ISSN 2350 - 0328

International Journal of AdvancedResearch in Science, Engineering and Technology

Vol. 3, Issue 9, September 2016

The Mathematical and Algorithmic Foundations of Identification of Dynamic Objects Based on Integral Equations.

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ABSTRACTThe article deals with the development of numerical algorithms for the identification of controlled dynamic objects, noise-resistantrelatively the signal values, as well as allowing the efficient organization of computing processes while creating a computer simulation tools, including hardware implementation. Questions of constructing the sampled models based on approximation methods are considered. The proposed algorithms are comprehensively investigated taking into account theerrors of initial data.

KEYWORDS:Automatic control systems, dynamic objects, mathematical models, identification algorithms, Volterra integral equations.

I. INTRODUCTION

Application of mathematical models and computer means realized in information technologies is especially characteristic to processes of development and functioning the automated and automatic control systems by the technological processes and many views of industrial objects, etc. Modern methods and control facilities in many cases are created by electronic systems, mobile objects based on using the mathematical models of managementobjects. One of the basic methods of reception the mathematical descriptions is construction of models of processes by experimental data that corresponds to an identification tasks.

Are the integrated equations, possessing property of universality, the effective mathematical device for modeling continuous objects, including solvingthe tasks of calculating theparameters of dynamic objects' models? They are especially effective when mathematical models are creating based on experimentally measured input and output signals. This shows that while solving many problems of numerical modeling it is possible to realize following advantages of integrated substitution: smoothing properties of integrated operators and high stability of numerical operations at integration. Therefore, methods of identification of the models, based on application of the integrated equations and their numerical realization, are effective for practical realization.

II. INTEGRATED MODELS.

As dynamic model we shall understand mathematical model of dynamic objects (DO) or systems of a view

$$A\,[Y\,(x,t);F\,(X,t);Q\,(x,t)] = 0, \qquad (1)$$

where $t \in [0,T], x \in \Omega \in R^m, Y \in B_1\ F \in B_2, Q \in B_3$ t —time, Ω — some compact set from R^m, $x = (x_1,x_2,...,x_m)$ — a vector of spatial coordinates of DO model, $B_i, i = 1,2,3$ -functional spaces, A — any, generally, the unknown operator certain on the Cartesian product of spaces $B1 \times B2 \times B3$, satisfying to a condition of existence of continuous

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implicitfunction $Y(F,Q)$, Y - a vector of output coordinates(signals) DO, F - a vector of input coordinates (signals) DO, Q -a vector of unknown parameters [1,2]describing non-stationary DOwith the concentrated parameters[7].

In case of integrated dynamic model (IDM) input or output variables enter under a sign on integral and, thus, there is no operation of differentiation of these signals (operator *A*in (1) integrated). In other words, model (1) we shall name IDM if the operator *A* is integrated.

Researches of last year's show, that in some cases it is expedient to consider more general integrated dynamic models of a view

$$A(t)y(t) + \int_{G(t)} K(t,\tau)y(\tau)d\tau = F(t), \tau \in G(t), \tag{2}$$

where $A(t)$ and $K(t,\tau)$, functions subject to definition, $y(t)$ - an output signal $F(t) := F(f;t)$ - the known function defined through values of input signal f, G - some final or infinite set.

III. INTEGRATED METHOD OF IDENTIFICATION OF DYNAMIC OBJECTS.

We shall consider the problem constructions based on square-law formulas of computing algorithms for definition of parameters of linear integrated dynamic models of a view

$$\begin{aligned} A_1(x,t)y(x,t) + \int_{D_1\otimes G_1(t)} K_1[x,t,s,\tau,y(s,\tau)dsd\tau + L_1(x,t) = \\ A_2(x,t)f(x,t) + \int_{D_2(x)\otimes G_2(t)} K_2[x,t,s,\tau)f(s,\tau)]dsd\tau + L_2(x,t) \end{aligned}, \tag{3}$$

where y (x, t) and f (x, t) —vectors of entrance and output variables respectively, $A_i(x,t)$ $K_i(x,s,t,\tau)$,$L_i(x,t)$ — the functions-matrixes defined structural and physical properties of modeled objects or systems; x, s — spatial coordinates: $x, s \in D = D_1 \cup D_2 \subset R^m$ (m=1,2,…)t,τ — time $t,\tau \in G_1(t) \cup G_2(t) \subset R^1$, D_i и $G_i(t), i=1,2$— some sets from Euclidean spaces respectively $R^m$and R^1.

