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# On Radio Mean D-Distance Number of Degree Splitting Graphs

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**ABSTRACT:** A Radio Mean D-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to ℕ such that for two distinct vertices u and v of G, $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G)$, where $d^D(u, v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio mean D-distance number of f, $rmn^D(f)$ is the maximum label assigned to any vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the minimum value of $rmn^D(f)$ taken over all radio mean D-distance labeling f of G. In this paper we find the radio mean D-distance number of degree splitting graphs.

**KEYWORDS:.** D-distance, radio D-distance coloring, radio D-distance number, radio mean D-distance, radio mean D-distance number, split graph, degree splitting graph.
**AMS Subject Classification.** 05C78.

## I. INTRODUCTION

By a graph G = (V, E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimum radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labeling (multi-level distance labeling) can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by Hale [6]. Chartrand et al. [2] introduced the concept of radio labeling of graph. Chartrand et al. [3] gave the upper bound for the radio number of Path. The exact value for the radio number of Path and Cycle was given by Liu and Zhu [10]. However Chartrand et al. [2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of Cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O.Togni [8]. M.M.Rivera et al. [21] gave the radio number of $C_n \times C_n$, the cartesian product of $C_n$. In [4] C.Fernandez et al. found the radio number for complete graph, star graph, complete bipartite graph, wheel graph and gear graph. M.T.Rahim and I.Tomescu [16] investigated the radio number of Helm Graph. The radio number for the generalized prism graphs were presented by Paul Martinez et.al. in [11].

The concept of D-distance was introduced by D. Reddy Babu et al. [17, 18, 19].If u, v are vertices of a connected graph *G*, the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s) + deg(v) + deg(u) + \sum deg(w)$ where the sum runs over all intermediate vertices *w* of *s* and $\ell(s)$ is the length of the path. The *D*-distance, $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{\ell(s) + deg(v) + deg(u) + \sum deg(w)\}$where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in *G*.

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In [12], we introduced the concept of Radio D-distance. The radio D-distance coloring is a function $f: V(G) \to \mathbb{N} \cup \{0\}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$. It is denoted by $rn^D(G)$. A radio D-distance coloring $f$ of G is a minimum radio D-distance coloring if $rn^D(f) = rn^D(G)$, where $rn^D(G)$ is called radio D-distance number.

Radio mean labeling was introduced by R. Ponraj et al [13,14, 15]. A radio mean labeling is a one to one mapping $f$ from V(G) to $\mathbb{N}$ satisfying the condition

$$d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(G). \quad (1.1)$$

for every $u, v \in V(G)$. The span of a labeling $f$ is the maximum integer that f maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labelings of the graph G. The condition (1.1) is called radio mean condition.

In [13], we introduce the concept of radio mean D-distance number. A radio mean D-distance labeling is a one to one mapping $f$ from V(G) to $\mathbb{N}$ satisfying the condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G). \quad (1.2)$$

for every $u, v \in V(G)$. The span of a labeling $f$ is the maximum integer that $f$ maps to a vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the lowest span taken over all radio mean D-distance labelings of the graph G. The condition (1.2) is called radio mean D-distance condition. In this paper we determine the radio mean D-distance number of some well-known graphs. The function $f: V(G) \to \mathbb{N}$ always represents injective map unless otherwise stated.

## II. MAIN RESULT

- For a graph G the split graph is obtained by adding to each vertex v, a new vertex v′ such that v′ is adjacent to each vertex that is adjacent to v in G. The resultant graph is denoted by spl(G).
- Let G = (V(G), E(G)) be a graph with $V(G) = S1 \cup S2 \cup,\dots,\cup St \cup T$, where each $S_i$ is a set of all the vertices having same degree with at least two elements and $T = V(G) \setminus \cup_{i=1}^{t} S_i$. The degree splitting graph DS(G) is obtained from G by adding vertices $w_1, w_2, \dots, w_t$ and joining to each vertex of $S_i$ for $1 \leq i \leq t$.

**Theorem 2.1.**
The radio mean D-distance number of a splitting graph of star, $rmn^D(spl(K_{1,n})) = 2n + 7$, $n \geq 2$.

