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Elimination-Minimization Principle: Fitting of Polynomial Curve to Numerical Data

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ABSTRACT: An approach of estimation of parameters had been introduced in 2011 where the usual principle of least squares is applied to each parameter separately. This is done, for each parameter, by obtaining one model, containing the single parameter to be estimated. Later on this approach was termed in 2014 as stepwise least squares method. The principle involved in the approach has been termed in 2015 as elimination-minimization principle. The approach has already been applied successfully in fitting linear and quadratic curves to numerical data. This paper describes the method of fitting of a polynomial curve to numerical data by the approach.

KEYWORDS: Elimination-minimization principle, polynomial curve, fitting to numerical data.

I. INTRODUCTION

The method of least squares is indispensible and is widely used method of curve fitting to numerical data. The method of least squares was first discovered by the French mathematician *Legendre* in 1805 [Crafton (1870), Glaisher (1872), *Mansfield* (1877*a* , 1877*b*), Stigler (1977 , 1981) et al} and it has been established with the works of the renowned statistician *Adrian* (1808), the German Astronomer *Gauss* {Gauss (1809*a* , 1809*b* , 1929), Hall (1970), Buhler (1981), Sheynin (1979), Sprott (1978), Stigler (1977), et al], the mathematicians viz. *Ivory* (1825), *Hagen* (1837), *Bassel* (1838), *Donkim* (1857), *Herscel* (1850), *Crofton* (1870) etc..

In fitting of a curve by the method of least squares, the parameters of the curve are estimated by solving the normal equations which are obtained by applying the principle of least squares with respect to all the parameters associated to the curve jointly (simultaneously). However, for a curve of higher degree polynomial and / or for a curve having many parameters, the calculation involved in the solution of the normal equations becomes more complicated as the number of normal equations then becomes larger. Moreover, in many situations, it is not possible to obtain normal equations by applying the principle of least squares with respect to all the parameters simultaneously. These lead to think of searching for some other approach of estimation of parameters. For this reason, an approach of estimation of parameters had been introduced (Dhritikesh, 2011) where the usual principle of least squares is applied to each parameter separately. This is done, for each parameter, by obtaining one model, containing the single parameter to be estimated. Later on, this approach was termed as stepwise least squares method (Dhritikesh, 2014). The principle involved in the approach has been termed in 2015 as elimination-minimization principle (Atwar & Dhritikesh, 2015*a*). The approach has already been applied successfully in fitting linear and quadratic curves to numerical data {Atwar & Dhritikesh (2015*a* , 2015*b* , 2015*c* , 2015*d*)}. In the current study, the approach has been applied in the fitting of polynomial curve to numerical data. This paper describes the method of fitting of a polynomial curve.

II. METHOD OF FITTING

Here the general polynomial curve specifically the curve defined by the polynomial

$$Y = a_0 + a_1 X + a_2 X^2 + \ldots\ldots\ldots\ldots + a_k X^k$$

in X of degree k where $a_0, a_1, a_2, \ldots\ldots\ldots\ldots, a_k$ are the parameters, is taken into consideration.

Here, the two operators namely Δ and R have been used with the following definitions:

$$\Delta Y_i = Y_{i+1} - Y_i \quad \& \quad R(Y : X) = Y/X \tag{2.1}$$

At first, let us consider the case where observations on Y are distributed around a constant (parameter) μ i.e. the observations y_i $(i = 1, 2, 3, \ldots\ldots\ldots, n)$ on Y satisfy

$$y_i = \mu + e_i \;, \quad (i = 1, 2, 3, \ldots\ldots\ldots, n). \tag{2.2}$$

where e_i is the deviation/error associated to the observation y_i .

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 5 , May 2016

In order to apply the principle of least squares to estimate μ, it is required to minimize S with respect to μ where

$$S = \sum_{i=1}^{n} e_i^{\,2} = \sum_{i=1}^{n} (y_i - \mu)^2$$

Application of this yields the least squares estimate $\hat{\mu}_{(NE)}$ of μ as

$$\hat{\mu}_{(NE)} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad (2.3)$$

Now, let $\{(x_i , y_i) , (i = 1 , 2 , 3 , \ldots\ldots\ldots\ldots , n)\}$ be n pairs of observations on (X , Y).
The problem is to find out a method, other than the usual method of least squares, of estimating the parameters of the polynomial curve described by Equation (2.1).