For such class of DO the model (2) will become:

$$a_1(t)y(t) + \int_{G_1(t)} K_1(t,\tau)y(\tau)d\tau + L_1(t) = a_2(t)f(t) + \int_{G_2(t)} K_2(t,\tau)f(\tau)d\tau + L_2(t), \tag{4}$$

where $a_i(t),\ K_i(t,\tau),\ L_i(t)\quad (i=1,2)$ parameters a subject definition,y and f—according to output and entrance signals, $G_i(t)$ variable, generally, unknown areas of integration. In the further for simplicity of a statement we shall believe, that $G_1(t) = G_2(t) = [0,t], t \in [0,T]$.

The simple algorithm for calculation of unknown parameters in (4) can be received, applying to calculation of integrals in (4) quadrature formulas of a view [3; 5]

$$\int_0^{t_i} x(\tau)d\tau = \sum_{j=0}^{N_i} W_{ij}x(t_i) + r_i[x], N_i = \overline{1,N}, \tag{5}$$

where W_{ij}— weights,$\boldsymbol{r_i[x]}$— a residual member square-law formulas, t_j —units of a view

$$0 \le t_1 \le t_2 \le \ldots < t_N \le T \tag{6}$$

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with some errors $\tilde{f}(t_i) = f(t_i) + \delta_i, \quad \max_{0\le i\le N} |\delta_i| = \delta,$ (7)

$$\tilde{y}(t_i) = y(t_i) + \varepsilon_i, \quad \max_{0\le i\le N} |\varepsilon_i| = \varepsilon, \quad (8)$$

where $\tilde{f}$ and $\tilde{y}$ - approximate, f and y–exact values of signal.

The essence of the quadrature algorithm of calculatingthe parameters of model (4) is that settlement expressions in it are formed based on digitization of integrals by means of square-law formulas of a view (4) with rejection of corresponding residual members. Digitizing, thus, the model (4) in points t_i, $i=\overline{0,N}$ of a view (6), we receive following system from $N+1$ –th linear algebraic equation concerning unknown parameters:

$$\begin{aligned} &a_1(0)y(0)+L_1(0)=a_2(0)f(0)+L_2(0),\\ &a_1(t_i)y(t_i)+\sum_{j=0}^{N_i} W_{ij}K_1(t_i,t_j)y(t_j)+L_1(t_i)= \\ &=a_2(t_i)y(t_i)+\sum_{j=0}^{M_i} W_{ij}K_2(t_i,t_j)f(t_j)+L_2(t_i),\\ &M_iN_i=\overline{1,N}, i=\overline{1,N}. \end{aligned} \quad (9)$$

At absence of any additional information of unknown parameters $a_\nu(t_i), K_\nu(t_i,t_j), L_\nu(t_i), \nu=1,2$ in system (9) will be, generally speaking, $2(N+1)(N+3)$ unknown persons, i.e. in this case there are known difficulties of the solving not predetermined SLAE. To avoid it is possible, for example, having assumed, that unknown parameters in (4) have polynomial a view, i.e.

$$a_\nu(t)=\sum_{k=1}^{m_\nu}\alpha_{\nu k}\beta_k^\nu(t), \quad (10)$$

$$L_\nu(t)=\sum_{k=1}^{l_\nu}\lambda_{\nu k}\gamma_k^\nu(t), \quad (11)$$

$$K_\nu(t,\tau)=\sum_{k=1}^{P_\nu}\sum_{s=1}^{Q_\nu}C_{\nu rs}\varphi_r^\nu(t)\psi_s^\nu(\tau), \quad (12)$$

where $\alpha_{\nu k}, \lambda_{\nu k}, c_{\nu rs}$ — unknown constant factors, and $\{\beta_k^\nu\}_{k=1}^{m_\nu}, \{\gamma_k^\nu\}_{k=1}^{l_\nu}, \{\varphi_r^\nu\}_{r=1}^{P_\nu}, \{\psi_s^\nu\}_{s=1}^{Q_\nu}$ - some systems of linearly independent functions, $\nu=1,2$.

It is obvious, that, if

$$N=p=m_1+m_2+l_1+l_2+p_1Q_1+p_2Q_2-1, \quad (13)$$

thannumber of the equations in (9) will be equal to number of unknown parameters. Certainly, in this case, questions of choice of functions $\beta_k^\nu, \gamma_k^\nu, \varphi_k^\nu, \psi_k^\nu$; questions of existence and uniqueness of SLAE decision (9), and also a question of impact of errors both $\delta_i\ \varepsilon_i$ in (7) and (8) on accuracy of calculation of unknown parameters are opened up. These questions in the certain degree can be solved in particular, but enough important case when required parameters $a_\nu(t) L_\nu(t)$ and $K_\nu(t,s)$ are defined by following parities:

$$a_1(t)\equiv 1, a_2(t)\equiv 0, L_1(t)\equiv 0, \quad (14)$$

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$$K_1(t,s) = K(t-s) = \sum_{j=1}^{m} q_j \frac{(t-s)^{j-1}}{(j-1)!}, m \in N, \tag{15}$$

$$K_2(t,s) = \frac{(t-s)^{m-1}}{(m-1)!}, \tag{16}$$

$$L_2(t) = \sum_{j=1}^{m-1}\left(c_j \frac{t_j}{j!} + q_j \sum_{k=0}^{m-j-1} c_k \frac{t^{k+j}}{(k+j)!} \right), \tag{17}$$

where q_i — unknown parameter, and c_i — known constants.

