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Solving a Transportation Problem using a Heptagonal Fuzzy Number

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ABSTRACT: In this paper, we consider a Fuzzy Transportation Problem (FTP) in which the values of transportation costs are represented as heptagonal fuzzy numbers. We use the proposed method to solve the FTP. FTP can be converted into a crisp valued Transportation Problem using a new ranking method.

KEYWORDS: Fuzzy Transportation problem, Heptagonal fuzzy number and ranking

I INTRODUCTION

The Fuzzy Transportation Problem (FTP) is one of the special kinds of fuzzy linear programming problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Transportation problem was originally introduced and developed by Hitchcock in 1941, in which the parameters like transportation cost, demand and supply are crisp values. But in the present world the transportation parameters may be uncertain due to many uncontrolled factors. So to deal the problems with imprecise information Zadeh [12] introduced the concept of fuzziness. Many authors discussed the solution of FTP with various fuzzy numbers. Chen [2] introduced the concept of generalized fuzzy numbers to deal problems with unusual membership function. Pandian and Natarajan [8] proposed a new algorithm namely fuzzy zero point method to find optimal solution of a FTP with trapezoidal fuzzy numbers.Many researchers applied generalized fuzzy numbers to solve the real life problems. Kaur and Kumar [5, 6] solved FTP with generalized trapezoidal fuzzy numbers. Chandrasekaran.S, Kokila.G, Junu Saju [3] proposed a Ranking of Heptagon Number using Zero Suffix Method. In the present paper a FTP with heptagonal fuzzy numbers is introduced using Russell's Method, North West corner Method and Least Cost Method with a comparative study.

II PRELIMINARIES

A. Fuzzy set: [10]

A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval [0, 1]. (i.e.) $\mu_A : X \to [0,1]$

B. Normal fuzzy set: [9]

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that $\mu_A(x) = 1$.

C. Support of a fuzzy set:[10]

The support of a fuzzy set in the universal set X is the set that contains all the elements of X that have a non- zero membership grade in $\tilde{A}$. i.e.,Supp($\tilde{A}$) = {x ϵX/$\mu_{\tilde{A}}$ (x) > 0}.

D. α - Cut: [9]

Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, the $\alpha -$ cut, α_A ,is the crisp set $\alpha_A = \{x / A(x) \geq \alpha\}$

E. Fuzzy Number: [4]

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following properties

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i) A must be a normal fuzzy set.

ii) α_A must be a closed interval for every $\alpha \epsilon (0,1]$

iii) The support of A , O^{+}_{A} must be bounded.

III. HEPTAGONAL FUZZY NUMBERS (HFN)

In this section, a new form of fuzzy number called as Heptagonal Fuzzy number (HFN) is introduced which can be effectively used in solving many decision making problem. A fuzzy number $\widetilde{H}$ in R is said to be a heptagonal fuzzy number if its membership function $\mu_H : R \to [0,1]$ has the following characteristics.

We denote the heptagonal fuzzy number by $\widetilde{H}$ = ($h_1,h_2,h_3,h_4,h_5,h_6,h_7$)

$$\mu_{\widetilde{H}}(x) = \begin{cases} 0 & \text{for } x < h_1 \\ \frac{1}{2}\left(\frac{x-h_1}{h_2-h_1}\right) & \text{for } h_1 \le x \le h_2 \\ \frac{1}{2} & \text{for } h_2 \le x \le h_3 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-h_3}{h_4-h_3}\right) & \text{for } h_3 \le x \le h_4 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{h_5-x}{h_5-h_4}\right) & \text{for } h_4 \le x \le h_5 \\ \frac{1}{2} & \text{for } h_5 \le x \le h_6 \\ \frac{1}{2}\left(\frac{h_7-x}{h_7-h_6}\right) & \text{for } h_6 \le x \le h_7 \\ 0 & \text{for } x \ge h_7 \end{cases}$$