First, let us consider the case of fitting of linear curve.

The mathematical curve considered here is of the form

$$Y = \mu + \lambda X \qquad (2.4)$$

where μ & λ are the parameters to be estimated on the basis of the observed data.

If the number of observations is more than 2, they need not necessarily satisfy the theoretical linear curve properly. Moreover, observations may suffer from error. Thus the observations satisfy the model

$$y_i = \mu + \lambda x_i + e_i \ , \ (i = 1 , 2 , 3 , \ldots\ldots\ldots\ldots , n) \qquad (2.5)$$

where e_i is the deviation/error component.

This yield $$\Delta y_i = \lambda \Delta x_i + \Delta e_i$$

i.e. $$y_i(1) = \lambda + e_i(1) \qquad (2.6)$$

where $y_i(1) = R(\Delta y_i : \Delta x_i)$ & $e_i(1) = R(\Delta e_i : \Delta x_i)$, $(i = 1 , 2 , 3 , \ldots\ldots\ldots\ldots , n - 1)$.

This is of the form (4.2).

Therefore, the least squares estimate of λ is found as

$$\hat{\lambda}_{(EM)} = \frac{1}{n-1}\sum_{i=1}^{n-1} y_i(1) \qquad (2.7)$$

$$= \text{some value } v_1, \text{ say}$$

Substituting the estimate $\hat{\lambda}_{(EM)}$ of λ in (4.6), one obtains that

$$y_i = \mu + v_1 x_i + e_i$$

i.e. $$y_i(x_i , v_1) = \mu + e_i \qquad (2.8)$$

where $$y_i(x_i , v_1) = y_i - v_1 x_i \ ,$$

for $(i = 1, 2, 3\ldots\ldots\ldots\ldots. n)$.

This is also of the form (4.2).

Therefore by the same logic as in the case of (4.2) in **Case-1**, the estimate $\hat{\mu}_{(EM)}$ of μ becomes

$$\hat{\mu}_{(EM)} = \frac{1}{n}\sum_{i=1}^{n} y_i(x_i , v_1) \qquad (2.9)$$

Next, let us consider the case of fitting of quadratic curve.

The mathematical curve considered here is of the form

$$Y = \alpha + \beta X + \gamma X^2 \qquad (2.10)$$

where α, β & γ are the parameters to be estimated on the basis of observed data.

If the number of observations is more than 3, they need not necessarily satisfy the theoretical quadratic curve properly. Moreover, observations may suffer from error. Thus the observations satisfy the model

$$y_i = \alpha + \beta x_i + \gamma x_i^2 + e_i \qquad (2.11)$$

for $(i = 1, 2, 3\ldots\ldots\ldots\ldots n)$.

where e_i is the deviation/error component.

ISSN: 2350-0328
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Vol. 3, Issue 5 , May 2016

This yield $\Delta y_i = \beta \Delta x_i + \gamma \Delta x_i^2 + \Delta e_i$

i.e. $y_i(1) = \beta + \gamma\ x_i^2(1) + e_i(1)$ (2.12)

where
$$y_i(1) = R(\Delta y_i : \Delta x_i),$$
$$x_i^2(1) = R(\Delta x_i^2 : \Delta x_i)$$
$$\& \quad e_i(1) = R(\Delta e_i : \Delta x_i)$$
for $(i = 1, 2, 3 \ldots\ldots\ldots n - 1)$.

This is a linear equation in x.

This further yield $y_i(2) = \gamma + e_i(2)$ (2.13)

where
$$y_i(2) = R(\Delta y_i(1) : \Delta x_i^2(1))$$
$$\& \quad e_i(2) = R(\Delta e_i(1) : \Delta x_i^2(1))$$
for $(i = 1, 2, 3 \ldots\ldots\ldots n - 2)$.

This is of the form (4.2).

Therefore, the estimate of γ becomes
$$\hat{\gamma}_{(EM)} = \frac{1}{n-2}\sum_{i=1}^{n-2} y_i(2) \tag{2.14}$$
= some value v_2, say

Substituting the estimate $\hat{\gamma}_{(EM)}$ of γ in (4.13), one obtains that
$$y_i(1) = \beta + v_2\ x_i^2(1) + e_i(1)$$
i.e. $y_i(x_i^2(1), v_2) = \beta + e_i$ (2.15)

where $y_i(x_i^2(1), v_2) = y_i(1) - v_2\ x_i^2(1)$
$(i = 1, 2, 3 \ldots\ldots\ldots n - 1)$.

