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Regularized Recurrent-Iterative Algorithms of Weight Matrix Estimation of Nonlinear Dynamic Filters

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ABSTRACT: The formation and construction of regularized recurrent-iterative algorithms for estimating the weight matrix of nonlinear dynamic filters are considered. Various estimation algorithms are analyzed using explicit and implicit iterative procedures. The algorithms given for synthesis of filter to be included in the control loop of a nonlinear system make it possible to perform a simple filtration procedure and provide high accuracy of control using regular iterative procedures in the case when the equations of the control object and the probabilistic characteristics of the perturbations are known.

KEYWORDS: nonlinear dynamic filter, weight matrix, recurrent-iterative algorithm, estimation, regularization.

I. INTRODUCTION

The practical solution of the overwhelming majority of filtration problems is currently based on the use of linear estimation methods. As it is known [1, 2], a linear estimation is optimal for Gaussian random processes described by a linear auto regression model. In a number of problems [2-4], which are important from the application point of view, the accuracy of the linear estimates is insufficient. The question of increasing the accuracy of estimates is especially actual in problems where it is required to minimize the duration of the observation time necessary to achieve a given level by the error variance of the estimates. In order to improve the accuracy of estimates, and consequently, increase the rate of convergence, it is obvious to refer to the nonlinear methods of filtration.

Nowadays, the problem of constructing nonlinear estimates has developed significantly. It is worth to emphasize the results of [2,3,5], which provide considerable approaches to the construction of recurrent nonlinear filters for random processes described by the nonlinear recurrence equations. At the same time, suboptimal filtering algorithms have been developed noticeably [6, 7], which make it possible to construct estimates with accuracy exceeding the accuracy of linear estimates; this relates to those problems wherein linear estimates are not optimal. The increasing of estimation accuracy can be achieved by involving regular procedures to the evaluation, especially when there are unavoidable errors in stating the model of the control object used in the filtering algorithm.

II. TEXT DETECTION

Let the equations describing the dynamics of the control object have the form:

$$x_k = F_k(x_{k-1}, u_{k-1}) \quad (k = 1,2,...), \tag{1}$$

where $x_k = (x_k^1, x_k^2, ..., x_k^n)$ is the n-dimensional vector of the coordinates of the object state at a discrete time moment k; $F_k = (F_k^1, F_k^2, ..., F_k^n)$ is the known vector function of its arguments; u_k - control action. It is possible to include conditionally into the vector x_k also the vector of object parameters.

The equation of the measurement process has the form

$$z_k = z_k(x_k, v_k) \quad (k = 1,2, ...), \tag{2}$$

where – $z_k = (z_k^1, z_k^2, ..., z_k^n)$ is m-dimensional vector of the measured coordinates; v_k is the m-dimensional measurement error vector. The densities of the distribution of vectors $x_0, v_k \quad (k = 1,2, ...)$ to be known a priori.

The control algorithm is given in the form

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$$u_k = u_k(\hat{x}_k), \tag{3}$$

where $\hat{x}_k$ is an estimation of the state coordinate vector xk obtained at the k-th time moment by the recurrent filtration algorithm of the Kalman type

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k(y_k - \hat{z}_{k/k-1}). \tag{4}$$

In the algorithm (4)

$$\hat{x}_{k/k-1} = \hat{x}_{k/k-1}(\hat{x}_{k/k-1}, u_{k-1}), \quad \hat{z}_{k/k-1} = \hat{z}_{k/k-1}(\hat{x}_{k-1}, u_{k-1})$$

- are estimations of vectors x_k, z_k, determined by data $\hat{x}_{k-1}, u_{k-1}$ on the management process; K_k is the coefficient of filter gain.

When solving the problem of synthesizing a system for constructing the control algorithm (3), the simplified object equations are used of the form:

$$x_k = F_k^*(x_{k-1}, u_{k-1}). \tag{5}$$

In this case the vector function $F_k^* = (F_k^{*1}, F_k^{*2}, ..., F_k^{*n})$ is chosen as simple as possible, so that the errors in determining the coordinates of the object from the substitutions of F_k to F_k^* do not exceed the prescribed values.

