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Sustainable algorithms for dynamic filtering, taking into account the inertia of the measuring device

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ABSTRACT*:* The problems of forming and building a stable filtering algorithms based on inertial measurement unit based on dynamic filter of Kalman type. The various filtering algorithms for dynamic analysis in terms of their computational stability. Televisions regularization procedure of filtering solutions of the equation on the basis of the regularization method and singular value decomposition. The proposed algorithm can improve the computational stability of dynamic filtering algorithms based on the inertia of the measuring device and can be used in information processing and measuring the results of observations systems.

KEYWORDS*:* dynamic filtering algorithms, inertial measurement unit, stability, regularization, singular value decomposition.

I. INTRODUCTION

In applied problems often arise issues of synthesis of information-measuring systems of processing of the results of observations of the state of a dynamic system. Similar problems arise on various stages of design and the most important gain in the processing of the measurement results of the experimental data.

Consider the issues of formation of the state of the procedure of construction of models of dynamic systems and filtering algorithms, taking into account the inertia of the measuring device [1-3]. Such cases occur most often in applications where the observed signal is controlled by the inertial measuring device and is exposed to the measurement noise. In these cases, the general filtering conditions are violated, leading to a need for methods of converting vector-matrix equations describing the dynamics of the original dynamical system and measuring devices. For the general filtering conditions in this case it is equivalent to an extended dynamic system comprising a random process with correlated values [2-6]. Thus, in theory and dynamic filtering prediction Kalman-Bucy particularly represent Gaussian Markov Random processes and random sequences. Practical use of Gaussian Markov processes is determined by several factors, which primarily include the possibility of approximation with high accuracy real Gaussian random processes, Markov processes; the possibility of reducing by linear transformations of random Gaussian random process with a finite number of derivatives to the equivalent Gaussian Markov random processes; feasibility of approximation of non-Gaussian random process Gaussian Markov random process [3,5,7,8]. These factors determine the pervasiveness of Gaussian Markov processes are the most important issues in the application of building models of linear dynamical systems [1,2,5].

Statistical data processing, based on the theory of Kalman-Bucy optimal filtering, assumes the technical implementation on the basis of computers. In this connection of great practical value acquiring filtering algorithms using recursive methods of statistical processing. Consequently, the particular interest in applications has a discrete optimal filtering [1,3,5]. The mathematical description of the digital filter is carried out as part of the difference equation or recursion, which is closely linked with the differential equations, which form the basis of the classic study of continuous processes. Difference equations, allowing to explore the sequence of states of discrete systems can be easily implemented using a computer, and a discrete Kalman filter defines only the data processing algorithm intended for implementation on a computer, and does not characterize the type of computer and the necessary software.

In the work deals with the formation and building sustainable filtration algorithms taking into account the inertia of the measuring device on the basis of dynamic filter of Kalman type.

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II. TEXT DETECTION

Consider the problem of dynamic assessment:

$$x_{i+1} = Ax_i + \Gamma w_i, \quad i = 0,1,...,k-1,$$

where x_i -*n*-vector of state, w_i - *r*-dimensional Gaussian purely random sequence with zero mean and covariance matrix Q.

The measurements described by the equation

$$z_i = Hx_i + v_i,\ i = 0,1,...,k, \tag{1}$$

where z_i - *p*-dimensional vector of measurements, v_i - *p*-dimensional Gaussian Markov sequence, which can be obtained using multistep shaping filter

$$v_{i+1} = \Psi v_i + \xi_i.$$

Here ξ_i - *p*-dimensional Gaussian purely random sequence with zero mean and covariance matrix Q^*.

Based on the principle of expansion models [2] considered the estimation problem can be reduced to the form

$$y_{i+1} = A^\alpha y_i + \Gamma^\alpha \eta_i, \tag{2}$$

$$z_i = H^\alpha y_i, \tag{3}$$

where

$$A^\alpha = \begin{bmatrix} A & 0 \\ 0 & \Psi \end{bmatrix},\ \Gamma^\alpha = \begin{bmatrix} \Gamma & 0 \\ 0 & E \end{bmatrix}, \quad \eta_i = \begin{bmatrix} w_i \\ \xi_i \end{bmatrix},\ H^\alpha = [H \mid E].$$

$$y_i \overset{\Delta}{=} \begin{bmatrix} x_i \\ v_i \end{bmatrix}.$$

From the equations (2) and (3) it is seen that the measurements (1) are "precise" in relation to the vector y_i, i.e. they do not contain pure random noise.

Thus, the filtering feature of this problem is the assumption that correlation measurement noise v_i. In this case, the additional assumption of Gaussian $x_0, \{w_i\}, \{v_i\}$ get [3,5]:

$$\hat{x}_k = E\{x_k | z_0, z_1,..., z_k\}.$$

Assessment $\hat{x}_k$ has the following properties [2]:

1.unbiasedness:

$$E\{\hat{x}_k\} = x_k;$$

2.minimum variance:

$$trE\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\} = \min_{\tilde{x}_k} trE\{(\tilde{x}_k - x_k)(\tilde{x}_k - x_k)^T\},$$

where $\tilde{x}_k$ - any unbiased, linear $z_0, z_1,..., z_k$ to x_k score; *tr* - trace of the matrix. These properties are characterized $\hat{x}_k$ as the best linear unbiased estimate.

