\ \left[M_y+R_3+M(\varphi_1)\right]\delta\alpha\big|_x=0. \qquad (22)$$

Based on Hooke's law the expression N_x, Q_3, M_y in displacements is obtained from the relationship (2)

$$\sigma_{11}=Ee_{11}=E\frac{\partial u_1}{\partial x}=E\frac{\partial}{\partial x}(u-z\alpha)=E\frac{\partial u}{\partial x}-Ez\frac{\partial \alpha}{\partial x};$$

$$\sigma_{13}=Ge_{13}=\frac{E}{2(1+\mu)}\left(\frac{\partial u_1}{\partial z}+\frac{\partial u_3}{\partial x}\right)=\frac{E}{2(1+\mu)}\left((-\alpha)+\frac{\partial w}{\partial x}\right);$$

$$N_x=\int_y\int_z\sigma_{11}dzdy=\int_y\int_z Ee_{11}dzdy=E\int_y\int_z\frac{\partial u_1}{\partial x}dzdy=E\int_y\int_z\frac{\partial}{\partial x}(u-z\alpha)dzdy=$$

$$=E\int_y\int_z\left(\frac{\partial u}{\partial x}-z\frac{\partial \alpha}{\partial x}\right)dzdy=E\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right);\qquad N_x=E\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right);$$

$$Q_3=\int_y\int_z\sigma_{13}dzdy=\int_y\int_z Ge_{13}dzdy=G\int_y\int_z\left(\frac{\partial u_1}{\partial z}+\frac{\partial u_3}{\partial x}\right)dzdy=G\int_y\int_z\left(-\alpha+\frac{\partial w}{\partial x}\right)dzdy;$$

$$Q_3=-GF\alpha+GF\frac{\partial w}{\partial x}=GF\left(-\alpha+\frac{\partial w}{\partial x}\right).$$

$$M_y=\int_y\int_z\sigma_{11}\cdot zdzdy=\int_y\int_z Eze_{11}dzdy=E\int_y\int_z z\frac{\partial u_1}{\partial x}dzdy=E\int_y\int_z z\frac{\partial}{\partial x}(u-z\alpha)dzdy=$$

$$=E\int_y\int_z\left(z\frac{\partial u}{\partial x}-z^2\frac{\partial \alpha}{\partial x}\right)dzdy=E\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial \alpha}{\partial x}\right);\ M_y=E\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial \alpha}{\partial x}\right).$$

Now define the nonlinear part of the equations

$$R_1=\int_F\left(\frac{\partial u_1}{\partial x}\sigma_{11}-\alpha\sigma_{13}\right)dF=\int_F\left\{\frac{\partial u_1}{\partial x}\left[E\frac{\partial u_1}{\partial x}\right]-\alpha\frac{E}{2(1+\mu)}\left[\frac{\partial u_1}{\partial z}+\frac{\partial u_3}{\partial x}\right]\right\}dF;$$

$$R_1=\int_F\left\{\frac{\partial u_1}{\partial x}\left[E\frac{\partial}{\partial x}(u-z\alpha)\right]-\alpha\left[\frac{E}{2(1+\mu)}\left(-\alpha+\frac{\partial w}{\partial x}\right)\right]\right\}dF;$$

$$R_1=\int_F\left\{E\frac{\partial}{\partial x}(u-z\alpha)\left[\frac{\partial u}{\partial x}-z\frac{\partial \alpha}{\partial x}\right]dF\right\}-\frac{E}{2(1+\mu)}\left\{\int_F\left[\alpha\left(-\alpha+\frac{\partial w}{\partial x}\right)\right]dF\right\};$$

$$R_1=\int_F\left\{E\left[\left(\frac{\partial u}{\partial x}\right)^2-2z\frac{\partial \alpha}{\partial x}\frac{\partial u}{\partial x}+z^2\left(\frac{\partial \alpha}{\partial x}\right)^2\right]dF-\frac{E}{2(1+\mu)}\int_F\left[-\alpha^2+\alpha\frac{\partial w}{\partial x}\right]dF\right\};$$

$$R_1=E\left\{F\left[\left(\frac{\partial u}{\partial x}\right)^2-2S_y\frac{\partial \alpha}{\partial x}\frac{\partial u}{\partial x}+I_y\left(\frac{\partial \alpha}{\partial x}\right)^2\right]-\frac{1}{2(1+\mu)}\left[-F\left(\alpha^2+\alpha\frac{\partial w}{\partial x}\right)\right]\right\};$$

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$$R_1 = E\left\{\frac{\partial u}{\partial x}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right) + \frac{\partial \alpha}{\partial x}\left[-S_y\frac{\partial u}{\partial x} + I_y\frac{\partial \alpha}{\partial x}\right] - \frac{\alpha}{2(1+\mu)}\left[-F\alpha + F\frac{\partial w}{\partial x}\right]\right\};$$

$$\frac{\partial R_1}{\partial x} = \frac{\partial}{\partial x}\left\{E\left[\frac{\partial u}{\partial x}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right) + \frac{\partial \alpha}{\partial x}\left(-S_y\frac{\partial u}{\partial x} + I_y\frac{\partial \alpha}{\partial x}\right) - \frac{\alpha}{2(1+\mu)}\left(-F\alpha + F\frac{\partial w}{\partial x}\right)\right]\right\} =$$

$$= E\left\{\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left[\frac{\partial \alpha}{\partial x}\left(-S_y\frac{\partial u}{\partial x} + I_y\frac{\partial \alpha}{\partial x}\right)\right] - \frac{\partial}{\partial x}\left[\frac{\alpha}{2(1+\mu)}\left(-F\alpha + F\frac{\partial w}{\partial x}\right)\right]\right\} =$$

$$= E\left\{\left[\frac{\partial^2 u}{\partial x^2}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right)\right] + \left[\frac{\partial u}{\partial x}\left(F\frac{\partial^2 u}{\partial x^2} - S_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right] + \left[\frac{\partial^2 \alpha}{\partial x^2}\left(-S_y\frac{\partial u}{\partial x} + I_y\frac{\partial \alpha}{\partial x}\right)\right] + \right.$$

$$\left. + \left[\frac{\partial \alpha}{\partial x}\left(-S_y\frac{\partial^2 u}{\partial x^2} + J_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right] - \frac{1}{2(1+\mu)}\left[\frac{\partial \alpha}{\partial x}\left(-F\alpha + F\frac{\partial w}{\partial x}\right) + \alpha\left(-F\frac{\partial \alpha}{\partial x} + F\frac{\partial^2 w}{\partial x^2}\right)\right]\right\};$$

$$R_2 = \int_F \frac{\partial w}{\partial x}\sigma_{11} dF = \int_F \frac{\partial w}{\partial x}\left(E\frac{\partial u}{\partial x} - Ez\frac{\partial \alpha}{\partial x}\right)dF = E\left(F\frac{\partial w}{\partial x}\frac{\partial u}{\partial x} - S_y\frac{\partial w}{\partial x}\frac{\partial \alpha}{\partial x}\right) = E\left[\frac{\partial w}{\partial x}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right)\right];$$

$$\frac{\partial R_2}{\partial x} = \frac{\partial}{\partial x}\left[E\frac{\partial w}{\partial x}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right)\right] = E\left\{\frac{\partial}{\partial x}\left[\frac{\partial w}{\partial x}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right)\right]\right\} =$$

$$= E\left\{\frac{\partial^2 w}{\partial x^2}\left(F\frac{\partial u}{\partial x} - S_y\frac{\partial \alpha}{\partial x}\right) + \frac{\partial w}{\partial x}\left(F\frac{\partial^2 u}{\partial x^2} - S_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right\};$$

$$R_3 = \int_F \left(\frac{\partial u_1}{\partial x}z\sigma_{11} - \alpha z\sigma_{13}\right)dF = \int_F \left\{\frac{\partial u_1}{\partial x}z\left[E\frac{\partial u_1}{\partial x}\right] - \alpha\left[z\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right)\right]\right\}dF;$$