**Proof.**

It is obvious that $diam^D(spl(K_{1,n})) = 3n + 6$. Let $V(spl(K_{1,n})) = \{v_i, u_i, x_j / i = 1, 2, \dots, n, j = 1, 2\}$ and $E = \{x_1v_i, x_ju_i / i = 1, 2, \dots, n, j = 1, 2\}$. Since $spl(K_{1,n})$ has 2n + 2 vertices and hence it requires 2n + 2 labels. The D-distance is $d^D(v_i, v_j) = 2n + 4$, $f(v_1) = 6$ then the label 5 is forbidden.

$$rmn^D(spl(K_{1,n})) \geq 5 + 2n + 2$$
$$\geq 2n + 7$$

Now we shall give the following label to set the equality. Let $f(x_1) = 2n + 7$, $f(x_2) = 2n + 6$, $f(v_i) = 5 + i$, $1 \leq i \leq n$, $f(u_i) = n + 5 + i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(spl(K_{1,n})) + 1 = 3n + 7, \text{ for every pair of vertices } (u, v) \text{ where } u \neq v.$$

$$d^D(v_i, u_i) + \left\lceil \frac{f(v_i)+f(u_i)}{2} \right\rceil \geq 2n + 5 + \left\lceil \frac{5 + i + n + 5 + i}{2} \right\rceil \geq 3n + 7.$$
$$d^D(v_i, x_1) + \left\lceil \frac{f(v_i)+f(x_1)}{2} \right\rceil \geq 2n + 2 + \left\lceil \frac{5 + i + 2n + 7}{2} \right\rceil \geq 3n + 7.$$
$$d^D(v_i, x_2) + \left\lceil \frac{f(v_i)+f(x_2)}{2} \right\rceil \geq 3n + 6 + \left\lceil \frac{5 + i + 2n + 6}{2} \right\rceil \geq 3n + 7.$$
$$d^D(x_1, x_2) + \left\lceil \frac{f(x_1)+f(x_2)}{2} \right\rceil \geq 3n + 4 + \left\lceil \frac{2n + 7 + 2n + 6}{2} \right\rceil \geq 3n + 7.$$
$$d^D(u_i, x_1) + \left\lceil \frac{f(u_i)+f(x_1)}{2} \right\rceil \geq 2n + 3 + \left\lceil \frac{n + 5 + i + 2n + 7}{2} \right\rceil \geq 3n + 7.$$
$$d^D(u_i, x_2) + \left\lceil \frac{f(u_i)+f(x_2)}{2} \right\rceil \geq n + 3 + \left\lceil \frac{n + 5 + i + 2n + 6}{2} \right\rceil \geq 3n + 7.$$
$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq n + 6 + \left\lceil \frac{n + 5 + i + n + 5 + j}{2} \right\rceil \geq 3n + 7.$$
$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 2n + 4 + \left\lceil \frac{5 + i + 5 + j}{2} \right\rceil \geq 3n + 7.$$

Therefore, $f(x_1) = 2n + 7$ is the largest label.

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Hence, $rmn^D(spl(K_{1,n})) = 2n + 7$ if $n \geq 2$. ■

- ❖ The middle graph M(G) of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

**Theorem 2.2.**

The radio mean D-distance number of a middle graph of path, $rmn^D(M(P_n)) = 6n - 9, n \geq 3$.
Proof.

It is obvious that $diam^D(M(P_n)) = 5n - 4$. Let $V(M(P_n)) = \{v_i, u_j / i = 1, 2, \ldots, n, j = 1, 2, 3, \ldots, n - 1\}$ and $E = \{ v_iu_j / i = 1, 2, \ldots, n, j = 1, 2, 3, \ldots, n - 1\}$. Since $M(P_n)$ has $2n - 1$ vertices it requires $2n - 1$ labels. The D-distance is $d^D(v_1, v_2) = 8$, $f(v_1) = 4n - 7$ then the label $4n - 8$ is forbidden.

$$rmn^D(M(P_n)) \geq 4n - 8 + 2n - 1$$
$$\geq 6n - 9$$

Now we shall give the following label to set the equality. Let $f(v_i) = 4n - 8 + i, 1 \leq i \leq n$, $f(u_i) = 6n - 8 - i, 1 \leq i \leq n$.
We shall check the radio mean D-distance condition

$$d^D(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq diam^D(M(P_n)) + 1 = 5n - 3, \text{ for every pair of vertices } (u, v) \text{ where } u \neq v.$$

$$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 5n - 4 + \left\lceil\frac{4n - 8 + i + 4n - 8 + j}{2}\right\rceil \geq 5n - 3.$$

$$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 8 + \left\lceil\frac{4n - 8 + i + 4n - 8 + j}{2}\right\rceil \geq 5n - 3.$$

$$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 5n - 7 + \left\lceil\frac{4n - 8 + i + 4n - 8 + j}{2}\right\rceil \geq 5n - 3.$$