It is easy to notice, that the equation (4) at the chosen values of parameters $a_v(t), L_v(t), K_v(t,s)$ will be equal to the differential equation of a view

$$y^{(m)}(t) + \sum_{j=1}^{m} q_j y^{(m-j)}(t) = f(t), y^{(k)}(0) = c_k, \quad k = \overline{0, m-1}.$$

For formation of system of the linear algebraic equations concerning unknown factors $q_j,\ j = \overline{1, m}$ we shall transform the equation (9),taking into account (10)- 17), to a view

$$\sum_{j=1}^{m} q_j \left[\int_0^t \frac{(t-s)^{j-1}}{(j-1)!} y(s)ds - \sum_{k=0}^{m-j-1} c_k \frac{t^{k+j}}{(k+j)!} \right] = \tag{18}$$
$$= \int_0^t \frac{(t-s)^{m-1}}{(m-1)!} f(s)ds + \sum_{j=0}^{m-1} c_j \frac{t_j}{j!} - y(t).$$

From here for points of fixing (measurement) $t_i\ (i = \overline{0, N})$ of a view (6), believing, that m = n we receive system of the linear algebraic equations concerning unknown factors $q_i, j = \overline{1, m}$

$$A.q = b, \tag{19}$$

where $q = (q_1, ..., q_m)^T, b = (b_1, ..., b_m)^T, A = [A_{ij}]_{i,j=1}^{m},$

$$A_{ij} = \int_0^{t_i} \frac{(t_i - s)^{j-1}}{(j-1)!} y(s)ds + \sum_{k=0}^{m-j-1} c_k \frac{t_i^{k+j}}{(k+j)!}, \tag{20}$$

$$b_j = \int_0^{t_i} \frac{(t_i - s)^{m-1}}{(m-1)!} f(s)ds - y(t_i) + \sum_{v=0}^{m-1} c_v \frac{t_i^v}{v!}, \tag{21}$$

Let's apply for calculation of integrals in (20) and (21) the quadrature formulas of a view (5) which for any integrated on Remand functions $u(t)$, will become:

$$\int_0^{t_i} (t_i - s)^v y(s)ds = \sum_{j=0}^{L_i} W_{ij} (t_i - t_j)^v y(t_j) + r_{iv}[n], \tag{22}$$

where $1 \le L_i \le N, r_{iv}$ - residual members of this formula, and other sizes are certain in (5).

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Rejecting the residual terms of square-law formulas, $r_{ij}[y]$ and $r_{im}[f]$ respectively, and considering the fact that according to (7) and (8) values of input and output signals are set experimentally with some errors, from system of the equation (19) we come to following system of the equations concerning the approximate values ofvector'scomponents: $\tilde{q} = (\tilde{q}_1,...,\tilde{q}_m)^T$:

$$\tilde{A}\tilde{q} = \tilde{b}, \quad (23)$$

where $\tilde{A} = \left[A_{ij}\right]_{i,j=1}^{m}, \tilde{b} = \left[\tilde{b}_1,...,\tilde{b}_m\right]^T$, (24)

$$\tilde{A}_{ij} = \frac{1}{(j-1)!}\sum_{k=0}^{N_i} W_{ik}(t_i - t_k)^{j-1}\tilde{y}(t_k) - \sum_{l=0}^{m-j-1} c_l \frac{t_i^{l+j}}{(l+j)!}, 1 \le N_i \le N, \quad (25)$$

$$\tilde{b}_i = \frac{1}{(m-1)!}\sum_{k=0}^{M_i} W_{ik}\left|t_i - t_k\right|^{m-1}\tilde{f}(t_k) + \sum_{v=0}^{m-1} c_v \frac{t_i^{v}}{v!} - \tilde{y}(t_i), \ 1 \le M_i \le N`. \quad (26)$$

Thus, we have received final system for calculation of parameters $q_i(i = \overline{1,n})$. The block diagram quadrature algorithm is shown on fig.1