Correspondingly, the equation of measurement (2) is simplified:

$$z_i = z_i^*(x_i, v_i). \tag{6}$$

The problem considered is usually solved on the basis of the known linearization methods, representing the functions (5), (6) in the form [1,6]

$$x_k = \theta_k(\hat{x}_{k-1}, u_{k-1}) + C_k x_{k-1},$$
$$z_k = E_k(\hat{x}_{k/k-1}) + H_k x_k + L_k v_k,$$

where $\theta_k(\hat{x}_{k-1}, u_{k-1}) = F_k^*(\hat{x}_{k-1}, u_{k-1}) - C_k \hat{x}_{k-1}$, $E_k(\hat{x}_{k/k-1}) = z_k^*(\hat{x}_{k/k-1}, v_k = 0) - H_k x_{k/k-1}$ is the matrix of order n with elements $c_k^{st} = \partial F_k^{*s}(\hat{x}_{k-1}, u_{k-1}) / \partial \hat{x}_{k-1}^t$ (s, t = 1,2,…,n); $H_k = [h_k^{rt}]$ is the matrix of size $m \times n$ with elements $h_k^{rt} = \partial z_k^{*r}(\hat{x}_{k/k-1}, v_k = 0) / \partial \hat{x}_{k/k-1}^t$; $L_k = [l_k^{rp}]$ is the matrix of order m with elements $l_k^{rp} = \partial z_k^{*r}(\hat{x}_k, v_k = 0) / \partial v_k^p$).

In some cases a "compromise" version of the filter, built on the basis of a linearized model is applicable for practical use

$$x_k = \theta_{0k}(\hat{x}_{k-1}, u_{k-1}) + C_{0k} x_{k-1}, \tag{7}$$
$$z_k = E_{0k}(\hat{x}_{k/k-1}) + H_{0k} x_k + L_{0k} v_k,$$

where $\theta_{0k}(\hat{x}_{k-1}, u_{k-1}) = F_k^*(\hat{x}_{k-1}, u_{k-1}) - C_{0k}\hat{x}_{k-1}$, $C_{0k} = [c_{0k}{}^{st}]$,

$$E_{0k}(\hat{x}_{k/k-1}) = z_k^*(\hat{x}_{k/k-1}, v_k = 0) - H_{0k}\hat{x}_{k/k-1}, \quad H_{0k} = [h_{0k}^{rt}], \quad L_{0k} = [l_{0k}^{rp}].$$

The drawback of the filter variants mentioned above is that the accuracy of the control is lowered anticipatorily due to of the residual between F_k and F_k^*, z_k and z_k^*. Therefore, it seems reasonable to determine the weight matrix K_i directly from the minimum risk condition [6]:

$$\mu_k^s = \min_{k_k^{st}(t=1,2,...,m)} \left\{ M(x_k^s - \hat{x}_{k/k-1}^s) - 2\sum_{t=1}^{m} k_k^{st} d_k^{st} + \sum_{t_1,t=1}^{m} k_k^{st_1} k_k^{st} \gamma_k^{t_1 t} \right\}, \tag{8}$$

assuming in the algorithm (4)

$$\hat{x}_{k/k-1} = F_k^*(\hat{x}_{k-1}, u_{k-1}), \ \hat{z}_{k/k-1} = z_k^*(x_{k/k-1}, v_k = 0).$$

In the equation (8) d_k^{st} and $\gamma_k^{t_1 t}$ are the elements of matrices

$$D_k = [d_k^{st}] = M\left\{[F_k(x_{k-1}, u_{k-1}) - F_k^*(\hat{x}_{k-1}, u_{k-1})][z_k(x_k, v_k) - z_k^*(\hat{x}_{k/k-1}, v_k = 0)]^T\right\} \tag{9}$$

and

$$\Gamma_k = [\gamma_k^{t_1 t}] = M\left\{[z_k(x_k, v_k) - \hat{z}_k^*(\hat{x}_{k/k-1}, v_k = 0)][z_k(x_k, v_k) - \hat{z}_k^*(\hat{x}_{k/k-1}, v_k = 0)]^T\right\} \tag{10}$$

of mixed initial moments the second order of vectors $x_k - \hat{x}_{k/k-1}$ and $z_k - \hat{z}_{k/k-1}$, and vectors $z_k - \hat{z}_{k/k-1}$ accordingly, k_k^{st} are elements of the matrix K_k.

