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# Elimination-Minimization Principle: Fitting of Gompertz Curve to Numerical Data

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**ABSTRACT**: The elimination-minimization approach of fitting of mathematical curve to numerical data, introduced by Chakrabarty in 2011, where the usual principle of least squares is applied in estimating each of the parameters involved in the curve to be fitted separately has already been applied successfully in fitting of polynomial curve and exponential curve to numerical data. In this study, attempt has been made on fitting of a non-polynomial curve namely Gompertz curve to numerical data by the same approach. This paper describes the derivation of the method of fitting of Gompertz curve by the approach and numerical example in order to show the application of the method to numerical data.

**KEYWORDS**: Elimination-minimization principle, Gompertz curve, fitting to numerical data.

## I. INTRODUCTION

The method of least squares is indispensible and is widely used method of curve fitting to numerical data. The method of *Mansfield* (1877*a* , 1877*b*), Stigler (1977 , 1981) et al} and it has been established with the works of the renowned statistician *Adrian* (1808), the German Astronomer *Gauss* {Gauss (1809*a* , 1809*b* , 1929), Hall (1970), Buhler (1981), Sheynin (1979), Sprott (1978), Stigler (1977), et al], the mathematicians viz. *Ivory* (1825), *Hagen* (1837), *Bassel* (1838), *Donkim* (1857), *Herscel* (1850), *Crofton* (1870) etc..

In fitting of a curve by the method of least squares, the parameters of the curve are estimated by solving the normal equations which are obtained by applying the principle of least squares with respect to all the parameters associated to the curve jointly (simultaneously). However, for a curve of higher degree polynomial and/or for a curve having many parameters, the calculation involved in the solution of the normal equations becomes more complicated as the number of normal equations then becomes larger. Moreover, in many situations, it is not possible to obtain normal equations by applying the principle of least squares with respect to all the parameters simultaneously. These lead to think of searching for some other approach of estimation of parameters. For this reason, an approach of estimation of parameters that can be termed as **elimination-minimization approach** had been introduced (Chakrabarty, 2011) where the usual principle of least squares is applied to each parameter separately. This is done, for each parameter, by obtaining one model, containing the single parameter to be estimated. Later on, this approach was termed as stepwise least squares method (Chakrabarty, 2014). The principle involved in the approach has been termed in 2015 as elimination-minimization principle (Rahman & Chakrabarty, 2015*a*). The approach has already been applied successfully in fitting of polynomial curve to numerical data {Rahman & Chakrabarty (2015*a*, 2015*b*, 2015*c*, 2015*d*), Chakrabarty (2016*a*)}. Moreover, the same has also been applied successfully in fitting of exponential curve to numerical data Chakrabarty (2016*b*). In this study, attempt has been made on fitting of a non-polynomial curve namely Gompertz curve to numerical data by the same approach. This paper describes the derivation of the method of fitting of Gompertz curve by the approach and numerical example in order to show the application of the method to numerical data.

## II. FITTING OF GOMPERTZ CURVE

Let

$$Y_1 , Y_2 , Y_3 , \ldots\ldots\ldots\ldots\ldots , Y_n$$

be the observed values assumed by the dependent variable $Y$ corresponding to the values

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$$X_1\ ,X_2\ ,X_3\ ,\ \ldots\ldots\ldots\ldots\ldots\ ,X_n$$

of the independent variable $X$.
Hence forward, one operator namely $\Delta$ is used with the following definition:

$$\Delta Y_i = Y_{i+1} - Y_i$$
$$\&\ \Delta X_i = X_{i+1} - X_i$$
$$(\text{for } i = 1, 2, 3, \ldots\ldots\ldots\ldots).$$

Before discussing the method of fitting of Gompertz curve, let us discuss the method of fitting of the two forms of modified exponential curve.

**First form of the modified exponential curve:**
The first form of the modified exponential curve is expressed as

$$Y = \mu + \alpha .exp(\beta X) \qquad (2.1)$$

where $\mu$ , $\alpha$ and $\beta$ are the three parameters.