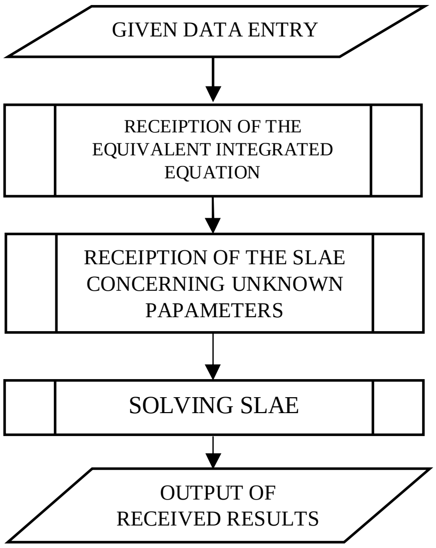

Fig. 1. The block diagram of quadrature algorithm

The analysis of the operations from given algorithm, allows assume, that at calculation of parameters of dynamic models of a view (4) it possesses high speed and stability. Besides,in its simplicity it allows to synthesize high-efficiency computers. To be convinced of working efficiency of method, we shall consider some examples of the decision of some test problems

Example. Input signal: $u(t) = 1 - e^{-2t}$ $t \in [0;2]$ $h = 0.01$

Output signal: $f(t) = -14e^{-2t} - 0.2$

Entry conditions: $C_1 = 0, \quad C_2 = 2, \quad C_3 = -4, C_4 = 8, C_5 = -16$

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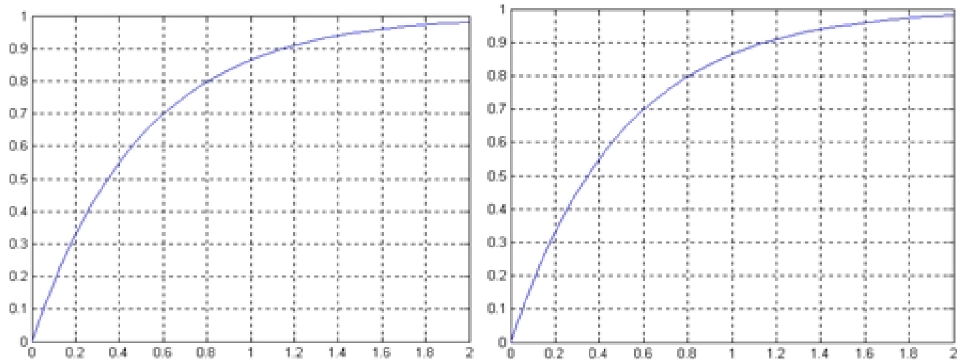
Problem: to define factors p_i of the equivalent differential equation

$$u^{(5)}(t) + p_1u^{(4)}(t) + p_2u^{(3)}(t) + p_3u^{(2)}(t) + p_4u^{(1)}(t) + p_5u(t) = f(t), \quad (27)$$
$$u^{(i-1)}(0) = C_i, \ i = \overline{1,5}.$$

The exact decision: $p_1 = 1.2,\ p_2 = -2,\ p_3 = 3.1,\ p_4 = 0.7,\ p_5 = -0.2$.



a ***b***

Fig. 2. The decision of the equation (27)

Using expressions (21)—(23) and the quadrature formula of trapezes for approximation of the integrals entering into expressions (22), (23), we receive SLAE concerning unknown factors p_i, and the given system is not joint. Applying to its decision a method of the least squares, we receive following values of required factors:

$p_1 = 1.8930,\ p_2 = -1.8640,\ p_3 = 3.0249,\ p_4 = 5.5493,\ p_5 = -0.1997$.

On fig. 2,the decision of the equation (27) is presented accordingly at exact values of factors (a) and at factors received as a result of calculation (b). A root-mean-square mistake $\Delta = 3.4\times10^{-6}$.

Let's add to a output signal*f*the casual handicap distributed under the normal law. The values of factors received at various values of handicaps are presentedin table 1.

Size in % from an output signal	Root-mean square mistake	p_1	p_2	p_3	p_4	p_5
1	4.0×10^{-5}	2.01	-2.19	5.16	12.09	-0.20
5	8.8×10^{-4}	-4.77	4.04	8.09	-61.30	-0.21
10	3.1×10^{-3}	41.17	-36.85	-15.41	422.95	-0.25

Table 1.The values of factors received at various values of handicaps.

Results of solving the considered examples (and many other things) tells about such properties of an integrated method of identification, as high enough stability, efficiency in sense of expenses of machine time and volume of calculations, simplicity of realization. Thus, the prospective method can be effectively used at the decision of problems of parametrical identification, which are characterized by presence of an error in initial data.