$$R_3 = \int_F \left[\frac{\partial}{\partial x}(u - z\alpha)\cdot z\left[E\left(\frac{\partial u}{\partial x} - z\frac{\partial \alpha}{\partial x}\right)\right]\right]dF - \int_F \alpha\cdot z\frac{E}{2(1+\mu)}\left(-\alpha + \frac{\partial w}{\partial x}\right)dF;$$

$$R_3 = E\int_F z\left[\left(\frac{\partial u}{\partial x}\right)^2 - 2z\frac{\partial u}{\partial x}\frac{\partial \alpha}{\partial x} + z^2\left(\frac{\partial \alpha}{\partial x}\right)^2\right]dF - \int_F \alpha\cdot z\frac{E}{2(1+\mu)}\left(-\alpha + \frac{\partial w}{\partial x}\right)dF;$$

$$R_3 = E\int_F z\left[\frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial x} - z\frac{\partial \alpha}{\partial x}\right) + \frac{\partial \alpha}{\partial x}\left(-z\frac{\partial u}{\partial x} + z^2\frac{\partial \alpha}{\partial x}\right)\right]dF - \frac{E\alpha S_y}{2(1+\mu)}\left(-\alpha + \frac{\partial w}{\partial x}\right);$$

$$R_3 = E\left[\frac{\partial u}{\partial x}\left(S_y\frac{\partial u}{\partial x} - I_y\frac{\partial \alpha}{\partial x}\right) + \frac{\partial \alpha}{\partial x}\left(-I_y\frac{\partial u}{\partial x} + A\left(z^3\right)\frac{\partial \alpha}{\partial x}\right) - \frac{\alpha}{2(1+\mu)}\left(-S_y\alpha + S_y\frac{\partial w}{\partial x}\right)\right];$$

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$$\frac{\partial R_3}{\partial x}=\frac{\partial}{\partial x}\left\{E\left[\frac{\partial u}{\partial x}\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial \alpha}{\partial x}\right)+\frac{\partial \alpha}{\partial x}\left(-I_y\frac{\partial u}{\partial x}+A(z^3)\frac{\partial \alpha}{\partial x}\right)-\frac{\alpha}{2(1+\mu)}\left(-S_y\alpha+S_y\frac{\partial w}{\partial x}\right)\right]\right\}=$$

$$=E\left\{\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial \alpha}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left[\frac{\partial \alpha}{\partial x}\left(-I_y\frac{\partial u}{\partial x}+A(z^3)\frac{\partial \alpha}{\partial x}\right)\right]-\frac{\partial}{\partial x}\left[\frac{\alpha}{2(1+\mu)}\left(-S_y\alpha+S_y\frac{\partial w}{\partial x}\right)\right]\right\}=$$

$$=E\left\{\left[\frac{\partial^2 u}{\partial x^2}\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial \alpha}{\partial x}\right)\right]+\frac{\partial u}{\partial x}\left(S_y\frac{\partial^2 u}{\partial x^2}-I_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right]+\left[\frac{\partial^2 \alpha}{\partial x^2}\left(-I_y\frac{\partial u}{\partial x}+A(z^3)\frac{\partial \alpha}{\partial x}\right)\right]+$$

$$+\left[\frac{\partial \alpha}{\partial x}\left(-I_y\frac{\partial^2 u}{\partial x^2}+A(z^3)\frac{\partial^2 \alpha}{\partial x^2}\right)\right]-\frac{1}{2(1+\mu)}\left[\frac{\partial \alpha}{\partial x}\left(-S_y\alpha+S_y\frac{\partial w}{\partial x}\right)+\alpha\left(-S_y\frac{\partial \alpha}{\partial x}+S_y\frac{\partial^2 w}{\partial x^2}\right)\right];$$

$$R_4=\int_F\frac{\partial u_1}{\partial x}\sigma_{13}dF=\int_F\left[\frac{\partial}{\partial x}(u-z\alpha)\left[\frac{E}{2(1+\mu)}\left(\frac{\partial u_1}{\partial z}+\frac{\partial u_3}{\partial x}\right)\right]\right]dF;$$

$$R_4=\frac{E}{2(1+\mu)}\int_F\left[\left(\frac{\partial u}{\partial x}-z\frac{\partial \alpha}{\partial x}\right)\left((-\alpha)+\frac{\partial w}{\partial x}\right)\right]dF=\frac{E}{2(1+\mu)}\int_F\left[-\alpha\frac{\partial u}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}+z\alpha\frac{\partial \alpha}{\partial x}-z\frac{\partial \alpha}{\partial x}\frac{\partial w}{\partial x}\right]dF;$$

$$R_4=\frac{E}{2(1+\mu)}\left[-\alpha\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right)+\frac{\partial w}{\partial x}\left(S_y\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right)\right];$$

Introducing the values of N_x, Q_3 , M_y R_1, R_2, R_3, R_4, $\partial R_1/\partial x$, $\partial R_2/\partial x$ *and* $\partial R_3/\partial x$ into the equations (20) and the boundary conditions (21), one gets:

$$-\rho F\frac{\partial^2 u}{\partial t^2}+\rho S_y\frac{\partial^2 \alpha}{\partial t^2}+EF\frac{\partial^2 u}{\partial x^2}-ES_y\frac{\partial^2 \alpha}{\partial x^2}+$$

$$+E\left\{\left[\frac{\partial^2 u}{\partial x^2}\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right)\right]+\left[\frac{\partial u}{\partial x}\left(F\frac{\partial^2 u}{\partial x^2}-S_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right]+\left[\frac{\partial^2 \alpha}{\partial x^2}\left(-S_y\frac{\partial u}{\partial x}+I_y\frac{\partial \alpha}{\partial x}\right)\right]+\right.$$

$$\left.+\left[\frac{\partial \alpha}{\partial x}\left(-S_y\frac{\partial^2 u}{\partial x^2}+I_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right]-\frac{1}{2(1+\mu)}\left[\frac{\partial \alpha}{\partial x}\left(-F\alpha+F\frac{\partial w}{\partial x}\right)+\alpha\left(-F\frac{\partial \alpha}{\partial x}+F\frac{\partial^2 w}{\partial x^2}\right)\right]\right\}+(f_1+\overline{q}_1)=0;$$

$$-\rho F\frac{\partial^2 w}{\partial t^2}+GF\left(-\frac{\partial \alpha}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+E\left\{\frac{\partial^2 w}{\partial x^2}\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right)+\frac{\partial w}{\partial x}\left(F\frac{\partial^2 u}{\partial x^2}-S_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right\}+(f_3+\overline{q}_3)=0;$$

$$\rho S_y\frac{\partial^2 u}{\partial t^2}-\rho I_y\frac{\partial^2 \alpha}{\partial t^2}+E\left(S_y\frac{\partial^2 u}{\partial x^2}-I_y\frac{\partial^2 \alpha}{\partial x^2}\right)+$$

$$+E\left\{\left[\frac{\partial^2 u}{\partial x^2}\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial \alpha}{\partial x}\right)\right]+\frac{\partial u}{\partial x}\left(S_y\frac{\partial^2 u}{\partial x^2}-I_y\frac{\partial^2 \alpha}{\partial x^2}\right)\right]+\left[\frac{\partial^2 \alpha}{\partial x^2}\left(-I_y\frac{\partial u}{\partial x}+A(z^3)\frac{\partial \alpha}{\partial x}\right)\right]+$$

$$+\left[\frac{\partial \alpha}{\partial x}\left(-I_y\frac{\partial^2 u}{\partial x^2}+A(z^3)\frac{\partial^2 \alpha}{\partial x^2}\right)\right]-\frac{1}{2(1+\mu)}\left[\frac{\partial \alpha}{\partial x}\left(-S_y\alpha+S_y\frac{\partial w}{\partial x}\right)+\alpha\left(-S_y\frac{\partial \alpha}{\partial x}+S_y\frac{\partial w}{\partial x}\right)\right]+ \tag{23}$$

$$+\frac{E}{2(1+\mu)}\left[-\alpha\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right)+\frac{\partial w}{\partial x}\left(S_y\frac{\partial u}{\partial x}-S_y\frac{\partial \alpha}{\partial x}\right)\right]-\left(M(f_1)+M(\overline{q}_1)\right)=0.$$