Having equate to zero the derivative of the expression in the curly brackets (8) with respect to k_k^{st}, we obtain

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$$\sum_{t_1=1}^{m} k_k^{st} \gamma_k^{t_1 t} = d_k^{st} \quad (t = 1,2,...,m;\ s = 1,2,...,n),$$

or

$$K_k \Gamma_k = D_k \quad (k = 1,2,...). \tag{11}$$

In the filter described by equations (4), (8), (9), (10), all calculations in the control process are performed on the basis of model (6) using the gain factor K_k, which can be determined a priori by the relation (11).

In the equation (11) the operator Γ_k is a bounded positive by the definite symmetric matrix operator acting in the Hilbert space H under the assumption that the zero belongs to the spectrum of this operator; however, generally speaking, it is not its eigenvalue. Under the assumptions made, the problem of the solvability of equation (11) is incorrect [8]. If the solution of equation (11) still exists and is unique, then it is natural to try to apply various regular methods to find it [8-14].

The solution of the equation (11) can be expressed in the form $k_{k,\alpha}^j = (\alpha I + \Gamma_k)^{-1} d_k^j = g_\alpha(\Gamma_k) d_k^j$, where $g_\alpha(\lambda) = (\alpha + \lambda)^{-1}$, λ is the spectral parameter, $0 \le \lambda < \infty$.

To find an approximate solution of (11), we will use the iteration scheme [10]:

$$k_{k,r}^j = k_{k,r-1}^j - g(\Gamma_k)(\Gamma_k k_{k,r-1}^j - d_k^j), \quad r \ge 1,$$

where $k_{k,r}^j$ and d_k^j are j-th columns of matrix K_k^T and D_k^T, accordingly ($j = 1,2,..n$).

Due to inaccuracies in measurement and random rounding errors, the more realistic scheme is

$$k_{k,r}^j = k_{k,r-1}^j - g(\tilde{\Gamma}_k)(\tilde{\Gamma}_k k_{k,r-1}^j - \tilde{d}_k^j), \quad r \ge 1, \tag{12}$$

where g is the bounded and nonnegative function on the numerical line: $g(\tilde{\Gamma}_k)$ is the function of the operator;

$$\left\| \tilde{\Gamma}_k - \Gamma_k \right\| \le \eta, \quad \left\| \tilde{d}_k^j - d_k^j \right\| \le \delta.$$

In particularly, from the equation (12) at $g \equiv 1$ we obtain the simplest explicit iterative scheme $k_{k,r}^j = k_{k,r-1}^j - (\tilde{\Gamma}_k k_{k,r-1}^j - \tilde{d}_k^j)$; at $g(\lambda) = (\alpha + \lambda)^{-1}$, $\alpha > 0$, we achieve an implicit scheme $(\alpha I + \tilde{\Gamma}_k)(k_{k,r}^j - k_{k,r-1}^j) = -(\tilde{\Gamma}_k k_{k,r-1}^j - \tilde{d}_k^j)$ or $(\alpha I + \tilde{\Gamma}_k) k_{k,r}^j = \alpha k_{k,r-1}^j + \tilde{d}_k^j$, I is identity operator.

We will use the following a posteriori stopping rule for the residual:

$$\tau = \min\left(r \ge 0 : \left\| \tilde{\Gamma}_k k_{k,r}^j - \tilde{d}_k^j \right\| \le \mu \right).$$

Here τ is the stopping point; $k_{k,0}^j$ is the initial approximation; $\mu > 0$ is stopping level. It is considered that the values η, δ are small, and the task is stated so to conform the stopping level μ with these small parameters, i.e. choose $\mu = \mu(\eta, \delta)$, in the limit η, $\delta \to 0$, to obtain a regularization of the problem or convergence on the average to an exact solution of the original problem.

It is possible to demonstrate [10-12], that if the following relation is hold true

$$\mu \ge b\left(\left\| k_{k,*}^j \right\| \eta + \delta \right), \ b > 1 \tag{13}$$

and besides, $\mu = \mu(\eta, \delta) \to 0$ at $\eta, \delta \to 0$, then

$$\lim_{\eta, \delta \to 0} E \left\| k_{k,\tau}^j - k_{k,*}^j \right\| = 0,$$

where $k_{k,*}^j$ - is an exact solution of the problem (11), the nearest to $k_{k,0}^j$ by norm H.