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IV. PARAMETERIZATION OF INTEGRATED MODELS THROUGH THE USE OF POWER SERIES

In connection with that in practice, as a rule, the number of measurements (points of quantization) and number of unknown parameters $M := \sum_{i=0}^{N} m_i$ are equal, and also in connection with necessity of effective calculation of integrals in (18), we investigate a question of reception of normal systems of the linear algebraic equations rather $\{q_{ij}\}\begin{matrix} j=\overline{1,m_i} \\ i=0,r \end{matrix}$ on the basis of preliminary approximation of initial data $\tilde{y}(t_i)$ and $\tilde{f}(t_i), i=\overline{1,N}$ by means of adder operators [4]. Not limiting a generality of reasoning's, further we shall believe that in initial model (18) every $y_k = y_k(0), k=\overline{0,r-1}$ and $\varphi_j(t)$ functions such, that integrals of a view

$$\int_0^t \tau^v \varphi_k^{(j)}(s)ds, \; v \in N \qquad (28)$$

are calculated precisely.

Let's consider algorithm of calculation of parameters q_{ij} which consists in the following.

Representing by means of the adder operator $U_{n,N}(t)$ entrance and output signals in the form of equation Volterra

$$A_0(t)y(t) + \int_0^t k(t,\tau)y(\tau)d\tau = F(t), \qquad (29)$$

in which

$$K(t,\tau) = \sum_{j=1}^{r}(-1)^{j-1}\binom{r}{j}\frac{(t-s)^{j-1}}{(j-1)!}A_0^{(J)}(s) +$$

$$\sum_{\tau=1}^{r}\sum_{j=0}^{r-\tau}(-1)^{j-1}\binom{r-\tau}{j}\frac{(t-s)^{\tau+j-1}}{(\tau+j-1)!}A_\tau^{(j)}(s), \qquad (30)$$

$$\binom{p}{n} = \frac{p!}{(p-n!n!)},$$

$$F(t) = \sum_{\tau=0}^{r-1}\sum_{j=0}^{r-\tau-1}\frac{B_j^{(r-\tau)}(A_{r-\tau};y)}{(j+\tau)}t^{j+\tau} + I_r(f;t);$$

$$B_j\binom{r-\tau}{j}(A_{r-j};y) = \frac{(r-\tau)}{j!}\sum_{v=0}\binom{j}{v}A_{r-\varepsilon}^{(v)}(0)\times$$
$$\times y_{j-v}\sum_{i=0}^{n}(-1)^i\frac{(j-i)!}{(r-\tau-v)!}\binom{v}{i} \qquad (31)$$

Through $Ir\ (f;t)$ r-th integral from inputsignal is designated:

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$$I_r(f;t)=\frac{1}{(r-1)!}\int_0^t (t-s)^{r-1} f(s)ds,$$
$$Y_k := Y^{(k)}(0), k=\overline{0,r-1}, \tag{32}$$

we pass from the equation (29) to corresponding approached integrated model. Further, the given algorithm of calculation of parameters of dynamic model (29) we shall name integral-adder (IA).

Scheme of IA algorithm for calculation of parameters of dynamic model (14)—(17) stationary DO is the same, and system (21) will become $A\tilde{q}=u,$ where

$$A=\left[\alpha_{ij}\right]_{i,j=1}^{m,m}, q=[\tilde{q}_1,\tilde{q}_2,...,\tilde{q}_m]^T, U=[u_1,u_2,...,u_m]^T,$$

$$A_{ij}=\frac{1}{(j-1)!}\int_0^{t_i}(t_i-s)^{j-1}U_n(\tilde{y},s)ds-\sum_{\nu=0}^{m-j-1}c_\nu\frac{t_i^{\nu+j}}{(\nu+j)!},\quad c_{-1}=0, \tag{33}$$

$$F_i=\frac{1}{(m-1)!}\int_0^{t_i}(t_i-s)^{m-1}U_n(\tilde{f};s)ds+\sum_{\nu=0}^{m-1}c_\nu\frac{t_i^\nu}{\nu!}-U_n(\tilde{y};t_i) \tag{34}$$

Question of realization IA of algorithm we shall consider on an example of stationary DO, described in the form of the differential equation of the second order

$$y''(t)+q_1y'(t)+q_2y=f(t),$$
$$y(0)=c_0, y'(0)=c_1. \tag{35}$$

For the differential equation (35)the equivalent integrated equation is

$$y(t)+\int_0^t[q_1+q_2(t-s)]y(s)ds=\int_0^t(t-s)f(s)ds+c_0+c_1t+q_1c_0t. \tag{36}$$

Let's write this equation in the convenient form for formation SLAE concerning unknown parameters

$$q_1(\int_0^t y(s)ds-c_0t)+q_2\int_0^t(t-s)y(s)ds=\int_0^t(t-s)f(s)ds+c_0+c_1t-y(t).$$