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$$\left[-E\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial\alpha}{\partial x}\right)-E\left\{\frac{\partial u}{\partial x}\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial\alpha}{\partial x}\right)+\frac{\partial\alpha}{\partial x}\left[-S_y\frac{\partial u}{\partial x}+I_y\frac{\partial\alpha}{\partial x}\right]-\frac{\alpha}{2(1+\mu)}\left[-F\alpha+F\frac{\partial w}{\partial x}\right]\right\}+\overline{\varphi}_1\right]\delta u\Big|_x=0;$$

$$\left[-GF\left(-\alpha+\frac{\partial w}{\partial x}\right)-E\left[\frac{\partial w}{\partial x}\left(F\frac{\partial u}{\partial x}-S_y\frac{\partial\alpha}{\partial x}\right)\right]+\overline{\varphi}_3\right]\delta w\Big|_x=0; \qquad (24)$$

$$\left[E\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial\alpha}{\partial x}\right)+E\left[\frac{\partial u}{\partial x}\left(S_y\frac{\partial u}{\partial x}-I_y\frac{\partial\alpha}{\partial x}\right)+\frac{\partial\alpha}{\partial x}\left(-I_y\frac{\partial u}{\partial x}+A(z^3)\frac{\partial\alpha}{\partial x}\right)-\frac{\alpha}{2(1+\mu)}\left(-S_y\alpha+S_y\frac{\partial w}{\partial x}\right)\right]+M(\varphi_1)\right]\delta\alpha\Big|_x=0.$$

Initial conditions:

$$\left[\rho F\frac{\partial u}{\partial t}-\rho S_y\frac{\partial\alpha}{\partial t}\right]\delta u\Big|_t=0;\ \left[\rho F\frac{\partial w}{\partial t}\right]\delta w\Big|_t=0;\ \left[-\rho S_y\frac{\partial u}{\partial t}+\rho I_y\frac{\partial\alpha}{\partial t}\right]\delta\alpha\Big|_t=0. \quad (25)$$

C. Solution Algorithm.

Introduce dimensionless parameters: $u=a\overline{u},\ w=a\overline{w},\ t=t_0\overline{t},\ x=l\overline{x}$. Considering introduced dimensionless parameters, the system of equations (23), boundary (24) and initial conditions (25) have the form:

$$-\rho F\frac{a^2\partial^2\overline{u}}{t_0{}^2\partial\overline{t}^2}+\rho S_y\frac{a\partial^2\alpha}{t_0{}^2\partial\overline{t}^2}+EF\frac{a^2\partial^2\overline{u}}{l^2\partial\overline{x}^2}-ES_y\frac{a\partial^2\alpha}{l^2\partial\overline{x}^2}+$$

$$+\frac{\partial^2\overline{u}}{\partial\overline{x}^2}\left(\frac{EFa^3}{l^3}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{ES_ya^2}{l^3}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\overline{u}}{\partial\overline{x}}\left(\frac{EFa^3}{l^3}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{ES_ya^2}{l^3}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)+$$

$$+\frac{\partial^2\alpha}{\partial\overline{x}^2}\left(-\frac{ES_ya^2}{l^3}\frac{\partial\overline{u}}{\partial\overline{x}}+\frac{EI_ya}{l^3}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{ES_ya^2}{l^3}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}+\frac{EI_ya}{l^3}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)-$$

$$+\frac{1}{2(1+\mu)}\left[\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{EFa}{l}\alpha+\frac{EFa^2}{l^2}\frac{\partial\overline{w}}{\partial\overline{x}}\right)+\alpha\left(-\frac{EFa}{l}\frac{\partial\alpha}{\partial\overline{x}}+\frac{EFa^2}{l^2}\frac{\partial^2 w}{\partial\overline{x}^2}\right)\right]+a\left(f_1+\overline{q}_1\right)=0;$$

$$-\rho F\frac{a^2\partial^2\overline{w}}{t_0{}^2\partial\overline{t}^2}-\frac{EF}{2(1+\mu)}\frac{a\partial\alpha}{l\partial\overline{x}}+\frac{EF}{2(1+\mu)}\frac{a^2\partial^2 w}{l^2\partial\overline{x}^2}+$$

$$+\frac{\partial^2\overline{w}}{\partial\overline{x}^2}\left(\frac{EFa^3}{l^3}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{ES_ya^2}{l^3}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\overline{w}}{\partial\overline{x}}\left(\frac{EFa^3}{l^3}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{ES_ya^2}{l^3}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)+a\left(f_3+\overline{q}_3\right)=0;$$

$$\rho S_y\frac{a\partial^2\overline{u}}{t_0{}^2\partial\overline{t}^2}-\rho I_y\frac{\partial^2\alpha}{t_0{}^2\partial\overline{t}^2}+ES_y\frac{a\partial^2\overline{u}}{l^2\partial\overline{x}^2}-EI_y\frac{\partial^2\alpha}{l^2\partial\overline{x}^2}+ \qquad (26)$$

$$+\frac{\partial^2\overline{u}}{\partial\overline{x}^2}\left(\frac{Ea^2S_y}{l^3}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{EI_ya}{l^3}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\overline{u}}{\partial\overline{x}}\left(\frac{ES_ya^2}{l^3}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{EI_ya}{l^3}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)+\frac{\partial^2\alpha}{\partial\overline{x}^2}\left(-\frac{EI_ya}{l^3}\frac{\partial\overline{u}}{\partial\overline{x}}+\frac{EA(z^3)}{l^3}\frac{\partial\alpha}{\partial\overline{x}}\right)+$$

$$+\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{EaI_y}{l^3}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}+\frac{EA(z^3)}{l^3}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)-\frac{1}{2(1+\mu)}\left[\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{ES_y}{l}\alpha+\frac{EaS_y}{l^2}\frac{\partial\overline{w}}{\partial\overline{x}}\right)\right]-\frac{1}{2(1+\mu)}\left[-\alpha\left(\frac{ES_y}{l}\frac{\partial\alpha}{\partial\overline{x}}+\right.\right.$$

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$$+\frac{EaS_y}{l^2}\frac{\partial^2\overline{w}}{\partial\overline{x}}\Bigg)\Bigg]+\frac{1}{2(1+\mu)}\Bigg[-\alpha\Bigg(\frac{EaF}{l}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{ES_y}{l}\frac{\partial\alpha}{\partial\overline{x}}\Bigg)\Bigg]+\frac{1}{2(1+\mu)}\Bigg[\frac{\partial\overline{w}}{\partial\overline{x}}\Bigg(\frac{Ea^2S_y}{l^2}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{EaS_y}{l^2}\frac{\partial\alpha}{\partial\overline{x}}\Bigg)\Bigg]-$$
$$-\left(M\left(f_1\right)+M\left(\overline{q}_1\right)\right)=0.$$