III. EXPERIMENTAL RESULTS

Comparison of implicit algorithms with known explicit iterative algorithms [10-14] shows that the orders of their optimal estimates are the same. The advantage of explicit algorithms is that they do not require the operator's inversion, but require only calculation of operator values on successive approximations. However, implicit algorithms have the following important advantages. In an explicit algorithm, a parameter α is imposed by upper bound [10,12],

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which in practice can lead to the need for a large number of iterations. In implicit algorithms, there is no upper bounds on $\alpha > 0$. This allows us to consider $\alpha > 0$ to be arbitrarily large independently of $\|\Gamma_k\|$. In this respect, the optimal estimation for the implicit algorithm can be obtained already at the first steps of the iterations. This explains the wide use, along with explicit iterative algorithms, of implicit iterative schemes in the practice of solving operator equations. To solve equation (11), we can also use the following implicit iteration algorithm [12]:

$$(I + \alpha\tilde{\Gamma}_k)k^j_{k,r(\delta,\eta)} = k^j_{k,(r-1)(\delta,\eta)} + \alpha\tilde{d}^j_k, \quad k^j_{k,0(\delta,\eta)} = 0. \tag{14}$$

The iterative algorithm (14) is written in the form

$$k^j_{k,r(\delta,\eta)} = g_r(\tilde{\Gamma}_k)\tilde{d}^j_k,$$

where $g_r(\lambda) = \lambda^{-1}[1-(1+\alpha\lambda)^{-r}] \geq 0$.

On the basis of the iterative regularization principle, it can be shown [10-12] that if the stopping time $\tau(\delta,\eta)$ of the iterative process (13) is selected based on the rule

$$\left.\begin{array}{c}\left\|\tilde{\Gamma}_k k^j_{k,r(\delta,\eta)} - \tilde{d}^j_k\right\| > \varepsilon, \ (r<\tau), \\ \left\|\tilde{\Gamma}_k k^j_{k,\tau(\delta,\eta)} - \tilde{d}^j_k\right\| \leq \varepsilon,\end{array}\right\}, \quad \varepsilon = b\left(\delta + \left\|k^{j^*}_k\right\|\eta\right), \ b>1. \tag{15}$$

then the following limit relation holds $(\delta+\eta)^2\tau(\delta,\eta) \to 0$, $k^j_{k,\tau(\delta,\eta)} \to k^{j^*}_k$ at $\delta \to 0$, $\eta \to 0$. This follows from the fact that

$$\begin{gathered} k^j_{k,r(\delta,\eta)} - k^{j^*}_k = g_r(\tilde{\Gamma}_k)\tilde{d}^j_k - k^{j^*}_k = -\tilde{K}_r k^{j^*}_k + \tilde{K}_r k^{j^*}_k + g_r(\tilde{\Gamma}_k)\tilde{d}^j_k - k^{j^*}_k = \\ = -\tilde{K}_r k^{j^*}_k + (I - \tilde{\Gamma}_k g_r(\tilde{\Gamma}_k))k^{j^*}_k - k^{j^*}_k + g_r(\tilde{\Gamma}_k)\tilde{d}^j_k = \\ -\tilde{K}_r k^{j^*}_k + k^{j^*}_k - \tilde{\Gamma}_k g_r(\tilde{\Gamma}_k)k^{j^*}_k - k^{j^*}_k + g_r(\tilde{\Gamma}_k)\tilde{d}^j_k = -\tilde{K}_r k^{j^*}_k + g_r(\tilde{\Gamma}_k)(\tilde{d}^j_k - \tilde{\Gamma}_k k^{j^*}_k), \end{gathered} \tag{16}$$

where $k^j_{k,r(\delta,\eta)} = g_r(\tilde{\Gamma}_k)\tilde{d}^j_k$, $\tilde{K}_r = I - \tilde{\Gamma}_k g_r(\tilde{\Gamma}_k)$.