As the adder operator, we shall take interpolation a cubic spline of a view

$$s_3(x;\tilde{y})=d_1^j(x_j-x)^3+d_2^j(x-x_{j-1})^3+d_3^j(x_j-x)+d_4^j(x-x_{j-1}), \tag{37}$$
$$s_3(x;\tilde{f})=a_1^j(x_j-x)^3+a_2^j(x-x_{j-1})^3+a_3^j(x_j-x)+a_4^j(x-x_{j-1}). \tag{38}$$

Let s substitute (37) and (38) in (36), we shall receive:

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$$q_1\left(\sum_{j=1}^{i}\int_{t_{j-1}}^{t_j} s_3(s;\tilde{y})ds - c_0 t_j\right) + q_2\sum_{j=1}^{i}\int_{t_{j-1}}^{t_j}(s_j - s)s_3(s,\tilde{y})ds = \qquad (39)$$

$$= \sum_{j=1}^{i}\int_{t_{j-1}}^{t_j}(t_j - s).s_3(s,\tilde{f})ds + c_0 + c_1 t_j - s_3(s;\tilde{y}_j), \quad i=\overline{1,m}.$$

For formation of system rather q_1, also q_2 it is necessary to calculate integrals of a view

$$\int_{t_{j-1}}^{t_j}[d_1^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})]ds, \qquad (40)$$

$$\int_{t_{j-1}}^{t_j}(s_j - s)[d_i^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})]ds. \qquad (41)$$

First, we shall calculate integral

$$\int_{t_{j-1}}^{t_j}\left[d_1^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})\right]ds = d_1^j[(jh)^3 h -$$
$$-3j^3h^4 + \frac{3}{2}j^2h^4 + 3j^3h^4 - 3j^2h^4 + jh^4 + j^3h^4 - \frac{3}{2}j^2h^4 + jh^4 - \frac{h^4}{4}] + d_2^j[j^3h^4 -$$
$$-\frac{3}{2}j^3h^4 - \frac{3}{2}j^2h^4 + jh^4 + \frac{h^4}{4} - 3j^3h^4 + 6j^2h^4 - 4jh^4 + 3j^3h^4 - \frac{15}{2}j^2h^4 - 6j^4h^4 + \qquad (41a)$$
$$+\frac{3}{2}h^4 - j^3h^4 + 3j^2h^4 + h^4] + d_3^j(j^2h^2 - j^2h^2 + jh^2 + \frac{j^2h^2}{2} - \frac{j^2h^2}{2} + jh^2 - \frac{h}{2})^2 +$$
$$+ d_4^j[\frac{j^2h^2}{2} - \frac{j^2h^2}{2} + jh^2 - \frac{h^2}{2} - (j^2 - j)h^2 + (j^2 - 2j + 1)h^2] =$$
$$= d_1^j(2j^3 - 3j^2 + 2j - \frac{1}{4})h^4 + d_2^j(6j - 6j^4 + \frac{15}{4})h^4 + d_3^j(2j - \frac{1}{2})h^2 + d_4^j\frac{h^2}{2}.$$

Now we shall calculate integral (41), using (41a):

$$\int_{t_{i-1}}^{t_j}(s_j - s)[d_1^j(s_j - s)^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})]ds = s_j[d_1^j(2j^3 - 3j^2 + 2j - \frac{1}{4})h^4 +$$
$$+ d_2^j(6j - 6j^4 + \frac{15}{4})h^4 + d_3^j(2j - \frac{1}{2})h^2 + d_4^j\frac{h^2}{2}] - \int_{t_{j-1}}^{t_j} s[d_1^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 +$$
$$+ d_3^j(s_j - s) + d_4^j(s - s_{j-1})].$$

Further, we shall calculate integral

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$$\int_{t_{j-1}}^{t_j} s[d_1^j(s_j-s)^3+d_2^j(s-s_{j-1})^s+d_3^j(s_j-s)+d_4^j(s-s_{j-1})]ds= \tag{41b}$$

$$=\left(\frac{1}{4}h^5-\frac{h^5}{5}\right)d_1^j++(2j^3+\frac{1}{4}j-\frac{1}{20})h^5\cdot d_2^j+$$

$$=(2j^2h^3-\frac{3}{2}jh^3+\frac{h}{3})^3d_3^j+\frac{h^2}{2}d_4^j.$$

and, substituting (41) in (41a), we shall receive

$$\int_{t_{j-1}}^{t_j} s(s_j-s)[d_1^j(s_j-s)^3+d_2^j(s-s_{j-1})^3+d_3^j(s_j-s)+d_4^j(s-s_{j-1})]ds=$$

$$=[d_1^j(2j^4-3j^3-2j^2-\frac{1}{2}j+\frac{1}{5})h^5+d_2^j(-2j^3+6j^2+\frac{7}{2}j-6j^5+$$