A. Graphical Representation of Heptagonal fuzzy number

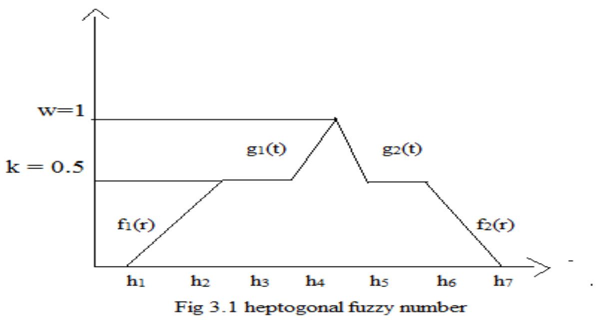

Fig 3.1 heptogonal fuzzy number

B. Parametric form of Heptagonal Fuzzy Number

The parametric form of Heptagonal Fuzzy Number is defined as $\widetilde{H} = (f_1(r), g_1(t), g_2(t), f_2(r))$ for $r \epsilon [0,k]$ & $t \epsilon [k,1]$ Where $f_1(r)$ &$g_1(t)$ are bounded left continuous non decreasing functions over $[0,w_1]$ &$[k,w_2]$ respectively and $f_2(r)$ &$g_2(t)$ are bounded left continuous non increasing functions over $[0,w_1]$ &$[k,w_2]$ respectively and $0 \le w_1 \le k$, $k \le w_2 \le 1$

C. Arithmetic Operations on Heptagonal Fuzzy numbers

Let $\tilde{A}_H$ = ($a_1, a_2, a_3, a_4, a_5, a_6, a_7$) & $\tilde{B}_H$ = ($b_1, b_2, b_3, b_4, b_5, b_6, b_7$) be two Heptagonal Fuzzy Numbers then addition and subtraction can be performed as

$\tilde{A}_H + \tilde{B}_H$ = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7)

$\tilde{A}_H - \tilde{B}_H$ = (a_1-b_7, a_2-b_6, a_3-b_5, a_4-b_4, a_5-b_3, a_6-b_2, a_7-b_1)

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IV. ROBUST'S RANKING

Robust's ranking which satisfy compensation, linearity, and additively properties and provides results which are consistent with human intuition. If $\tilde{A}_H$ is a fuzzy number then the ranking is defined by

$$R(\tilde{A}_H) = \int_0^1 0.5(a_{h\alpha}^L, a_{h\alpha}^U)\, d\alpha$$

Where $(a_{h\alpha}^L, a_{h\alpha}^U)$ is the α level cut of fuzzy number $\tilde{A}_H$.

A.. Proposed Ranking

If $(h_1,h_2,h_3,h_4,h_5,h_6,h_7)$ are a heptagonal fuzzy numbers then

$(a_{h\alpha}^L, a_{h\alpha}^U)$ = $\{(h_2 - h_1)\alpha + h_1,\ h_4 - (h_4 - h_3)\alpha,\ (h_6 - h_5)\alpha + h_5,\ h_7 - (h_7 - h_5)\alpha\}$

Hence, the fuzzy version of heptagonal ranking is

$$= \int_0^1 0.5\{(h_2 - h_1)\alpha + h_1,\ h_4 - (h_4 - h_3)\alpha, (h_6 - h_5)\alpha + h_5,\ h_7 - (h_7 - h_5)\alpha\}\, d\alpha \longrightarrow (4.1)$$

V. FUZZY VERSION OF PROPOSED METHOD

Even though there are many methods to find the basic feasible solution for a transportation problem we have analysed a fuzzy version of Russell's Method, North west corner Method and Least cost Method for solving transportation problem. .

Proposed Algorithm:

Step 1:

Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one, if it is not.

Step 2:

Using the above ranking as per equation (4.1), the fuzzy transportation problem is converted into a crisp value problem and solved using Russell's Method, North West corner Method and Least Cost Method**.**

Step 3:

Russell's Method:

(i) In the reduced FTP, identify the row and column difference considering the least two numbers of the respective row and column.

(ii) Select the maximum among the difference (if more than one, then selects any one) and allocate the respective demand/supply to the minimum value of the corresponding row or column.

(iii) We take the difference of the corresponding supply and demand of the allocated cell which leads either of one to zero, eliminating corresponding row or column (eliminate both row and column if both demand and supply is zero)

(iv) Repeat the steps ((i),(ii) and(iii) until all the rows and columns are eliminated.