**First, let us consider the situation where the observations perfectly follow the modified exponential low described by equation (2.1)**

In this situation,

$$Y_i = \mu + \alpha \,.\, exp(\beta X_i) \quad , \quad (i = 1, 2, \ldots\ldots\ldots, n) \qquad (2.2)$$

which implies $\Delta Y_i = \alpha\ \Delta\ exp(\beta X_i)$

$$= [exp\{(\beta \,.\, (X_i + h)\} - exp(\beta X_i)]$$

i.e. $\Delta Y_i = \alpha \,.\, exp(\beta X_i)\{exp(\beta h) - 1\}$

i.e. $Z_i = A \,.\, exp(\beta X_i) \quad , \quad (i = 1, 2, \ldots\ldots\ldots, n-1) \qquad (2.3)$

where $Z_i = \Delta Y_i$ , $A = \alpha \,.\, \{exp(\beta h) - 1\}$ & $h$ is the interval of the values of $X$.
Taking logarithm of both side of (2.3),

$$\log Z_i = \log A + \beta \,.\, X_i$$

i.e. $Z_i' = A' + \beta X_i \quad , \quad (i = 1, 2, \ldots\ldots\ldots, n-1) \qquad (2.4)$

where $Z_i' = \log Z_i$ and $A' = \log A$.
From (2.4),

$$\Delta Z_i' = \beta \,.\, \Delta X_i$$

i.e. $\beta = (\Delta Z_i') / (\Delta X_i)$ , $(i = 1, 2, \ldots\ldots\ldots, n-2) \qquad (2.5)$

Since the observed values perfectly follow the modified exponential law, all the values of

$$(\Delta Z_i') / (\Delta X_i) \quad , \quad (i = 1, 2, \ldots\ldots\ldots, n-2)$$

are equal and the common value is the value of the parameter $\beta$ .
Substituting this value of $\beta$ in (3.4) one obtains $(n-1)$ equations in $A' = \log A$.
Solving any one of these equations, the value of $A' = \log A$ can be found out and then taking the antilogarithm of it the value of $A$ can be obtained.
Consequently, $\alpha$ can be obtained from

$$A = \alpha \,.\, \{exp(\beta h) - 1\} \qquad (2.6)$$

Finally, the parameter $\mu$ can be obtained from any one of the equations (3.2).

**Now, let us consider the situation where the observations do not perfectly follow the modified exponential low described by equation (2.1)**
It may be the situation that the observed values may not perfectly follow the exponential law described by equation (2.1) and the observations may suffer from error.
In the situation, where either or both of the two possibilities do influence, the observed values will follow the model

$$Y_i = \mu + \alpha \,.\, exp(\beta X_i)\,\delta_i \ , \ (i = 1, 2, \ldots\ldots\ldots, n) \qquad (2.7)$$

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where $\delta_i$ is the error associated to $Y_i$ .
This implies,

$$Z_i = A \,.\, exp\,(\beta X_i) \times \varepsilon_i{}' \quad , \qquad (i = 1, 2, \ldots\ldots\ldots , n-1) \qquad (2.8)$$

where $\varepsilon_i{}'$ is the error associated to $Z_i$ .
Taking logarithm of both side of (2.8)

$$\log Z_i = \log A + \beta . X_i + \epsilon_i$$

$$\text{i.e. } Z_i{}' = A' + \beta X_i + \epsilon_i \;, \quad (i = 1, 2, \ldots\ldots\ldots , n-2) \qquad (2.9)$$

where $Z_i{}' = \log Z_i$ , $A' = \log A$ & $\epsilon_I = \log \varepsilon_i{}'$.
From (2.9)

$$\Delta Z_i' = \beta . \Delta X_i + \Delta \epsilon_i \quad , \quad (i = 1, 2, \ldots\ldots\ldots , n-2) \qquad (2.10)$$