Boundary conditions:

$$\left[-\frac{EFa^2\partial\overline{u}}{l\partial\overline{x}}-\frac{ES_ya\partial\alpha}{l\partial\overline{x}}-\frac{a^2\partial\overline{u}}{l\partial\overline{x}}\left(\frac{EFa\partial\overline{u}}{l\partial x}-\frac{ES_y\partial\alpha}{l\partial\overline{x}}\right)+\frac{a\partial\alpha}{l\partial\overline{x}}\left[-\frac{ES_ya\partial\overline{u}}{l\partial\overline{x}}+\frac{EI_y\partial\alpha}{l\partial\overline{x}}\right]-\frac{a\alpha}{2(1+\mu)}\left[-EF\alpha+\frac{EFa\partial\overline{w}}{l\partial\overline{x}}\right]+a\overline{\varphi}_1\right]\delta\overline{u}\Bigg|_{\overline{x}}=0;$$

$$\left[\frac{EaF}{2(1+\mu)}\alpha-\frac{Ea^2F}{2(1+\mu)}\frac{\partial\overline{w}}{l\partial\overline{x}}-\frac{a^2\partial\overline{w}}{l\partial\overline{x}}\left(\frac{EFa\partial\overline{u}}{l\partial\overline{x}}-\frac{ES_y\partial\alpha}{l\partial\overline{x}}\right)+a\overline{\varphi}_3\right]\delta\overline{w}\Bigg|_{\overline{x}}=0; \qquad (27)$$

$$\left[\frac{ES_ya\partial\overline{u}}{l\partial\overline{x}}-\frac{EJ_y\partial\alpha}{l\partial\overline{x}}+\frac{\partial\overline{u}}{\partial\overline{x}}\left(\frac{ES_ya^2\partial\overline{u}}{l^2\partial\overline{x}}-\frac{EI_xa\partial\alpha}{l^2\partial\overline{x}}\right)+\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{EI_ya\partial\overline{u}}{l^2\partial\overline{x}}+\frac{EA\left(z^3\right)\partial\alpha}{l^2\partial\overline{x}}\right)-\frac{\alpha}{2(1+\mu)}\left(-ES_y\alpha+\frac{ES_ya\partial\overline{w}}{l\partial\overline{x}}\right)+M\left(\varphi_1\right)\right]\delta\alpha\Bigg|_{\overline{x}}=0.$$

Initial conditions:

$$\left[\rho F\frac{a^2\partial\overline{u}}{t_0\partial\overline{t}}-\rho S_y\frac{a\partial\alpha}{t_0\partial\overline{t}}\right]\delta\overline{u}\Bigg|_{\overline{t}}=0;\ \left[\rho F\frac{a^2\partial\overline{w}}{t_0\partial\overline{t}}\right]\delta\overline{w}\Bigg|_{\overline{t}}=0;\ \left[-\rho S_y\frac{a\partial\overline{u}}{t_0\partial\overline{t}}+\rho I_y\frac{\partial\alpha}{t_0\partial t}\right]\delta\alpha\Bigg|_{\overline{t}}=0. \qquad (28)$$

The system of equations (26), boundary (27) and initial conditions (28) is divided by $EF\frac{a^2}{l^2}$ and the following expression $t_0^{\ 2}=l^2\frac{\rho}{E}$ is introduced.

$$-\frac{\partial^2\overline{u}}{\partial\overline{t}^2}+\frac{S_y}{Fa}\frac{\partial^2\alpha}{\partial\overline{t}^2}+\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{S_y}{Fa}\frac{\partial^2\alpha}{\partial\overline{x}^2}+\frac{\partial^2\overline{u}}{\partial\overline{x}^2}\left(\frac{a}{l}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{S_y}{lF}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\overline{u}}{\partial\overline{x}}\left(\frac{a}{l}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{S_y}{lF}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)+$$
$$+\frac{\partial^2\alpha}{\partial\overline{x}^2}\left(-\frac{S_y}{lF}\frac{\partial\overline{u}}{\partial\overline{x}}+\frac{I_y}{lFa}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{S_y}{lF}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}+\frac{I_y}{lFa}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)-$$
$$+\frac{1}{2(1+\mu)}\left[\frac{\partial\alpha}{\partial\overline{x}}\left(-\frac{l}{a}\alpha+\frac{\partial\overline{w}}{\partial\overline{x}}\right)+\alpha\left(-\frac{l}{a}\frac{\partial\alpha}{\partial\overline{x}}+\frac{\partial^2 w}{\partial\overline{x}^2}\right)\right]+\frac{l^2}{EFa}\left(f_1+\overline{q}_1\right)=0;$$

$$-\frac{\partial^2\overline{w}}{\partial\overline{t}^2}-\frac{1}{2(1+\mu)}\frac{l\partial\alpha}{a\partial\overline{x}}+\frac{1}{2(1+\mu)}\frac{\partial^2 w}{\partial\overline{x}^2}+$$
$$+\frac{\partial^2\overline{w}}{\partial\overline{x}^2}\left(\frac{l}{a}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{S_y}{lF}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\overline{w}}{\partial\overline{x}}\left(\frac{l}{a}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{S_y}{lF}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)+\frac{l^2}{EFa}\left(f_3+\overline{q}_3\right)=0;$$

$$\frac{S_y}{Fa}\frac{\partial^2\overline{u}}{\partial\overline{t}^2}-\frac{I_y}{Fa^2}\frac{\partial^2\alpha}{\partial\overline{t}^2}+\frac{S_y}{Fa}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{I_y}{Fa^2}\frac{\partial^2\alpha}{\partial\overline{x}^2}+$$
$$+\frac{\partial^2\overline{u}}{\partial\overline{x}^2}\left(\frac{S_y}{Fl}\frac{\partial\overline{u}}{\partial\overline{x}}-\frac{I_y}{lFa}\frac{\partial\alpha}{\partial\overline{x}}\right)+\frac{\partial\overline{u}}{\partial\overline{x}}\left(\frac{S_y}{Fl}\frac{\partial^2\overline{u}}{\partial\overline{x}^2}-\frac{I_y}{lFa}\frac{\partial^2\alpha}{\partial\overline{x}^2}\right)+\frac{\partial^2\alpha}{\partial\overline{x}^2}\left(-\frac{I_y}{lFa}\frac{\partial\overline{u}}{\partial\overline{x}}+\frac{A\left(z^3\right)}{a^2lF}\frac{\partial\alpha}{\partial\overline{x}}\right)+$$

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$$+\frac{\partial\alpha}{\partial\bar{x}}\left(-\frac{I_y}{lFa}\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{A\left(z^3\right)}{lFa^2}\frac{\partial^2\alpha}{\partial\bar{x}^2}\right)-\frac{1}{2(1+\mu)}\left[\frac{\partial\alpha}{\partial\bar{x}}\left(-\frac{S_y l}{Fa^2}\alpha+\frac{S_y}{Fa}\frac{\partial\bar{w}}{\partial\bar{x}}\right)\right]-\frac{1}{2(1+\mu)}\left[-\alpha\left(\frac{S_y l}{Fa^2}\frac{\partial\alpha}{\partial\bar{x}}+\frac{S_y}{Fa}\frac{\partial^2\bar{w}}{\partial\bar{x}}\right)\right]+\frac{1}{2(1+\mu)}\left[-\alpha\left(\frac{l}{a}\frac{\partial\bar{u}}{\partial\bar{x}}-\frac{S_y l}{Fa^2}\frac{\partial\alpha}{\partial\bar{x}}\right)\right]+\frac{1}{2(1+\mu)}\left[\frac{\partial\bar{w}}{\partial\bar{x}}\left(\frac{S_y}{F}\frac{\partial\bar{u}}{\partial\bar{x}}-\frac{S_y}{Fa}\frac{\partial\alpha}{\partial\bar{x}}\right)\right]-$$
$$-\frac{l}{EFa^2}\left(M\left(f_1\right)+M\left(\bar{q}_1\right)\right)=0.$$