Assume that $(k+1)$ -th observation is carried out without errors. Then

$$z_{k+1} = H_{k+1} x_{k+1}. \tag{4}$$

Equation (4) can be interpreted as follows:

$$z_{k+1} = H_{k+1} x_{k+1} + v_{k+1},$$

where v_{k+1} - random disturbance with zero mean and covariance matrix zero.

Then, based on the known equations of the Kalman filter [3,5,7], we can write the following equation for the multi-stage filter system extended (2) and (3):

$$\hat{y}_k = A^\alpha \hat{y}_{k-1} + K_k^\alpha \left(z_k - H^\alpha A^\alpha \hat{y}_{k-1}\right), \tag{5}$$

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$$K_k^\alpha = M_k^\alpha \left(H^\alpha\right)^T \left[H^\alpha M_k^\alpha \left(H^\alpha\right)^T\right]^{-1}, \tag{6}$$

$$M_{k+1}^\alpha = A^\alpha P_k^\alpha \left(A^\alpha\right)^T + \Gamma^\alpha Q^\alpha \left(\Gamma^\alpha\right)^T, \tag{7}$$

$$P_k^\alpha = M_k^\alpha - M_k^\alpha \left(H^\alpha\right)^T \left[H^\alpha M_k^\alpha \left(H^\alpha\right)^T\right]^{-1} H^\alpha M_k^\alpha, \tag{8}$$

where

$$Q^\alpha = \left[\begin{array}{c|c} Q & 0 \\ \hline 0 & Q^* \end{array}\right].$$

Analyzing the relationships (6) and (8) it can be seen that the calculations for these expressions have low computational stability. This is caused by that because the matrix $H^\alpha M_k^\alpha \left(H^\alpha\right)^T$ may be ill-conditioned, and thereby calculating the matrix inverse to $H^\alpha M_k^\alpha \left(H^\alpha\right)^T$ can be very inaccurate. Essentially, this is due to the fact that the filter described by the expressions (5) - (8) is a case where measurements are performed accurately, i.e. with out errors.
These difficulties can be avoided by using the following dynamic filter [1,3]:

$$\hat{x}_k = \bar{x}_k + K_k\left(\zeta_k - H^* \bar{x}_k\right), \tag{9}$$

$$\bar{x}_{k+1} = A\hat{x}_k + D\left(\zeta_k - H^* \hat{x}_k\right), \tag{10}$$

$$K_k = P_k \left(H^*\right)^T R^{-1}, \tag{11}$$

$$P_k = M_k - M_k \left(H^*\right)^T \left(H^* M_k \left(H^*\right)^T + R\right)^{-1} H^* M_k, \tag{12}$$

$$M_{k+1} = \left(A - DH^*\right) P_k \left(A - DH^*\right)^T + \Gamma Q \Gamma^T - DRD^T, \tag{13}$$

where R, D and S are defined by equations.

$$R = H\ \Gamma\ Q_k \Gamma^T H^T + Q^*,$$

$$D = \Gamma\ S\ R^{-1},$$

$$S = Q\ \Gamma^T H^T,$$

$$\zeta_k = z_{k+1} - \Psi\ z_k$$

or

$$\zeta_k = H^* x_k + \varepsilon_k,$$

where

$$H^* \equiv H\ A - \Psi\ H \quad \text{and} \quad \varepsilon_k \equiv H\ \Gamma\ w_k + \xi_k.$$

However, in this problem, there are two features: the selected measurement ζ_k with respect to the actual measurement of z_k is delayed by one step; noise in the measurement system and correlated. The dimension of the filter *n*, while dimension filter (5) - (8) is equal to (*n*+*p*) to the extended state vector. Measurements z_k can be viewed as p accurate measurements, containing $(n+p)$ and v_k of the x_k variables.
In case, if the equations (11) and (12) a nonsingular matrix $R \equiv H\Gamma Q \Gamma^T H^T + Q^*$, the calculation process of the component K_k and D is not poorly conditioned. In the case of a degenerate R is advisable to use a filter of the form [3]:

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$$\hat{x}_k = \bar{x}_k + K_k(\zeta_k - H^*\bar{x}_k),$$

$$\bar{x}_{k+1} = A\hat{x}_k + \Gamma S[H^* M_k (H^*)^T + R]^{-1}(\zeta_k - H^*\bar{x}_k), \quad (14)$$

$$K_k = M_k (H^*)^T [H^* M_k (H^*)^T + R]^{-1}, \quad (15)$$

$$P_k = (E - K_k H^*) M_k (E - K_k H^*) + K_k R K_k^T, \quad (16)$$

$$M_{k+1} = AP_k A^T + \Gamma Q \Gamma^T - \Gamma S[H^* M_k (H^*)^T + R]^{-1} S^T \Gamma^T - AK_k S^T \Gamma^T - \Gamma S K_k^T A^T. \quad (17)$$