Applying the elimination-minimization principle the estimate of the parameters $\beta$ and $A'$ can be obtained as

$$\hat{\beta}_{(EM)} = \frac{1}{h(n-2)} \sum_{i=1}^{n-2} \Delta Z_i{}' \qquad (2.11)$$

$$\& \quad \hat{A}' = \bar{Z}' - \hat{\beta}_{(EM)} \bar{X} \qquad (2.12)$$

Accordingly, $A = \text{antilog}\,(\hat{A}')$

$$\text{i.e. } A = anti\log(\bar{Z}' - \hat{\beta}_{(EM)} \bar{X}) \qquad (2.13)$$

Consequently, estimate of $\alpha$ is obtained as

$$\hat{\alpha}_{(EM)} = \frac{A}{(e^{\hat{\beta}_{(EM)}.h} - 1)}$$

$$\text{i.e. } \hat{\alpha}_{(EM)} = \frac{anti\log(\bar{Z}' - \hat{\beta}_{(EM)} \bar{X})}{(e^{\hat{\beta}_{(EM)}.h} - 1)} \qquad (2.14)$$

Finally from the equations (3.7), one can obtain the estimates of the parameter $\mu$ as

$$\hat{\mu}_{(EM)} = \bar{Y} - \frac{\hat{\alpha}_{(EM)}}{n} \sum_{i=1}^{n} e^{\hat{\beta}_{(EM)}.X_i} \qquad (2.15)$$

**Second form of the modified exponential curve:**

The second form of the modified exponential curve is expressed as

$$Z = P + Q \,.\, S^X \qquad (2.15)$$

where (i) Z is the dependent variable which depends upon the variable $X$
and (ii) $P$ , $Q$ & $R$ are the parameters.
Let

$$Z_1 , Z_2 , Z_3 , \ldots\ldots\ldots\ldots\ldots , Z_n$$

be the values assumed by the dependent variable $Z$ corresponding to the values

$$X_1 , X_2 , X_3 , \ldots\ldots\ldots\ldots\ldots , X_n$$

of the independent variable $X$.
Then

$$Z_i = P + Q \,.\, S^{Xi} \qquad (2.16)$$

The equation (2.15) can be expressed as

$$Z' = M . S^X \qquad (2.17)$$

where $Z' = \Delta.Z$ & $M = Q.(S^h - 1)$ .

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Again, equation (2.17) can be written as

$$Z'' = M' + S'.X \tag{2.18}$$

where $Z'' = \log Z'$ , $M' = \log M$ & $S' = \log S$ .

In order to obtain estimated value of $Y_{i+1}$ from the observed value of $Y_i$, the equation that can describes the dependence of $Y_{i+1}$ on $Y_i$ is to be known.
From equation (2.18)

$$Z_i'' = M' + S'.X_i \tag{2.19}$$

Which implies $Z_{i+1}'' - Z_i'' = S'(X_{i+1} - X_i)$

i.e. $$Z_{i+1}'' = Z_i'' + S'.\Delta X_i \tag{2.20}$$

where $\Delta X_i = X_{i+1} - X_i$

($i$ = 1 , 2 , ………… , $n$ - 1).

Equation (2.20) is the recurrence relation whose application can provide estimate of $Z_{i+1}''$ from observed $Z_i''$ . This equation consists of the single parameter $S'$ . The estimate of $S'$ obtained by applying the step-wise method [(Chakrabarty (2011 , 2014)] is given by

$$\hat{S}'_{(EM)} = \frac{\sum_{i=1}^{n-1} \frac{\Delta Z_i''}{\Delta X_i}}{n-1} \tag{2.21}$$