If $v_i$ and $u_j$ are adjacent,

$$d^D(v_i, u_j) + \left\lceil\frac{f(v_i)+f(u_j)}{2}\right\rceil \geq 5 + \left\lceil\frac{4n - 8 + i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

$$d^D(v_i, u_j) + \left\lceil\frac{f(v_i)+f(u_j)}{2}\right\rceil \geq 6 + \left\lceil\frac{4n - 8 + i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

$$d^D(v_i, u_j) + \left\lceil\frac{f(v_i)+f(u_j)}{2}\right\rceil \geq 7 + \left\lceil\frac{4n - 8 + i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

If $v_i$ and $u_j$ are not adjacent,

$$d^D(v_i, u_j) + \left\lceil\frac{f(v_i)+f(u_j)}{2}\right\rceil \geq 5n - 6 + \left\lceil\frac{4n - 8 + i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

If $u_i$ and $u_j$ are adjacent,

$$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 7 + \left\lceil\frac{6n - 8 - i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

$$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 8 + \left\lceil\frac{6n - 8 - i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

$$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 9 + \left\lceil\frac{6n - 8 - i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

If $u_i$ and $u_j$ are not adjacent,

$$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 5n - 8 + \left\lceil\frac{6n - 8 - i + 6n - 8 - j}{2}\right\rceil \geq 5n - 3.$$

Therefore, $f(u_1) = 6n - 9$ is the largest label.
Hence, $rmn^D(M(P_n)) = 6n - 9, n \geq 3$. ■

**Theorem 2.3.**

The radio mean D-distance number of a degree splitting wheel, $rmn^D(DS(W_n)) = 2n + 2, n \geq 3$.
Proof.

It is obvious that $diam^D(DS(W_n)) = 2n + 4$. Let $V(DS(W_n)) = \{v\} \cup \{u\} \cup \{u_i / i = 1, 2, \ldots, n\}$ and $E = \{ u_iv , u_iu / i = 1, 2, \ldots, n\}$. Since $DS(W_n)$ has $n + 2$ vertices and hence it requires $n + 2$ labels. The D-distance is $d^D(v, u_j) = n + 5$, $f(v) = n + 1$ then the label n is forbidden.

$$rmn^D(DS(W_n)) \geq n + n + 2$$
$$\geq 2n + 2$$

Now we shall give the following label to set the equality. Let $f(v) = n + 1$, $f(u_i) = n + 1 + i$, $1 \le i \le n$, $f(u) = 2n + 2$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge diam^D(DS(W_n))+1 = 2n + 5$, for every pair of vertices $(u, v)$ where $u \ne v$.

If $v_i$ and $v_j$ are adjacent,

$d^D(u_i, v) + \left\lceil \frac{f(u_i)+f(v)}{2} \right\rceil \ge n + 4 + \left\lceil \frac{n+1+i+n+1}{2} \right\rceil \ge 2n + 5.$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge 11 + \left\lceil \frac{n+1+i+n+1+j}{2} \right\rceil \ge 2n + 5.$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge n + 8 + \left\lceil \frac{n+1+i+n+1+j}{2} \right\rceil \ge 2n + 5.$

$d^D(u_i, u) + \left\lceil \frac{f(u_i)+f(u)}{2} \right\rceil \ge n + 4 + \left\lceil \frac{n+1+i+2n+2}{2} \right\rceil \ge 2n + 5.$

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge 2n + 4 + \left\lceil \frac{2n+2+n+1}{2} \right\rceil \ge 2n + 5.$

Therefore, $f(u) = 2n + 2$ is the largest label.

Hence, $rmn^D(DS(W_n)) \le 2n + 2$ if $n \ge 3$. ■

**Theorem 2.4.**

The radio mean D-distance number of a degree splitting path, $rmn^D(DS(P_n)) = \begin{cases} 5, & n = 3. \\ 11, & n = 4. \\ 12, & n = 5 \\ 2n + 1, & n \ge 6. \end{cases}$

Proof.