Boundary conditions:

$$\left[-\frac{l\partial\bar{u}}{\partial\bar{x}}-\frac{S_y l\partial\alpha}{Fa\partial\bar{x}}-\frac{\partial\bar{u}}{\partial\bar{x}}\left(\frac{a\partial\bar{u}}{\partial\bar{x}}-\frac{S_y\partial\alpha}{F\partial\bar{x}}\right)+\frac{\partial\alpha}{\partial\bar{x}}\left[-\frac{S_y\partial\bar{u}}{F\partial\bar{x}}+\frac{I_y\partial\alpha}{Fa\partial\bar{x}}\right]-\frac{1}{2(1+\mu)}\left[-\frac{l^2}{a}\alpha^2+l\alpha\frac{\partial\bar{w}}{\partial\bar{x}}\right]+\frac{l^2}{EFa}\bar{\phi}_1\right]\delta\bar{u}\Big|_{\bar{x}}=0;$$

$$\left[\frac{1}{2(1+\mu)}\left(\frac{l^2}{a}\alpha-l\frac{\partial\bar{w}}{\partial\bar{x}}\right)-\frac{\partial\bar{w}}{\partial\bar{x}}\left(a\frac{\partial\bar{u}}{\partial\bar{x}}-\frac{S_y\partial\alpha}{F\partial\bar{x}}\right)+\frac{l^2}{EFa}\bar{\varphi}_3\right]\delta\bar{w}\Big|_{\bar{x}}=0;$$

$$\left[\frac{S_y l\partial\bar{u}}{Fa\partial\bar{x}}-\frac{I_y l\partial\alpha}{Fa^2\partial\bar{x}}+\frac{\partial\bar{u}}{\partial\bar{x}}\left(\frac{S_y\partial\bar{u}}{F\partial\bar{x}}-\frac{I_x\partial\alpha}{Fa\partial\bar{x}}\right)+\frac{\partial\alpha}{\partial\bar{x}}\left(-\frac{I_y\partial\bar{u}}{Fa\partial\bar{x}}+\frac{A\left(z^3\right)\partial\alpha}{Fa^2\partial\bar{x}}\right)-\frac{\alpha}{2(1+\mu)}\left(-\frac{S_y l^2}{Fa^2}\alpha+\frac{S_y l}{Fa}\frac{\partial\bar{w}}{\partial\bar{x}}\right)+\frac{l^2}{EFa^2}M\left(\varphi_1\right)\right]\delta\alpha\Big|_{\bar{x}}=0.$$

Initial conditions:

$$\left[\frac{t_0\partial\bar{u}}{\partial\bar{t}}-\frac{t_0 S_y}{Fa}\frac{\partial\alpha}{\partial\bar{t}}\right]\delta\bar{u}\Big|_{\bar{t}}=0;\quad\left[\frac{t_0\partial\bar{w}}{\partial\bar{t}}\right]\delta\bar{w}\Big|_{\bar{t}}=0;\quad\left[-\frac{t_0 S_y}{Fa}\frac{\partial\bar{u}}{\partial\bar{t}}+\frac{t_0 I_y}{Fa^2}\frac{\partial\alpha}{\partial\bar{t}}\right]\delta\alpha\Big|_{\bar{t}}=0.$$

The system of equations of motion:

$$-\frac{\partial^2\bar{u}^k}{\partial\bar{t}^2}+\frac{S_y}{Fa}\frac{\partial^2\alpha^k}{\partial\bar{t}^2}+\frac{\partial^2\bar{u}^k}{\partial\bar{x}^2}-\frac{S_y}{Fa}\frac{\partial^2\alpha^k}{\partial\bar{x}^2}+\Phi_1^{k-1}+\frac{l^2}{EFa}\left(f_1+\bar{q}_1\right)=0; \quad (29)$$

$$-\frac{\partial^2\bar{w}^k}{\partial\bar{t}^2}-\frac{1}{2(1+\mu)}\frac{l\partial\alpha^k}{a\partial\bar{x}}+\frac{1}{2(1+\mu)}\frac{\partial^2 w^k}{\partial\bar{x}^2}+\Phi_2^{k-1}+\frac{l^2}{EFa}\left(f_3+\bar{q}_3\right)=0;$$

$$\frac{S_y}{Fa}\frac{\partial^2\bar{u}^k}{\partial\bar{t}^2}-\frac{I_y}{Fa^2}\frac{\partial^2\alpha^k}{\partial\bar{t}^2}+\frac{S_y}{Fa}\frac{\partial^2\bar{u}^k}{\partial\bar{x}^2}-\frac{I_y}{Fa^2}\frac{\partial^2\alpha^k}{\partial\bar{x}^2}+\Phi_3^{k-1}-\frac{l^2}{EFa^2}\left(M\left(f_1\right)+M\left(\bar{q}_1\right)\right)=0.$$

Here

$$\Phi_1^{k-1}=\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}\left(\frac{a}{l}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{S_y}{lF}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)+\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}\left(\frac{a}{l}\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}-\frac{S_y}{lF}\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\right)+$$
$$+\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\left(-\frac{S_y}{lF}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}+\frac{I_y}{lFa}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)+\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\left(-\frac{S_y}{lF}\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}+\frac{I_y}{lFa}\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\right)-$$
$$+\frac{1}{2(1+\mu)}\left[\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\left(-\frac{l}{a}\alpha^{k-1}+\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\right)+\alpha\left(-\frac{l}{a}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}+\frac{\partial^2 w^{k-1}}{\partial\bar{x}^2}\right)\right];$$

$$\Phi_2^{k-1}=\frac{\partial^2\bar{w}^{k-1}}{\partial\bar{x}^2}\left(\frac{l}{a}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{S_y}{lF}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)+\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\left(\frac{l}{a}\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}-\frac{S_y}{lF}\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\right);$$

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$$\Phi_3^{k-1}=\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}\left(\frac{S_y}{Fl}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{I_y}{lFa}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)+\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}\left(\frac{S_y}{Fl}\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}-\frac{I_y}{lFa}\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\right)+$$

$$+\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\left(-\frac{I_y}{lFa}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}+\frac{A(z^3)}{a^2lF}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)+\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\left(-\frac{I_y}{lFa}\frac{\partial^2\bar{u}^{k-1}}{\partial\bar{x}^2}+\frac{A(z^3)}{lFa^2}\frac{\partial^2\alpha^{k-1}}{\partial\bar{x}^2}\right)-$$

$$-\frac{1}{2(1+\mu)}\left[\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\left(-\frac{S_yl}{Fa^2}\alpha^{k-1}+\frac{S_y}{Fa}\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\right)\right]-\frac{1}{2(1+\mu)}\left[-\alpha^{k-1}\left(\frac{S_yl}{Fa^2}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}+\frac{S_y}{Fa}\frac{\partial^2\bar{w}^{k-1}}{\partial\bar{x}}\right)\right]+$$

$$+\frac{1}{2(1+\mu)}\left[-\alpha^{k-1}\left(\frac{l}{a}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{S_yl}{Fa^2}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)\right]+\frac{1}{2(1+\mu)}\left[\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\left(\frac{S_y}{F}\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{S_y}{Fa}\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\right)\right];$$

Boundary conditions:

$$\left[-\frac{l\partial\bar{u}^k}{\partial\bar{x}}-\frac{S_yl\partial\alpha^k}{Fa\partial\bar{x}}-\bar{\Phi}_1^{k-1}+\frac{l^2}{EFa}\bar{\varphi}_1\right]\delta\bar{u}\Big|_{\bar{x}}=0;$$

$$\left[\frac{1}{2(1+\mu)}\left(\frac{l^2}{a}\alpha^k-l\frac{\partial\bar{w}^k}{\partial\bar{x}}\right)-\bar{\Phi}_2^{k-1}+\frac{l^2}{EFa}\bar{\varphi}_3\right]\delta\bar{w}\Big|_{\bar{x}}=0; \qquad (30)$$

$$\left[\frac{S_yl\partial\bar{u}^k}{Fa\partial\bar{x}}-\frac{I_yl\partial\alpha^k}{Fa^2\partial\bar{x}}+\bar{\Phi}_3^{k-1}+\frac{l^2}{EFa^2}M(\varphi_1)\right]\delta\alpha\Big|_{\bar{x}}=0.$$