**Note:**
If the values of the independent variable are at equal interval of length $h$, the estimate of $S'$ becomes

$$\hat{S}'_{(EM)} = \frac{\sum_{i=1}^{n-1} \Delta Z_i''}{h.(n-1)}$$

Using this estimate in equation (2.18), estimate of the parameter $M'$ can be obtained by the same method. The estimate, so obtained, is found to be

$$\hat{M}' = \bar{Z}'' - \hat{S}'_{(EM)}.\bar{X} \tag{2.22}$$

Now,

$$M' = \log M \quad \& \quad S' = \log S .$$

Therefore, estimates of the parameters $S$ and $M$ are obtained as

$$\hat{S} = anti\log(S') \quad \& \quad \hat{M} = anti\log(M') \tag{2.23}$$

Consequently, estimates of the parameters $Q$ & $B$ are found to be

$$\hat{Q} = \frac{M}{S^h - 1} \quad \& \quad \hat{B} = anti\log(Q) \tag{2.24}$$

Finally, estimate of the parameter $P$ can be obtained from equation (2.16) by the same method and it is found to be

$$\hat{P} = \bar{Z} - \hat{Q}.\hat{S}^{\bar{X}} \tag{2.25}$$

from which estimate of $K$ is found as

$$\hat{K} = anti\log(P) \tag{2.26}$$

Equation (2.6) yields the recurrence relation

$$Z_{i+1} = Z_i + Q.(S^{X_{i+1}} - S^{X_i}) \tag{2.27}$$

The fitted recurrence relation becomes

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$$Z_{i+1} = Z_i + \hat{Q}.(\hat{S}^{X_{i+1}} - \hat{S}^{X_i}) \tag{2.28}$$

Estimated value of $Z_{i+1}$ can be calculated using the observed value of $Z_i$ in this fitted regression equation.
The estimated value of $Y_{i+1}$ then can be obtained on taking antilog of $Z_{i+1}$.

**Note:** In the case of usual method of estimation, estimated values of $Z_i$ are computed from equation (2.16) putting the corresponding values of $X_i$ in the equation and then the estimated values of $Y_i$ are obtained by taking the antilog of $Z_i$.

**Fitting of Gompertz curve:**

Let a variable $Y$ be dependent on another variable $X$ and the dependence follow the rule that can be represented by the Gompartz curve which is of the form

$$Y = K.B^{S^X} \tag{2.29}$$

where $K$ , $B$ and $S$ are the parameters.
This can be expressed as

$$Z = P + Q.S^X \tag{2.30}$$

where Z = log $Y$, log $K = P$, $Q$ = log $B$ .
The equation (2.2) can be expressed as

$$Z' = M.S^X \tag{2.31}$$

where $Z' = \Delta.Z$ & $M = Q.(S^h - 1)$ .
Again, equation (2.31) can be written as

$$Z'' = M' + S'.X \tag{2.32}$$

where $Z'' = \log Z'$ , $M' = \log M$ & $S' = \log S$ .
Let

$$Y_1 , Y_2 , Y_3 , \ldots\ldots\ldots\ldots\ldots , Y_n$$

be the values assumed by the dependent variable $Y$ corresponding to the values

$$X_1 , X_2 , X_3 , \ldots\ldots\ldots\ldots\ldots , X_n$$

of the independent variable $X$ .
In order to obtain estimated value of $Y_{i+1}$ from the observed value of $Y_i$, the equation that describes the dependence of $Y_{i+1}$ on $Y_i$ is to be known.
From equation (2.32)

$$Z_i'' = M' + S'.X_i \tag{2.34}$$

which implies $$Z_{i+1}'' - Z_i'' = S'(X_{i+1} - X_i)$$

i.e. $$Z_{i+1}'' = Z_i'' + S'.\Delta X_i \tag{2.35}$$

where $\Delta X_i = X_{i+1} - X_i$
($i$ = 1 , 2 , ………… , $n$ - 1).