(v) Finally total minimum cost is calculated as sum of the product of the cost and the allocated value.

Step 4:

North West corner Method:

(i) Allocate the maximum amount allowable by the supply and demand constraints to the variable x_{11} (i.e., the cell in the top left corner of the transportation of the transportation tableau.)

(ii) If a column (or row) is satisfied, cross it out. The remaining decision variables in that column (or row) are non-basic and are set equal to zero. If a row or column are satisfied simultaneously, cross only one out. (it does not matter which)

(iii) Adjust supply and demand for the non – crossed out rows and columns.

(iv) Allocate the maximum feasible solution amount to the first variable non crossed and element in the next column or (row).

(v) When exactly one row or column is left, all the remaining variables are basic and are assigned the only feasible allocation.

Step 5:

Least Cost Method:

(i) Assign as much as possible to the cell with the smallest unit cost in the entire tableau. If there is a tie then choose arbitrarily.

(ii) Cross out the row or column which has satisfied supply or demand. If a row or column is both satisfied

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then cross out only one of them.
(iii) Adjust the supply and demand for those rows and columns which are crossed out.
(iv) When exactly one row or column is left, all the remaining variables are basic and are assigned the only feasible allocation.

VI. NUMERICAL EXAMPLE

A. Consider the following fuzzy transportation problem.

A company has three sources $S_1, S_2, S_3,$ and three destinations $D_1, D_2, D_3,$ the fuzzy transportation cost for unit quantity of the product from i[th] source to j[th] destination is C_{ij}

$$\text{Where, } C_{ij} = \begin{bmatrix} (3,6,2,1,5,0,4) & (2,3,1,4,3,6,5) & (2,4,3,1,6,5,2) \\ (2,7,7,6,3,2,1) & (1,3,5,7,9,11,13) & (0,1,2,4,6,0,5) \\ (3,6,3,2,1,8,7) & (3,4,3,2,1,1,0) & (2,4,6,8,10,12,14) \end{bmatrix}$$

and the fuzzy availability of the supply are (2,2,1,2,1,1,0), (3,2,1,4,5,0,1), (2,4,3,1,6,5,2) and the fuzzy availability of the demand are (0,1,2,4,6,0,5), (0,4,6,4,6,2,0), (2,7,7,6,3,2,1) respectively.

Solution:

Step 1

Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one, if it is not.

Table -6.1 Fuzzy Transportation problem

Destinations / Sources	D_1	D_2	D_3	SUPPLY
S_1	(3,6,2,1,5,0,4)	(2,3,1,4,3,6,5)	(2,4,3,1,6,5,2)	**(2,2,1,2,1,1,0)**
S_2	(2,7,7,6,3,2,1)	(1,3,5,7,9,11,13)	(0,1,2,4,6,0,5)	**(3,2,1,4,5,0,1)**
S_3	(3,6,3,2,1,8,7)	(3,4,3,2,1,1,0)	(2,4,6,8,10,12,14)	**(2,4,3,1,6,5,2)**
Demand	**(0,1,2,4,6,0,5)**	**(0,4,6,4,6,2,0)**	**(2,7,7,6,3,2,1)**	

Step 2

Using the proposed ranking method, the fuzzy transportation problem is converted into a crisp value as

$$\begin{bmatrix} (3,6,2,1,5,0,4) & (2,3,1,4,3,6,5) & (2,4,3,1,6,5,2) \\ (2,7,7,6,3,2,1) & (1,3,5,7,9,11,13) & (0,1,2,4,6,0,5) \\ (3,6,3,2,1,8,7) & (3,4,3,2,1,1,0) & (2,4,6,8,10,12,14) \end{bmatrix}$$

$R(\tilde{A}_H) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u)d\alpha$

$= \int_0^1 0.5\{(h_2 - h_1)\alpha + h_1, h_4 - (h_4 - h_3)\alpha, (h_6 - h_5)\alpha + h_5, h_7 - (h_7 - h_5)\alpha\}d\alpha$