Here

$$\bar{\Phi}_1^{k-1}=\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}\left(\frac{a\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{S_y\partial\alpha^{k-1}}{F\partial\bar{x}}\right)+\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\left[-\frac{S_y\partial\bar{u}^{k-1}}{F\partial\bar{x}}+\frac{I_y\partial\alpha^{k-1}}{Fa\partial\bar{x}}\right]-\frac{1}{2(1+\mu)}\left[-\frac{l^2}{a}\alpha^{2k-1}+l\alpha\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\right];$$

$$\bar{\Phi}_2^{k-1}=\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\left(a\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}-\frac{S_y\partial\alpha^{k-1}}{F\partial\bar{x}}\right);$$

$$\bar{\Phi}_3^{k-1}=\frac{\partial\bar{u}^{k-1}}{\partial\bar{x}}\left(\frac{S_y\partial\bar{u}^{k-1}}{F\partial\bar{x}}-\frac{I_x\partial\alpha^{k-1}}{Fa\partial\bar{x}}\right)+\frac{\partial\alpha^{k-1}}{\partial\bar{x}}\left(-\frac{I_y\partial\bar{u}^{k-1}}{Fa\partial\bar{x}}+\frac{A(z^3)\partial\alpha^{k-1}}{Fa^2\partial\bar{x}}\right)-$$

$$-\frac{\alpha^{k-1}}{2(1+\mu)}\left(-\frac{S_yl^2}{Fa^2}\alpha^{k-1}+\frac{S_yl}{Fa}\frac{\partial\bar{w}^{k-1}}{\partial\bar{x}}\right).$$

Initial conditions:

$$\left[\frac{\partial\bar{u}^k}{\partial\bar{t}}-\frac{S_y}{Fa}\frac{\partial\alpha^k}{\partial\bar{t}}\right]t_0\delta\bar{u}\Big|_{\bar{t}}=0;\ \left[\frac{\partial\bar{w}^k}{\partial\bar{t}}\right]t_0\delta\bar{w}\Big|_{\bar{t}}=0;\ \left[-\frac{S_y}{Fa}\frac{\partial\bar{u}^k}{\partial\bar{t}}+\frac{I_y}{Fa^2}\frac{\partial\alpha^k}{\partial t}\right]t_0\delta\alpha\Big|_{\bar{t}}=0. \qquad (31)$$

The solutions of differential equations of motion (29) with corresponding initial (31) and boundary conditions (30), obtained from the variation principle in a scalar form, are quite difficult. Therefore, the system of differential equations, initial and boundary conditions can be represented in vector form.

Introduce the vectors

$$\vec{U}=[\bar{u},\bar{w},\alpha]^T;\ \vec{F}=[(f_1+\bar{q}_1),(f_3+\bar{q}_3),(M(f_1)+M(q_1))]^T;\ \vec{F}(\varphi)=[\bar{\varphi}_1,\bar{\varphi}_3,M(\varphi_1)]^T;$$

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$$\vec{\Phi}^{k-1} = \left[\Phi_1{}^{k-1}, \Phi_2{}^{k-1}, \Phi_3{}^{k-1}\right]^T; \qquad \vec{\overline{\Phi}}^{k-1} = \left[\overline{\Phi}_1{}^{k-1}, \overline{\Phi}_2{}^{k-1}, \overline{\Phi}_3{}^{k-1}\right]^T. \tag{32}$$

The system of equations (29), initial conditions (31) and boundary conditions (30) with introduced elements of the matrix are written in vector form

$$M\frac{\partial^2\vec{U}^k}{\partial t^2} + A\frac{\partial^2\vec{U}^k}{\partial x^2} + B\frac{\partial\vec{U}^k}{\partial x} + E\vec{\Phi}^{k-1} + D\vec{F} = 0; \quad (33) \qquad \left[-M\frac{\partial\vec{U}^k}{\partial t}\right]Et_0\delta\vec{U}\Big|_{\bar{t}} = 0; \quad (34)$$

$$\left[\overline{A}\frac{\partial\vec{U}^k}{\partial x} + B\vec{U}^k + E\vec{\overline{\Phi}}^{k-1} + \overline{D}\vec{F}(\varphi)\right]El\delta\vec{U}\Big|_{\bar{x}} = 0. \tag{35}$$

Here the elements of the matrix M from the system of equations have the following expressions:

$$m_{11} = -1;\; m_{13} = \frac{S_y}{Fa};\; m_{22} = -1;\; m_{31} = \frac{S_y}{Fa};\; m_{33} = -\frac{J_y}{Fa^2}.$$

The elements of the matrix A from the system of equations have the form

$$a_{11} = 1;\; a_{13} = -\frac{S_y}{Fa};\; a_{22} = \frac{G}{E};\; a_{31} = \frac{S_y}{Fa};\; a_{33} = \frac{J_y}{Fa^2}.$$

$$b_{23} = -\frac{G}{E}\frac{l}{a};\; d_{11} = d_{22} = \frac{l^2}{EFa};\; d_{33} = -\frac{l^2}{EFa}.$$

In initial conditions the elements of the matrix have the opposite signs relative to the elements of the matrix of the system of equations $\left(M_{н.у.} = -M_{c.y.}\right)$

At *k=1* a linear problem is solved, this is a zero approximation of the problem

$$M\frac{\partial^2\vec{U}^k}{\partial t^2} + A\frac{\partial^2\vec{U}^k}{\partial x^2} + B\frac{\partial\vec{U}^k}{\partial x} + D\vec{F} = 0; \tag{36}$$

$$\left[-M\frac{\partial\vec{U}^k}{\partial t}\right]Et_0\delta\vec{U}\Big|_{\bar{t}} = 0; \quad (37) \qquad \left[\overline{A}\frac{\partial\vec{U}^k}{\partial x} + B\vec{U}^k + \overline{D}\vec{F}(\varphi)\right]El\delta\vec{U}\Big|_{\bar{x}} = 0. \quad (38)$$

At *k=2* nonlinear items of a given equation are solved. This is the first approximation of the solution, further the process of iteration occurs

$$\left|\vec{U}_{ij}{}^k - U_{ij}{}^{k-1}\right| \le \varepsilon.$$

When constructing the computing algorithm for the system of differential equations (36) with initial (37) and boundary conditions (38), the central finite-difference correlations of the Finite difference method with the second order of accuracy is applied [27,28]:

$$M\frac{\partial^2\vec{U}}{\partial t^2} = \frac{1}{\tau^2}M\left[\vec{U}_{i,j+1} - 2\vec{U}_{i,j} + \vec{U}_{i,j-1}\right]; \; A\frac{\partial^2\vec{U}}{\partial x^2} = \frac{1}{h^2}A\left[\vec{U}_{i+1,j} - 2\vec{U}_{i,j} + \vec{U}_{i-1,j}\right];$$

$$B\frac{\partial\vec{U}}{\partial x} = \frac{1}{2h}B\left[\vec{U}_{i+1,j} - \vec{U}_{i-1,j}\right]; \; \frac{\partial\vec{U}}{\partial t} = \frac{1}{2\tau}\left[\vec{U}_{i,j+1} - \vec{U}_{i,j-1}\right] \tag{39}$$

Introducing (39) into (36) one obtains:

$$\frac{M}{\tau^2}\left[\vec{U}_{i,j+1} - 2\vec{U}_{i,j} + \vec{U}_{i,j-1}\right] + \frac{A}{h^2}\left[\vec{U}_{i+1,j} - 2\vec{U}_{i,j} + \vec{U}_{i-1,j}\right] + \frac{B}{2h}\left[\vec{U}_{i+1,j} - \vec{U}_{i-1,j}\right] + D\vec{F}_{i,j} = 0. \tag{40}$$

Divide the equation (40) by M/τ^2.

$$\left[\vec{U}_{i,j+1}-2\vec{U}_{i,j}+\vec{U}_{i,j-1}\right]+\frac{\tau^2 AM^{-1}}{h^2}\left[\vec{U}_{i+1,j}-2\vec{U}_{i,j}+\vec{U}_{i-1,j}\right]+ \\ +\frac{\tau^2 BM^{-1}}{2h}\left[\vec{U}_{i+1,j}-\vec{U}_{i-1,j}\right]+D\tau^2 M^{-1}\vec{F}_{i,j}=0. \quad (41)$$

Reduce the similar items

$$\vec{U}_{i,j+1}-\left[2+\frac{2\tau^2 AM^{-1}}{h^2}-\tau^2 CA^{-1}\right]\vec{U}_{i,j}+\left[\frac{\tau^2 AM^{-1}}{h^2}-\frac{\tau^2 BM^{-1}}{2h}\right]\vec{U}_{i-1,j}+ \\ +\left[\frac{\tau^2 AM^{-1}}{h^2}+\frac{\tau^2 BM^{-1}}{2h}\right]\vec{U}_{i+1,j}+\vec{U}_{i,j-1}+DM^{-1}\tau^2\vec{F}_{i,j}=0;$$