$$+\frac{1}{20})h^5+d_3^j(j-\frac{1}{6})h^3+d_4^j(-j^2+2j-\frac{5}{6})h^3].$$

For calculation of integrals $\int_{xj-1}^{xj}(x_j-x)s_3(x,f)dx$ we shall seize the expression above

$$\int_{t_{j-1}}^{t_j}(s_j-s).s_3(s,\tilde{f})ds=\int_{t_{j-1}}^{t_j}[(s_j-s)^3+a_2(s-s_{j-1})^3+a_3^j(s_j-s)+a_4^j\ (s-s_{j-1})]ds=$$

$$a_1^j(2j^4-3j^3+2j^2-\frac{1}{2}j+\frac{1}{5})h^5+a_2^j(-2j^3+6j^2+\frac{7}{2}j-6j^5+\frac{1}{20})h^5+$$

$$+a_3^j(j-\frac{1}{6})h^3+a_4^j(-j^2+2j-\frac{5}{6})h^3.$$

Now for formation of system about q_1 also q_2 we use expressions (40) and (41). Then, it is possible to write down (39) in the form of

$$q_1[\sum_{j=1}^{i}d_1^j(2j^3-3j^2+2j-\frac{1}{4})h^4+d_2^j(6j-6_j^4+\frac{15}{4})h^4+d_3^j(2j-\frac{1}{2})h^2+$$

$$+d_4^j\frac{h^2}{2}-C_0jh]+q_2\sum_{j=1}^{i}[d_1^j(2j^4-3j^3+2j^2-\frac{1}{2}j+\frac{1}{5})h^5+d_2^j(-2j^3+$$

$$+6j^2+\frac{7}{2}j-6j^5+\frac{1}{20})h^5+d_3^j(j-\frac{1}{3})h^3+d_4^j(-j^2+2j-\frac{5}{6}h^3]-$$

$$-\sum_{j=1}^{i}[a_1^j(2j^4-3j^3+2j^2]-\frac{1}{2}j+\frac{1}{5})h^5+a_2^j(-2j^3+6j^2+\frac{7}{2}j-$$

$$-6j^5+\frac{1}{20})h^5+a_3^j(j-\frac{1}{3})h^3+a_4^j(-j^2+2j-\frac{5}{6})h^3]+C_0+C_1jh-y_j.$$

So, the final view of SLAE about $q_1$and q_2 looks like

$$Aq\quad=\quad F$$

where

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$$A=\{A_{k1},A_{k2}\}_{k=\overline{1,m}},q=(q_1,q_2)^T,F=(F_1,F_2,...,F_m),$$

$$A_{k1}=\sum_{j=1}^{k}[d_1^j(2j^3-3j^2+2j-\frac{1}{4})h^4+d_2^j(6j+\frac{15}{4}-6j^4)h^4+d_3^jh^2(2j-\frac{1}{2})+$$

$$+d_4^j\frac{h^2}{2}-C_0jh],A_{k2}=\sum_{j=1}^{k}[d_1^j(2j^4-3j^3+2j^2-\frac{1}{2}j+\frac{1}{5})h^5+d_2^j(-2j^3+$$

$$+6j^2+\frac{7}{2}j-6j^5+\frac{1}{20})h^5+d_3^j(j-\frac{1}{3})h^3+d_4^j(-j^2+2j-\frac{5}{6})h^3],$$

$$F_k=\sum_{j=1}^{k}[a_1^k(2k^4-3k^3+2k^2-\frac{1}{2}k+\frac{1}{5})h^5+a_2^k(-2k^3+6k^2+\frac{7}{2}k-$$

$$-6k^5+\frac{1}{20})h^5+a^k(k-\frac{1}{3})h^3+a_4^k(-k^2+2k-\frac{5}{6})h^3].$$

So, the algorithm of calculation of parameters in case of when input and output signals are approximated by cubic splines, consists in the following:

1) the task of initial data $t_j=jh,h,yi,fi,j=\overline{1,n},c_0,c_1$;

2) calculation of factors of a cubic spline on each site (x_{j-1},x_j) approximating entrance a signal $f(t)$ and a output signal $y(t)$;

3) formation of matrix A ;

3) formation of the right part ***F*** **;**

4) solvingsystem(40).

The block diagram of a spline-integrated algorithm of calculation of parameters of dynamic models DO is illustrated on fig. 3.

Application of splines allows:

1. to raise accuracy of calculation of parameters q on the order concerning a step h in comparison with quadrature algorithm based on the formula of trapezes;
2. to receive additional points for formation of normal systems concerning counted parameters in case of the incomplete initial information

Thus, the considered algorithm possesses a number of properties, which are necessary for digital realization, in particular, it is noise proof concerning an error of initial data, and also suitable as a basis of the organization of software for the decision of problems of identification. Procedure of numerical realization of systems of the linear equations supposes representation of a matrix of system*A*in the form of the product of the top and bottom triangular matrix allowing parallelizing computing processes.