Equation (2.6) is the recurrence relation whose application can provide estimate of $Z_{i+1}''$ from observed $Z_i''$ .
This equation consists of the single parameter $S'$ . The estimate of $S'$ obtained by applying the step-wise method [(Chakrabarty (2011 , 2014)] is given by

$$\hat{S}'_{(EM)} = \frac{\sum_{i=1}^{n-1} \frac{\Delta Z_i''}{\Delta X_i}}{n-1} \tag{2.36}$$

**Note:**
If the values of the independent variable are at equal interval of length $h$, (say), the estimate of $S'$ becomes

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$$\hat{S}'_{(EM)} = \frac{\sum_{i=1}^{n-1} \Delta Z_i''}{h.(n-1)}$$

Using this estimate in equation (2.34), estimate of the parameter $M'$ can be obtained by the same method. The estimate, so obtained, is found to be

$$\hat{M}' = \bar{Z}'' - \hat{S}'_{(EM)}.\bar{X} \tag{2.37}$$

Now,

$$M' = \log M \quad \& \quad S' = \log S .$$

Therefore, estimates of the parameters $S$ and $M$ are obtained as

$$\hat{S} = anti\log(S') \quad \& \quad \hat{M} = anti\log(M') \tag{2.38}$$

Consequently, estimates of the parameters $Q$ & $B$ are found to be

$$\hat{Q} = \frac{M}{S^h - 1} \quad \& \quad \hat{B} = anti\log(Q) \tag{2.39}$$

Finally, estimate of the parameter $P$ can be obtained from equation (2.30) by the same method and it is found to be

$$\hat{P} = \bar{Z} - \hat{Q}.\hat{S}^{\bar{X}} \tag{2.40}$$

from which estimate of $K$ is found as

$$\hat{K} = anti\log(P) \tag{2.41}$$

Equation (2.30) yields the recurrence relation

$$Z_{i+1} = Z_i + Q.(S^{X_{i+1}} - S^{X_i}) \tag{2.42}$$

The fitted recurrence relation becomes

$$Z_{i+1} = Z_i + \hat{Q}.(\hat{S}^{X_{i+1}} - \hat{S}^{X_i}) \tag{2.42}$$

Estimated value of $Z_{i+1}$ can be calculated using the observed value of $Z_i$ in this fitted regression equation. The estimated value of $Y_{i+1}$ then can be obtained on taking antilog of $Z_{i+1}$.

## III. Numerical Application:

The method described here has been applied to estimate the total population of India from the data on total population of India (collected for the years from 1951 to 2001 from the census report published by the Registrar General of India) as shown in the following table (**Table-3.1**).

**Table-3.1**
(Observed Total Population of India)

| Year | 1951 | 1961 | 1971 | 1981 | 1991 | 2001 |
|---|---|---|---|---|---|---|
| Total Population | 361088090 | 439234771 | 548159652 | 683329097 | 846302688 | 1027015247 |

First, Gompertz curve [equation (2.29)] has been fitted to these data by the method explained in section II.
The estimated total population of India obtained from the fitted Gompertz curve, fitted by this method, has been shown In **Table-3.2**.
Next, the same has been fitted to these data by the usual least squares method
(i.e. estimation by solving normal equations, explained in section II).
The estimated total population of India obtained from the fitted Gompertz curve has been shown in in **Table-3.2**.

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**Table-3.2**
(Estimated Population of India by Gompertz curve)

| (1) | (2) | (3) | (4) |
|---|---|---|---|
| Year | Observed Total Population | Estimated Total Population (by the recurrence relation) | Estimated Total Population (by usual least squares method) |
| 1951 | 361088090 | 361834305 | 361395576 |
| 1961 | 439234771 | 445966754 | 446383775 |
| 1971 | 548159652 | 549661386 | 550668147 |
| 1981 | 683329097 | 677466732 | 678470320 |
| 1991 | 846302688 | 834988930 | 834899510 |
| 2001 | 1027015247 | 1029137639 | 1026131373 |