It is obvious that $diam^D(DS(P_n)) = n + 10$. Let $V(DS(P_n)) = \{v\} \cup \{u\} \cup \{u_i / i = 1, 2, \ldots, n\}$ and $E = \{ u_iv, u_iu, u_iu_j / i = 1, 2, \ldots, n\}$. Since $DS(P_n)$ has $n + 2 (n \ge 6)$ vertices it required $n + 2$ labels.
The D-distance is $d^D(v_2, u_{i+1}) = n + 2$, $f(v_2) = 6$ then the label $n - 1$ is forbidden.

$rmn^D(DS(P_n)) \ge n - 1 + n + 2$
$\ge 2n + 1$

Now we shall give the following label to set the equality.
Let $f(v_1) = 2n + 1$, $f(v_2) = 6$, $f(u_1) = 2n$, $f(u_i) = n + i - 1$, $2 \le i \le n$,
We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \ge diam^D(DS(P_n)) + 1 = n + 11$, for every pair of vertices $(u, v)$ where $u \ne v$.

If $v_i$ and $v_j$ are adjacent,

$d^D(u_i, v_2) + \left\lceil \frac{f(u_i)+f(v_2)}{2} \right\rceil \ge n + \left\lceil \frac{n+i-1+6}{2} \right\rceil \ge n + 11.$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge 6 + \left\lceil \frac{n+i-1+n+j-1}{2} \right\rceil \ge n + 11.$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge 7 + \left\lceil \frac{n+i-1+n+j-1}{2} \right\rceil \ge n + 11.$

$d^D(u_i, v_1) + \left\lceil \frac{f(u_i)+f(v_1)}{2} \right\rceil \ge 5 + \left\lceil \frac{n+i-1+2n+1}{2} \right\rceil \ge n + 11.$

If $v_i$ and $v_j$ are non adjacent,

$d^D(u_i, v_2) + \left\lceil \frac{f(u_i)+f(v_2)}{2} \right\rceil \ge n + 3 + \left\lceil \frac{n+i-1+6}{2} \right\rceil \ge n + 11.$

$d^D(u_i, v_1) + \left\lceil \frac{f(u_i)+f(v_1)}{2} \right\rceil \ge n + 3 + \left\lceil \frac{n+i-1+2n+1}{2} \right\rceil \ge n + 11.$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge n + 7 + \left\lceil \frac{n+i-1+n+j-1}{2} \right\rceil \ge n + 11.$

$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \ge n + 10 + \left\lceil \frac{n+i-1+n+j-1}{2} \right\rceil \ge n + 11.$

$d^D(v_1, v_2) + \left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil \ge n + 6 + \left\lceil \frac{2n+1+6}{2} \right\rceil \ge n + 11.$

Therefore, $f(v_2) = 2n + 1$ is the largest label.

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Hence, $rmn^D(DS(P_n)) = \begin{cases} 5, & n = 3. \\ 11, & n = 4. \\ 12, & n = 5 \\ 2n + 1, & n \geq 6. \end{cases}$ ■

- ❖ The total graph T(G) of a graph G is the graph whose vertex set is V(G) ∪ E(G) and two vertices are adjacent whenever they are either adjacent or incident in G.

**Theorem 2.5.**

The radio mean D-distance number of a total graph of path, $rmn^D(T(P_n)) = \begin{cases} 6, & n = 3. \\ 7n - 16, & n \geq 4. \end{cases}$

Proof.

It is obvious that $diam^D(T(P_n)) = 5n - 5$. Let $V(T(P_n)) = \{v_i, u_j / i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, n\}$ and $E = \{v_iv_{i+1}, v_iu_j, u_ju_{j+1} / i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, n\}$. Since $T(P_n)$ has $2n - 1$vertices it required $2n - 1$labels. The D-distance is $d^D(v_i, v_{i+1}) = 9$, $f(v_2) = 5n - 14$ then the label $5n - 15$ is forbidden.

$$rmn^D(T(P_n)) \geq 5n - 15 + 2n - 1$$
$$\geq 7n - 16$$

Now we shall give the following label to set the equality.

Let $f(v_i) = 5n - 16 + i$, $2 \leq i \leq n$, $f(v_1) = 7n - 16$, $f(u_i) = 6n + i - 16$, $1 \leq i \leq n$, We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(DS(P_n)) + 1 = 5n - 4$, for every pair of vertices (u, v) where $u \neq v$.