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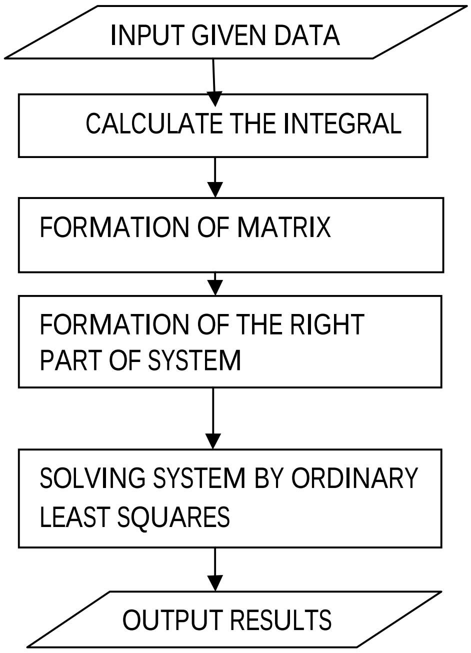

Fig.3. The block-diagram of spline-integrated algorithm.

Let us consider calculation of parameters of T-models [8] of non-stationary objects, which in effect adjoins an integrated method.
Consider initial mathematical model in the form of the equation

$$\sum_{i=0}^{n} a_i(t) y^{(i)}(t) = \sum_{j=0}^{s} b_j(t) f^{(j)}(t) \qquad (42)$$

where $y(t)$ and $f(/)$ — output and input signals respectively, $a_i(t)$and$b_j(t)$arevariable restored parameters.

As it hasbeen noted, the number of the basic difficulties arising at calculationerrors ofmeasurements values of entrance and output signals include the incorrectness of a problem of numerical differentiation of functions $y(t)$ and $f(t)$. One of effective ways of overcoming the specified difficulty is transition from model (42) to equivalent integrated model Volterra by M-fold (r = max(n,s)) integration of the left and right part (42) and repeated application of the formula of partial integration.

In that specific case, for example, when $r = n = s$in model (42) corresponds (to be convinced of it easily by analogy to the proof of the theorem of equivalence from [6]), in the assumption of sufficient smoothness of dependences$a_i(t)$ and$b_j(t)$, the following integrated, equivalent to itmodel of a view

$$a_r(t)y(t) + \int_0^t K_1(t,\tau)y(\tau)dt = b_r(t)x(t) + \int_0^t K_2(t,\tau)x(\tau)d\tau + F(t) \qquad (43)$$

where

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$$K_v(t,\tau)=\sum_{j=1}^{r}(-1)^{j-1}\binom{r}{j}\frac{(t-\tau)^{j-1}}{(j-1)!}C_{v0}^{(j)}(\tau)+$$
$$+\sum^{r}\sum_{j=0}^{r-l}(-1)^{j-1}\binom{r-l}{j}\frac{(t-\tau)^{l+j-1}}{(l+j-1)!}C_{vl}^{(j)}(t),\ v=1,2, \tag{44}$$

$$C_{1l}(t):=a_{r-l}(t),C_{2l}(t):=b_{r-l}(t),$$

$$F(t)=\sum_{l=0}^{r-1}\sum_{j=0}^{r-l-1}\frac{d_{jL}(x,y)}{(j+L)!}t^{j+s}, \tag{45}$$

$$d_{jl}(x,y)=\frac{(r-l)}{j!}\sum_{\mu=0}^{j}\binom{j}{\mu}[a_l^{\mu}(0)y^{(j-\mu)}0-$$
$$-b_l^{(\mu)}(0)x^{(j-\mu)}0]\sum_{i=0}^{\mu}(-1)^i\frac{(j-i)}{(r-l-v)!}\binom{\mu}{i}, \tag{46}$$

$$\binom{p}{q}:=\frac{p!}{(p-q)!q!}. \tag{47}$$

V. CONCLUSION.

The offered method of identification of dynamic objects on the basis of integrated equations Volterra allows to receive a set of square-law algorithms of calculating the parameters of mathematical models; the problem is reduced to the decision of algebraic systems, generally not joint which dimension is defined by character of initial experimental data and demanded accuracy of calculations. Research of algorithms allows drawing a conclusion on their high stability, efficiency in sense of expenses of machine time and volume of calculations, simplicity of realization.

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ISSN 2350 - 0328

International Journal of AdvancedResearch in Science, Engineering and Technology

Vol. 3, Issue 9, September 2016

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