Introduce the designations:

$$\tilde{A}=\frac{\tau^2 AM^{-1}}{h^2}-\frac{\tau^2 BM^{-1}}{2h};\ \tilde{B}=2+\frac{\tau^2 AM^{-1}}{h^2};\ \tilde{C}=\frac{\tau^2 AM^{-1}}{h^2}+\frac{\tau^2 BM^{-1}}{2h};\ \tilde{F}_{i,j}=D\tau^2 M^{-1}\vec{F}_{i,j}. \quad (42)$$

Introducing (42) into (41), one obtains

$$\vec{U}_{i,j+1}+\tilde{A}\vec{U}_{i-1,j}-\tilde{B}\vec{U}_{i,j}+\tilde{C}\vec{U}_{i+1,j}+\vec{U}_{i,j-1}+\tilde{F}_{i,j}=0.$$

The last equation is solved relative to the vector of functions $\vec{U}_{i,j+1}$.

$$\vec{U}_{i,j+1}=-\tilde{A}\vec{U}_{i-1,j}+\tilde{B}\vec{U}_{i,j}-\tilde{C}\vec{U}_{i+1,j}-\vec{U}_{i,j-1}-\tilde{F}_{i,j} \quad (43)$$

At $j=0, i=i$ consider the initial conditions

$$M\left.\frac{\partial\vec{U}}{\partial t}\right|_{\bar{t}=t_0}=\dot{\vec{U}}_{i,0}=\frac{M}{2\tau}\left(\vec{U}_{i,1}-\vec{U}_{i,-1}\right);\quad \left.\vec{U}\right|_{\bar{t}=t_0}=\vec{U}^0_{i,0}. \quad (44)$$

From initial conditions (44) define the vector of functions $\vec{U}_{i,-1}$.

$$\vec{U}_{i,-1}=\vec{U}_{i,1}-2\tau\cdot M^{-1}\dot{\vec{U}}_{i,0}\cdot A^{-1} \quad (45)$$

At $j=0, i=i$ equation (43) is solved

$$\vec{U}_{i,1}=-\tilde{A}\vec{U}_{i-1,0}+\tilde{B}\vec{U}_{i,0}-\tilde{C}\vec{U}_{i+1,0}-\left(\vec{U}_{i,1}-2\tau\cdot A^{-1}\dot{\vec{U}}^0_{i,0}\right)-\tilde{F}_{i,0}=0;$$

Reduce the similar items and the result is divided by 2.

$$\vec{U}_{i,1}=\frac{1}{2}\left[-\tilde{A}\vec{U}_{i-1,0}+\tilde{B}\vec{U}_{i,0}-\tilde{C}\vec{U}_{i+1,0}+2\tau\cdot A^{-1}\dot{\vec{U}}^0_{i,0}-\tilde{F}_{i,0}\right] \quad (46)$$

With initial conditions (44), equation (46) has a form

$$\vec{U}_{i,1}=\frac{1}{2}\left[-\tilde{A}\vec{U}_{i-1,0}{}^0+\tilde{B}\vec{U}_{i,0}{}^0-\tilde{C}\vec{U}_{i+1,0}{}^0+2\tau\cdot A^{-1}M^{-1}\dot{\vec{U}}^0_{i,0}-\tilde{F}_{i,0}\right] \quad (47)$$

At $j=1, i=i$ equation (43) with initial conditions has the form:

$$\vec{U}_{i,2}=-\tilde{A}\vec{U}_{i-1,1}+\tilde{B}\vec{U}_{i,1}-\tilde{C}\vec{U}_{i+1,1}-\vec{U}^0_{i,0}-\tilde{F}_{i,0} \quad (48)$$

Consider boundary conditions (38) at $i=0$, approximate by a step forward

$$\frac{\partial\vec{U}}{\partial x}=\frac{1}{2h}\left(-3\vec{U}_{0,j}+4\vec{U}_{1,j}-\vec{U}_{2,j}\right) \quad (49)$$

Introducing (49) into (38) one obtains:

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$$\left[-\frac{B}{2h}\left(-3\vec{U}_{0,j}+4\vec{U}_{1,j}-\vec{U}_{2,j}\right)+\overline{B}\vec{U}_{0,j}+\overline{D}\vec{F}(\varphi)_{0,j}\right]_{i=0}=0. \tag{50}$$

Reduce the similar items and solve relative to $\vec{U}_{0,j}$

$$\left[\left(-\frac{3B}{2h}+\overline{B}\right)\vec{U}_{0,j}+4\vec{U}_{1,j}-\vec{U}_{2,j}+\overline{D}\vec{F}(\varphi)_{0,j}\right]_{i=0}=0$$

$$\left[\vec{U}_{0,j}=\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}\cdot 4\vec{U}_{1,j}+\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}\vec{U}_{2,j}-\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}\overline{D}\vec{F}(\varphi)_{0,j}\right]_{i=0}=0 \tag{51}$$

at $j=j, i=1$ equation (43) is solved

$$\vec{U}_{1,j+1}=-\tilde{A}\vec{U}_{0,j}+\tilde{B}\vec{U}_{1,j}-\tilde{C}\vec{U}_{2,j}-\vec{U}_{1,j-1}-\tilde{F}_{1,j} \tag{52}$$

With boundary conditions (51)

$$\vec{U}_{i,j+1}=-\tilde{A}\left[\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}\left(4\vec{U}_{1,j}+\vec{U}_{2,j}-\overline{D}\vec{F}(\varphi)_{0,j}\right)\right]+\tilde{B}\vec{U}_{1,j}-\tilde{C}\vec{U}_{2,j}-\vec{U}_{1,j-1}-\tilde{F}_{1,j}$$

Reduce the similar items

$$\vec{U}_{i,j+1}=\left(4\tilde{A}\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}+\tilde{B}\right)\vec{U}_{1,j}+\left(-4\tilde{A}\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}+\tilde{C}\right)\vec{U}_{2,j}-$$
$$-\vec{U}_{1,j-1}+\tilde{A}\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}\overline{D}\vec{F}(\varphi)_{0,j}-\tilde{F}_{1,j} \tag{53}$$

Introduce the designations:

$$A_1=4\tilde{A}\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}+\tilde{B}\,;B_1=-4\tilde{A}\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}+\tilde{C}\,;$$

$$P_{0,j}=\tilde{A}\left(-\frac{3B}{2h}+\overline{B}\right)^{-1}\overline{D}\vec{F}(\varphi)_{0,j}. \tag{54}$$

With the introduced designations (54) equation (53) has the form:

$$\vec{U}_{1,j+1}=A_1\vec{U}_{1,j}+B_1\vec{U}_{2,j}-\vec{U}_{1,j-1}+P_{0,j}-\tilde{F}_{1,j} \tag{55}$$

At $j=j, i=N$ consider boundary conditions (39). Here the derivatives $\frac{\partial\vec{U}}{\partial x}$ are approximated by a step backward

$$\frac{\partial\vec{U}}{\partial x}=\frac{1}{2h}\left(3\vec{U}_{N,j}-4\vec{U}_{N-1,j}+\vec{U}_{N-2,j}\right)$$

Entering this equation into (38), one obtains:

$$-\frac{A}{2h}\left(3\vec{U}_{N,j}-4\vec{U}_{N-1,j}+\vec{U}_{N-2,j}\right)+\overline{B}\vec{U}_{N,j}+\overline{D}\vec{F}(\varphi)_{N,j}=0 \tag{56}$$

Reduce the similar items

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$$\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)\vec{U}_{N,j}+\frac{4\bar{A}}{2h}\vec{U}_{N-1,j}-\frac{\bar{A}}{2h}\vec{U}_{N-2,j}+\bar{D}\vec{F}(\varphi)_{N,j}=0. \quad (57)$$