If $v_i$ and $v_j$ are adjacent,

$$d^D(v_i, v_1) + \left\lceil \frac{f(v_i)+f(v_1)}{2} \right\rceil \geq 7 + \left\lceil \frac{5n - 16 + i + 7n - 16}{2} \right\rceil \geq 5n - 4.$$

If $v_i$ and $v_j$ are non adjacent,

$$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 5n - 5 + \left\lceil \frac{5n - 16 + i + 5n - 16 + j}{2} \right\rceil \geq 5n - 4.$$

$$d^D(v_i, v_1) + \left\lceil \frac{f(v_i)+f(v_1)}{2} \right\rceil \geq 5n - 8 + \left\lceil \frac{5n - 16 + i + 7n - 16}{2} \right\rceil \geq 5n - 4.$$

If $u_i$ and $u_j$ are adjacent,

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 8 + \left\lceil \frac{6n + i - 16 + 6n + j - 16}{2} \right\rceil \geq 5n - 4.$$

If $u_i$ and $u_j$ are non adjacent,

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 5n - 8 + \left\lceil \frac{6n + i - 16 + 6n + j - 16}{2} \right\rceil \geq 5n - 4.$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 5n - 12 + \left\lceil \frac{6n + i - 16 + 6n + j - 16}{2} \right\rceil \geq 5n - 4.$$

If $v_i$ and $u_j$ are adjacent,

$$d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 6 + \left\lceil \frac{6n + i - 16 + 5n - 16 + j}{2} \right\rceil \geq 5n - 4.$$

If $v_i$ and $u_j$ are non adjacent,

$$d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 5n - 4 + \left\lceil \frac{6n + i - 16 + 5n - 16 + j}{2} \right\rceil \geq 5n - 4.$$

Therefore, $f(v_2) = 2n + 1$ is the largest label.

Hence, $rmn^D(T(P_n)) = \begin{cases} 6, & n = 3. \\ 7n - 16, & n \geq 4. \end{cases}$ ■

- ❖ The double fan $DF_n$ is given by $P_n + 2K\,1$ .

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**Theorem 2.6.**

The radio mean D-distance number of a double fan, $rmn^D(D(F_n)) = 2n + 2, n \geq 2$.

Proof.

It is obvious that $diam^D(D(F_n)) = 2n + 5$. Let $V(D(F_n)) = \{v_i, u_j / i = 1, 2$ and $j = 1, 2, \ldots, n\}$ and $E = \{v_i u_j, u_j u_{j+1} / i = 1, 2$ and $j = 1, 2, \ldots, n\}$. Since $D(F_n)$ has n+ 2vertices it required n + 2 labels.

The D-distance is $d^D(v_1, u_i) = n + 4, i = 1$ or $n$, $f(v_1) = n + 1$ ($d^D(v_2, u_i) = n + 4, i = 1$ or $n$, $f(v_2) = n + 1$) then the label n is forbidden.

$$rmn^D(D(F_n)) \geq n + n + 2$$
$$\geq 2n + 2$$

Now we shall give the following label to set the equality. Let $f(v_1) = n + 1$, $f(v_2) = n + 2$, $f(u_i) = n + i + 2$, $1 \leq i \leq n$, We shall check the radio mean D-distance condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(D(F_n)) + 1 = 2n + 6, \text{ for every pair of vertices } (u, v) \text{ where } u \neq v.$$

$$d^D(u_i, v_1) + \left\lceil \frac{f(u_i)+f(v_1)}{2} \right\rceil \geq n + 4 + \left\lceil \frac{n+i+2+n+1}{2} \right\rceil \geq 2n + 6.$$

$$d^D(u_i, v_2) + \left\lceil \frac{f(u_i)+f(v_2)}{2} \right\rceil \geq n + 4 + \left\lceil \frac{n+i+2+n+2}{2} \right\rceil \geq 2n + 6.$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 7 + \left\lceil \frac{n+i+2+n+j+2}{2} \right\rceil \geq 2n + 6.$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq n + 8 + \left\lceil \frac{n+i+2+n+j+2}{2} \right\rceil \geq 2n + 6.$$

$$d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq n + 9 + \left\lceil \frac{n+i+2+n+j+2}{2} \right\rceil \geq 2n + 6.$$

$$d^D(v_1, v_2) + \left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil \geq 2n + 5 + \left\lceil \frac{n+1+n+2}{2} \right\rceil \geq 2n + 6.$$

Therefore, $f(u_n) = 2n + 2$ is the largest label.

Hence, $rmn^D(D(F_n)) = 2n + 2, n \geq 2$.

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