The $\alpha -$ Cut of the fuzzy number(3,6,2,1,5,0,4) is(a_α^l, a_α^u) = (3α+ 3,1-1α,-5α+5,4+1α)

$$= \int_0^1 0.5(3\alpha + 3,\ 1 - 1\alpha, -5\alpha + 5, 4 + 1\alpha)d\alpha$$

= 0.5(12) = 6

$R(a_{1,1}) = R(3,6,2,1,5,0,4) = 6$

Similarly, we get $R(a_{1,2}) = 6.75$, $R(a_{1,3}) = 7.25$, $R(a_{1,4}) = 2.5$, $R(a_{2,1}) = 8.25$, $R(a_{2,2}) = 14.5$
$R(a_{2,3}) = 7$, $R(a_{2,4}) = 10.5$, $R(a_{3,1}) = 7.75$, $R(a_{3,2}) = 16.5$, $R(a_{3,3}) = 10.5$, $R(a_{3,4}) = 14.5$,
$R(a_{4,1}) = 6$, $R(a_{4,2}) = 7$, $R(a_{4,3}) = 7.5$.

Hence, the crisp value is given in table (6.2)

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Table-6.2 Crisp Transportation Problem

Destinations / Sources	D_1	D_2	D_3	SUPPLY
S_1	6	6.75	7.25	2.5
S_2	8.25	14.5	7	10.5
S_3	7.75	16.5	10.5	14.5
DEMAND	6	7	7.5	

Step 3

Using the dummy variable, Repeating the same procedure we get the following table,

Table-6.3 IBFS by using fuzzy version of Russell's method

Destinations / Sources	D_1	D_2	D_3	Dummy variable	SUPPLY	Row difference
S_1	6	7 **6.75**	7.25	**0**	~~2.5~~ 0	0.75
S_2	8.25	14.5	7	**0**	10.5	1.25
S_3	7.75	16.5	10.5	**0**	14.5	2.75
DEMAND	6	7 ~~4.5~~	7.5	**7**	**Eliminate row A**	
Column difference	1.75	**7.75**	0.25	**0**		

Therefore,The total transportation cost using **Russell's method** is

Minimize Z = 7(6.75) +(14.5)3+(16.5)1.5+(8.5)4+(14.5)7.75+7(7.5)

= 43.5+24.75+112.37+52.5+47.25

= **280.375**

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Step 4

Table-6.4 IBFS by using fuzzy version of North West corner method

Destinations / Sources	D_1	D_2	D_3	Dummy variable	SUPPLY
S_1	**2.5** **6**	6.75	7.25	**0**	**2.5**
S_2	**3.5** **8.25**	**7.0** **14.5**	**7**	**0**	**10.5**
S_3	**7.75**	**16.5**	**7.5** **10.5**	**7.0** **0**	**14.5**
DEMAND	**6**	**7**	**7.5**	**7**	

Therefore, The total transportation cost using **North West corner method is**

Minimize Z = (2.5) 6+ (3.5) 8.25 +(7.0)14.5 +(7.5) 10.5 +(7.0) 0
= 15 + 28.875 + 101.5 + 78.75
= **224.125**

Step 5

Table-6.5 IBFS by using fuzzy version of Least Cost method

Destinations / Sources	D_1	D_2	D_3	Dummy variable	SUPPLY
S_1	6	6.75	7.25	**2.5** ~~0~~	2.5
S_2	8.25	14.5	7	**4.5** ~~0~~	~~10.5~~ **6.0**
S_3	7.75	16.5	10.5	0	14.5
DEMAND	6	7	7.5	7	

.

Therefore, The total transportation cost using **Least Cost method is**

Minimize Z = (2.5) 0 + (4.5) 0 + (6) 7 + (10.5) 1.5 + (16.5) 7 + (6) 7.75
= 15.75 + 115.5 +46.5 + 42=**219.75**