III.SOLUTION OF THE TASK

To ensure the stability of the filtration procedure is desirable instead of equations (6) and (8) and (14), (15) and (17) use the following relations, respectively:

$$K_k^\alpha = M_k^\alpha (H^\alpha)^T \cdot g_\alpha^a (C_k),$$

$$P_k^\alpha = M_k^\alpha - M_k^\alpha (H^\alpha)^T \cdot g_\alpha^a (C_k) \cdot H^\alpha M_k^\alpha,$$

$$g_\alpha^a (C_k) = (C_k + \alpha I)^{-1}, \, C_k = [H^\alpha M_k^\alpha (H^\alpha)^T];$$

$$\bar{x}_{k+1} = A\hat{x}_k + \Gamma S \cdot g_\alpha (C'_k)(\zeta_k - H^* \bar{x}_k),$$

$$K_k = M_k (H^*)^T \cdot g_\alpha (C'_k),$$

$$M_{k+1} = AP_k A^T + \Gamma Q \Gamma^T - \Gamma S \cdot g_\alpha (C'_k) S^T \Gamma^T - AK_k S^T \Gamma^T - \Gamma S K_k^T A^T,$$

$$g_\alpha (\mathrm{C}_k) = (\mathrm{C}'_k + \alpha I)^{-1}, \quad C'_k = [H^* M_k (H^*)^T + R].$$

where $g_\alpha^a (C_k)$ and $g_\alpha (\mathrm{C}'_k)$ - generating functions of the system for the regularization method [9,10], α - regularization parameter, I - identity matrix. There regularization parameter α wise to choose based on the method of model examples and simulations [11].

If the $(H^\alpha M_k^\alpha (H^\alpha)^T)^{-1}$ matrix does not exist, this means that in (4) includes some equations (equations $p - r$, namely, r - the rank of the H_{k+1} matrix) is linearly dependent on the remaining r. Therefore, these can be eliminated from equation (4). After exclusion instead get - z_{k+1} $\overset{*}{z}_{k+1}$ - r-dimensional vector, and now we can assume there is a similar inverse matrix in the new problem.

To realize a sustainable filtering algorithms (5) - (8) and (14) - (17) is also appropriate to use the concept of singular value decomposition [12-14]. Singular decomposition allows you to find different orthogonal bases vector spaces degradable matrix. It uses the different properties of singular value decomposition, for example, the ability to show the rank of the matrix, the matrix of the approximate rank, compute the inverse and pseudoinverse matrices.

Consider steady filtering process on the basis of equations (5) - (8) to use orthogonal s_k matrix [12,14]. Here are C_k to diagonal form

$$s_k^T \mathrm{C}_k s_k = \Lambda_k,$$

$$\Lambda_k = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_p, \ 0, ..., 0) = \begin{pmatrix} \tilde{\Lambda} & \vdots & 0 \\ \cdots & & \cdots \\ 0 & \vdots & 0 \end{pmatrix}.$$

Based on the Λ_k matrix can write

$$z_{k,1}^{(1)} = H_{k,1}^{(1)} x_k + v_k^{(1)},$$

$$z_{k,2}^{(1)} = H_{k,2}^{(1)} x_k,$$

where $z_{k,i}^{(1)}, H_{k,i}^{(1)}$ $(i = 1, 2)$ determined based on equations [4,12]

$$z_k^{(1)} = s_k^T z_k = \left(z_{k,1}^{T(1)}, z_{k,2}^{T(1)}\right)^T$$

and

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$$H_k^{(1)} = s_k^T H_k = \left(H_{k,1}^{T(1)}, H_{k,2}^{T(1)}\right)^T$$

respectively.

Taking into account that $s_k^T C_k s_k = \Lambda_k$ can write $C_k = s_k \Lambda_k s_k^T = s_{k,1} \tilde{\Lambda} s_{k,1}^T$. Then can write $C_k^+ = s_{k,1} \tilde{\Lambda}^{-1} s_{k,1}^T$ that satisfies all of the following equations, which are only to define the pseudoinverse C_k matrix:

$$C_k^+ C_k = I,$$

$$C_k C_k^+ = I,$$

$$C_k^+ C_k C_k^+ = C_k^+,$$

$$C_k C_k^+ C_k = C_k.$$

Thus, one can arrive at an estimate of the form:

$$\tilde{x}_i = F_i^{(1)} \tilde{x}_{i-1} + F_i^{(2)} z_i, \quad F_i = \left(\underbrace{F_i^{(1)}}_{n}, \underbrace{F_i^{(2)}}_{p},\right),$$

which depends on the $\tilde{x}_{i-1}$ and z_i.

IV. CONCLUSION

The above algorithms make it possible to increase the computational stability of dynamic filtering algorithms based on the inertia of the measuring device and can be used in information-measuring systems of processing the results of observations of the state of a dynamic system.

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