In the equation (16)

$$\left\|\tilde{K}_r k^{j^*}_k\right\| \to 0, \ r \to \infty, \ \eta \to 0.$$

Suppose that for the initial approximation $k^j_{k,0(\delta,\eta)}$ the residual is larger than the stopping level ε, i.e. $\left\|\Gamma_k k^j_{k,0(\delta,\eta)} - \tilde{d}^j_k\right\| > \varepsilon$.

The following transformation takes place

$$\begin{gathered} \left\|\tilde{d}^j_k - \tilde{\Gamma}_k k^{j^*}_k\right\| \leq \left\|\tilde{d}^j_k - d^j_k\right\| + \left\|d^j_k - \tilde{\Gamma}_k k^{j^*}_k\right\| = \left\|\tilde{d}^j_k - d^j_k\right\| + \left\|\Gamma_k k^{j^*}_k - \tilde{\Gamma}_k k^{j^*}_k\right\| \leq \\ \leq \delta + \left\|(\Gamma_k - \tilde{\Gamma}_k)k^{j^*}_k\right\| \leq \delta + \left\|k^{j^*}_k\right\|\eta. \end{gathered}$$

Then at $r = \tau(\delta,\eta)$ and $r < \tau(\delta,\eta)$ it can be written [10,12], correspondingly

$$\begin{gathered} \left\|\tilde{\Gamma}_k \tilde{K}_\tau k^{j^*}_k\right\| \leq \left\|\tilde{\Gamma}_k k^j_{k,\tau(\delta,\eta)} - \tilde{d}^j_k\right\| + \left\|\tilde{K}_\tau(\tilde{d}^j_k - \tilde{\Gamma}_k k^{j^*}_k)\right\| \leq \\ \leq b(\delta + \left\|k^{j^*}_k\right\|\eta) + (\delta + \left\|k^{j^*}_k\right\|\eta) = (b+1)(\delta + \left\|k^{j^*}_k\right\|\eta), \\ \left\|\tilde{\Gamma}_k \tilde{K}_n k^{j^*}_k\right\| \geq \left\|\tilde{\Gamma}_k k^j_{k,r(\delta,\eta)} - \tilde{d}^j_k\right\| - \left\|\tilde{K}_n(\tilde{d}^j_k - \tilde{\Gamma}_k k^{j^*}_k)\right\| \geq (b-1)(\delta + \left\|k^{j^*}_k\right\|\eta). \end{gathered} \tag{17}$$

Using expressions (13), (14), (17), it seems reasonable to use the following modification of the stopping rule

$$\begin{gathered} \left\|\tilde{\Gamma}_k k^j_{k,r} - \tilde{d}^j_k\right\| \leq b(\delta + \left\|k^j_{k,r}\right\|\eta), \quad b>1, \\ \left\|\tilde{\Gamma}_k k^j_{k,r} - \tilde{d}^j_k\right\| \geq b(\delta + \left\|k^j_{k,r'}\right\|\eta). \end{gathered}$$

To solve equation (11), nonlinear iterative algorithms are also applicable

$$k^j_{k,r+1,\delta} = k^j_{k,r,\delta} - \chi_r\xi_r, \quad \xi_r = \tilde{\Gamma}_k k^j_{k,r,\delta} - \tilde{d}^j_k, \quad k^j_{k,0,\delta} = 0,$$

$$\chi_r = \chi_r(\alpha) = \left(\left(\tilde{\Gamma}_k\right)^\alpha \xi_r, \xi_r\right) \Big/ \left(\left(\tilde{\Gamma}_k\right)^{\alpha+1}\xi_r, \xi_r\right),$$

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which are called α - methods [12,15]. At $\alpha \geq 1$ a posteriori choice with respect to the residual of the first r for which $\|\xi_r\| \leq b\delta + \zeta\eta = \eta(\delta,\eta)$ with given numbers $b > 1$, $\zeta > \|k_k^j\|$ transforms these methods into regularizing algorithms.

IV. CONCLUSION

The above algorithms for filter synthesis included into the control loop of a nonlinear system make it possible to perform a simple filtration procedure and provide high accuracy of control using regular procedures in the case when the equations of the control object and the probabilistic characteristics of the disturbances are fully known.

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