Equation (57) is solved relative to the vector of the function $\vec{U}_{N,j}$,

$$\vec{U}_{N,j}=\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\left(-\frac{4\bar{A}}{2h}\vec{U}_{N-1,j}+\frac{\bar{A}}{2h}\vec{U}_{N-2,j}-\bar{D}\vec{F}(\varphi)_{N,j}\right) \quad (58)$$

At $j=j,\ i=N-1$ equation (43) gas the form:

$$\vec{U}_{N-1,j+1}=-\tilde{A}\vec{U}_{N-2,j}+\tilde{B}\vec{U}_{N-1,j}-\tilde{C}\vec{U}_{N,j}-\vec{U}_{N-1,j-1}-\tilde{F}_{N-1,j}$$

With correlations (58)

$$\vec{U}_{N-1,j+1}=-\tilde{A}\vec{U}_{N-2,j}+\tilde{B}\vec{U}_{N-1,j}-\tilde{C}\left[\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\left(-\frac{4\bar{A}}{2h}\vec{U}_{N-1,j}+\frac{\bar{A}}{2h}\vec{U}_{N-2,j}-\bar{D}\vec{F}(\varphi)_{N,j}\right)\right]-\vec{U}_{N-1,j-1}-\tilde{F}_{N-1,j}$$

Reduce the similar items

$$\vec{U}_{N-1,j+1}=\left(-\tilde{A}-\tilde{C}\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\cdot\frac{\bar{A}}{2h}\right)\vec{U}_{N-2,j}+\left(\tilde{B}+\tilde{C}\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\cdot\frac{4\bar{A}}{2h}\right)\vec{U}_{N-1,j}-$$
$$-\vec{U}_{N-1,j-1}+\tilde{C}\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\cdot\bar{D}\vec{F}(\varphi)_{N,j}-\tilde{F}_{N-1,j} \quad (59)$$

Introduce the designations

$$\bar{A}_1=-\tilde{A}-\tilde{C}\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\cdot\frac{\bar{A}}{2h};\ \bar{B}_1=\tilde{B}+4\tilde{C}\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\cdot\frac{\bar{A}}{2h};$$
$$\bar{P}_{N,j}=\tilde{C}\left(-\frac{3\bar{A}}{2h}+\bar{B}\right)^{-1}\cdot\bar{D}\vec{F}(\varphi)_{N,j}; \quad (60)$$

With the introduced designations (60) equation (59) has the form:

$$\vec{U}_{N-1,j+1}=\bar{A}_1\vec{U}_{N-2,j}+\bar{B}_1\vec{U}_{N-1,j}-\vec{U}_{N-1,j-1}+\bar{P}_{N,j}-\tilde{F}_{N-1,j} \quad (61)$$

Equation (43) is written with $j=0,\ i=1$

$$\vec{U}_{1,1}=A_1\vec{U}_{1,0}+B_1\vec{U}_{2,0}-\vec{U}_{1,-1}+\mathrm{P}_{0,0}-\tilde{F}_{1,0}$$

With correlation (45).

$$\vec{U}_{1,1}=A_1\vec{U}_{1,0}+B_1\vec{U}_{2,0}-\vec{U}_{1,-1}+2\tau M^{-1}\dot{\vec{U}}^{0}_{1,0}+\mathrm{P}_{0,0}-\tilde{F}_{1,0}$$

Reduce the similar items and with initial conditions (43)

$$\vec{U}_{1,1}=\frac{1}{2}\left[A_1\vec{U}_{1,0}+B_1\vec{U}_{2,0}{}^{0}+2\tau M^{-1}\dot{\vec{U}}^{0}_{1,0}+\mathrm{P}_{0,0}-\tilde{F}_{1,0}\right] \quad (62)$$

At $j=1,\ i=1$ equation (43) has the form:

$$\vec{U}_{1,2}=-\tilde{A}\vec{U}_{0,1}+\tilde{B}\vec{U}_{1,1}-\tilde{C}\vec{U}_{2,1}-\vec{U}_{1,0}-\tilde{F}_{1,1}$$

With initial conditions (44), equation (55) has the form:

$$\vec{U}_{1,2}=A_1\vec{U}_{1,1}+B_1\vec{U}_{2,1}\vec{U}_{1,0}{}^{0}+\mathrm{P}_{0,1}-\tilde{F}_{1,1} \quad (63)$$

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At $i = N-1,\ j = 0$ equation (43) is written in the form:

$$\vec{U}_{N-1,1} = -\tilde{A}\vec{U}_{N-2,0} + \tilde{B}\vec{U}_{N-1,0} - \tilde{C}\vec{U}_{N,0} - \vec{U}_{N-1,-1} - \tilde{F}_{N-1,0}$$

With initial conditions (44) equation (61) has the form

$$\vec{U}_{N-1,1} = \bar{A}_1\vec{U}_{N-2,0} + \bar{B}_1\vec{U}_{N-1,0} - \vec{U}_{N-1,-1} + \bar{P}_{N,0} - \tilde{F}_{N-1,0}$$

With correlation (45), reduce the similar items. The last equation has the form.

$$\vec{U}_{N-1,1} = \frac{1}{2}\left[\bar{A}_1\vec{U}^0{}_{N-2,0} + \bar{B}_1\vec{U}^0{}_{N-1,0} + 2\tau A^{-1}\dot{\vec{U}}^0_{N-1,0} + \bar{\mathrm{P}}_{N,0} - \tilde{F}_{N-1,0}\right] \quad (64)$$

At $i = N-1,\ j = 1$ consider equation (43)

$$\vec{U}_{N-1,2} = -\tilde{A}\vec{U}_{N-2,1} + \tilde{B}\vec{U}_{N-1,1} - \tilde{C}\vec{U}_{N,1} - \vec{U}_{N-1,0} - \tilde{F}_{N-1,1}$$

Here consider initial conditions (44). Equation (61) has the form

$$\vec{U}_{N-1,2} = \bar{A}_1\vec{U}_{N-2,1} + \bar{B}_1\vec{U}^0{}_{N-1,1} - \vec{U}^0{}_{N-1,1} + \bar{P}_{N,1} - \tilde{F}_{N-1,1} \quad (65)$$

II. CONCLUSION

The order of formulated solution of the problem

0. $k = 1\ E\bar{\Phi}^0 = 0$
1. $i = 1,\ j = 0:$ equation (62) is solved
2. $i = i,\ j = 0:$ equation (47) is solved
3. $i = N-1,\ j = 0:$ equation (64) is solved
4. $i = 1,\ j = 1:$ equation (63) is solved
5. $i = i,\ j = 1:$ equation (48) is solved
6. $i = N-1,\ j = 1:$ equation (65) is solved
7. $i = 1,\ j = j:$ equation (55) is solved
8. $i = i,\ j = j:$ equation (43) is solved
9. $i = N-1,\ j = j:$ equation (61) is solved
10. $k = k+1$

1) $k = 1$;
2) $i = 1,\ j = 0:$ equation (62) is solved
3) $i = i+1$;
4) $i \ge N-2,$ If conditions are not satisfied to proceed to point 2, otherwise – to point 3;
5) $i = 1,\ j = 1:$ equation (48) is solved;
6) $i = i+1$;
7) $i \ge N-2.$ If conditions are not satisfied to proceed to point 5, otherwise – to point 6;
8) $i = N-1,\ j = 1:$ equation (65) is solved;
9) $i = 1,\ j = j:$ equation (55) is solved;
10) $i = i+1$;
11) $i \ge N-2.$ If conditions are not satisfied to proceed to point 9, otherwise – to point 10;
12) $i = N-1,\ j = j:$ equation (61) is solved;

13) $j = j+1$;

14) $j > \tilde{M}$. If conditions are not satisfied to proceed to point 12, otherwise – to point 8;

15) $k = k+1$;

16) Calculation of nonlinear items;

17) Proceed to point 1;

18) $\left|\vec{U}_{i\,j}^{\,k} - \vec{U}_{i\,j}^{\,k-1}\right| \le \varepsilon$. If conditions are satisfied to proceed to point 19, otherwise – to point $n-1$;

19) The end.

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