## Test of Significance of Difference between Observed and Estimated Values

Next, two statistical tests namely **Sandler's A-Test** and **Paired sample t-test** have been applied to test the significance of the difference between the observed values and the corresponding estimated values.
To test the n**ull hypothesis**
**$H_0$**: There is no significance difference between the observed population and the estimated population.
**Sandler's A-statistic** is given by

$$A = \frac{\sum_{i=1}^{11} D_i^{\,2}}{\left[\sum_{i=1}^{11} D_i\right]^2} \qquad \text{with (n-1) 4 degrees of freedom}$$

and the p**aired sample t-statistic** is given by

$$\text{t-test} = \frac{\overline{D}}{\frac{S}{\sqrt{n}}} \qquad \text{with (n-1) degrees of freedom}$$

where $\overline{D}$ = Mean difference,
$S$ = Standard deviation
& $n$ = Size of sample

The observed values of **A** are **12888.81829** and **4.109739923** for the method discussed in section II and for the usual least squares method respectively while the corresponding theoretical value of A is **0.30351431** for 4 degrees of freedom at 5% level of significance.

The observed value of the statistic A is greater than that of corresponding tabulated value of A at 5% level of significance for 4 degrees of freedom Therefore the null hypothesis is not rejected. This means, there is no significance difference between the observed values and the corresponding estimated values in the case of both the methods of fitting.

**Similarly, t**he observed values of ***t*** are 0.008 and 0.460 for the method discussed in section II and for the usual least squares method respectively while the corresponding theoretical value of *t* is 2.78 for 4 degrees of freedom at 5% level of significance.

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The observed value of the statistic *t* is less than that of corresponding tabulated value of *t* at 5% level of significance for 4 degrees of freedom Therefore the null hypothesis is not rejected. This means, there is no significance difference between the observed values and the corresponding estimated values in the case of both the methods of fitting.

## IV. CONCLUSION

(1) The elimination-minimization approach has already been found suitable for finding of method of fitting of a polynomial curve of any finite order to numerical data.
(2) This study establishes that this approach can be suitable for fitting of Gompertz curves to numerical data.
(3) It is yet to be investigated whether this approach can be applicable in finding of suitable method of fitting of other types of curves to numerical data.
(4) It is yet to be search for whether the estimates of parameter obtained by this method and those obtained by usual method of least squares are identical.

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**AUTHOR'S BIOGRAPHY**

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class and the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.
Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.
He attended five of orientation/refresher course held in Gauhati University, Indian Statistical Institute, University of Calicut and Cochin University of Science & Technology sponsored/organized by University Grants Commission/Indian Academy of Science. He also attended/participated eleven workshops/training programmes of different fields at various institutes.
Dr. Dhritikesh Chakrabarty joined the Department of Statistics of Handique Girls' College, Gauhati University, as a Lecturer on December 09, 1987 and has been serving the institution continuously since then. Currently he is in the position of Associate Professor (& Ex Head) of the same Department of the same College. He has also been serving the National Institute of Pharmaceutical Education & Research (NIPER), Guwahati, as a Guest Faculty continuously from May 02, 2010. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years. Dr. Chakrabarty has been working as an independent researcher for the last more than twenty five years. He has already published ninety research papers in various research journals mostly of international level and eight research papers in conference proceedings. Sixty research papers based on his research works have already been presented in research conferences/seminars of national and international levels both within and outside India. He has written two books titled (i) Statistics for Beginners and (ii) Selection of Random Samples: Drawing of Random Numbers. He is also one author of the Assamese Science Dictionary titled "Vigyan Jeuti" published by Assam Science Society. Moreover, he is one author of the research book **"**BIODIVERSITY- Threats and Conservation (ISBN-978-93-81563-48-9)" published by the Global Publishing House. He delivered invited talks/lectures in several seminars He acted as chair person in some seminars. He visited U.S.A. in

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2007, Canada in 2011 and U.K. in 2014. He has already completed one post doctoral research project (2002─05) and one minor research project (2010─11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Referee of the Journal of Assam Science Society (JASS) and a Member of the Editorial Boards of the two Journals namely (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET) and (2) Journal of Mathematics and System Science. Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.