A. Comparative study:

Table-6.1.1 Comparison Table

METHOD	OPTIMUM SOLUTION
Russell's Method	280.375

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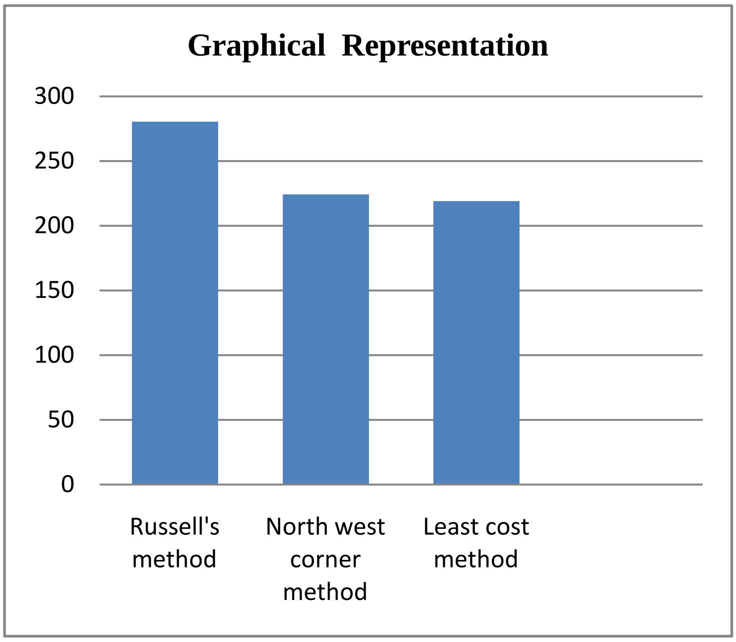
North-west corner Method	224.125
Least cost method	219.75

Fig 6.1 Graphical Representation



VII. CONCLUSION

In the above problem we have obtained an optimal solution for a FTP using heptagonal fuzzy number. A new approach called Fuzzy version of Russell's Method, North West corner and Least cost method is analysed . By comparing these methods, least cost method gives a better result than the other methods. This method is an easy approach compared to other methods to solve a fuzzy transportation problem.

REFERENCES

[1] Annie Christi, M.S., Kasthuri,B., "Transportation Problem with Pentagonal Intuitionistic Fuzzy Numbers Solved Using Ranking Technique and Russell's Method", International Journal of Engineering Research and Applications,Vol.6, pp.82-86 ,2016.

[2] Chen, S.H., "Operations on fuzzy numbers with function principle", Tamkang Journal of Management Sciences, vol.6, pp.13-25,1985.

[3] Chandrasekaran, S., Kokila, G.,and Junu Saju., "Ranking of Heptagon Number using Zero Suffix Method", International Journal of Science and Research, Vol. 4,pp.2256-2257, 2015.

[4] Chandrasekaran, S., Kokila, G.,and Junu Saju.," Fuzzy Transportation Problem of Hexagon Number with α– cut and Ranking Technique", International Journal of Scientific Engineering and AppliedScience, Vol.1,pp.530-538, 2015

[5] Kaur, A., and Kumar, A., "A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers", Applied soft computing, Vol.12,pp. 1201-1213,2012.

[6] Kaur, A.,and Kumar, A., "A method for solving fuzzy transportation problems using ranking function", Applied Mathematical Modelling, Vol.35 , pp.5652-5661,2011.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 4, Issue 1 , January 2017

[7] Malini, P., and Ananthanarayanan, P., "Solving Fuzzy Transportation Problem using Ranking of Trapezoidal Fuzzy Numbers", IOSR Journal of Mathematics , Vol. 11, pp. 08-13,2015.
[8] Pandian, P., and.Natarajan, G., "A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem", Applied Mathematical Sciences, Vol.4,pp.79-90, 2010
[9] Rajarajeswari, P., Sahaya Sudha, A., and Karthika, R., "A New Operation on Hexagonal Fuzzy Number", International Journal of Fuzzy Logic Systems, Vol.3,pp.15-26,2013.
[10] Rathi, K., Balamohan, S., "Representation and Ranking of Fuzzy Numbers with Heptagonal Membership Function value and Ambiguity Index", Applied Mathematical Sciences, Vol. 8, pp.4309 – 4321, 2014.
[11] Thamaraiselvi, A., Santhi, R., "Solving Fuzzy Transportation problem with Generalized Hexagonal Fuzzy Numbers", IOSR Journal of Mathematics ,Vol. 11, pp 08-13, 2015.
[12] Zadeh, L.A., " Fuzzy sets", Information and Control, Vol.8, pp.338-353,1965.