This is also of the form (4.2).

Therefore, the estimate of β becomes
$$\hat{\beta}_{(EM)} = \frac{1}{n-1}\sum_{i=1}^{n-1} y_i(x_i^2(1), v_2) \tag{2.16}$$
= some value v_1, say

Again, substituting the estimate $\hat{\beta}_{(EM)}$ of β in (4.12), one obtains that
$$y_i = \alpha + v_1 x_i + v_2 x_i^2 + e_i$$
i.e. $y_i(x_i, x_i^2, v_1, v_2) = \hat{\alpha} + e_i$ (2.17)

where $y_i(x_i, x_i^2, v_1, v_2) = y_i - v_1 x_i - v_2 x_i^2$
for $(i = 1, 2, 3 \ldots\ldots\ldots n)$.

This is also of the form (4.2).

Therefore, the estimate of α becomes
$$\hat{\alpha}_{(EM)} = \frac{1}{n}\sum_{i=1}^{n} y_i(x_i, x_i^2, v_1, v_2) \tag{2.18}$$
= some value v_0, say

Next, let us consider the case of fitting of cubic curve.

The mathematical curve considered here is of the form
$$Y = \theta + \phi\ X + \delta\ X^2 + \eta\ X^3 + e \tag{2.19}$$
where θ, ϕ, δ & η are the parameters to be estimated on the basis of observed data.

If the number of observations is more than 4, they need not necessarily satisfy the theoretical linear curve properly. Moreover, observations may suffer from error. Thus the observations satisfy the model
$$y_i = \theta + \phi\ x_i + \delta\ x_i^2 + \eta\ x_i^3 + e_i \tag{2.20}$$

ISSN: 2350-0328
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Vol. 3, Issue 5 , May 2016

for ($i = 1, 2, 3 \ldots\ldots\ldots\ldots n$).
where e_i is the deviation/error component.
This yield $\Delta y_i = \phi \Delta x_i + \delta \Delta x_i^2 + \eta \Delta x_i^3 + \Delta e_i$

i.e. $$y_i(1) = \phi + \delta \, x_i^2(1) + \eta \, x_i^3(1) + e_i(1) \qquad (2.21)$$

where $y_i(1) = R(\Delta y_i : \Delta x_i)$,
$x_i^2(1) = R(\Delta x_i^2 : \Delta x_i)$,
$x_i^3(1) = R(\Delta x_i^3 : \Delta x_i)$
& $e_i(1) = R(\Delta e_i : \Delta x_i)$
for ($i = 1, 2, 3 \ldots\ldots\ldots\ldots n$-1).
This yield $\Delta y_i(1) = \delta \, \Delta x_i^2(1) + \eta \Delta x_i^3(1) + \Delta e_i(1)$

i.e. $$y_i(2) = \delta + \eta \, x_i^3(2) + e_i(2) \qquad (2.22)$$

where $y_i(2) = R(\Delta y_i(1) : \Delta x_i^2(1))$,
$x_i^3(2) = R(\Delta x_i^3(1) : \Delta x_i^2(1))$
& $e_i(2) = R(\Delta e_i(1) : \Delta x_i^2(1))$
for ($i = 1, 2, 3 \ldots\ldots\ldots\ldots n$--2).
This further yield
$$\Delta y_i(2) = \eta \, \Delta x_i^3(2) + \Delta e_i(2)$$

i.e. $$y_i(3) = \eta + e_i(3) \qquad (2.23)$$

where $y_i(3) = R(\Delta y_i(2) : \Delta x_i^2(2))$
& $e_i(3) = R(\Delta e_i(2) : \Delta x_i^2(2))$
for ($i = 1, 2, 3 \ldots\ldots\ldots\ldots n$ - 3).
This is of the form (4.2).
Therefore, the estimate of η becomes

$$\hat{\eta}_{(EM)}) = \frac{1}{n-3}\sum_{i=1}^{n-3} y_i(3) \qquad (2.24)$$
= some value v_3, say

Substituting the estimate $\hat{\eta}_{(EM)}$ of a_3 in (4.22), one obtains that

$$y_i(2) = \delta + v_3 x_i^3(2) + e_i(2)$$

i.e. $$y_i(x_i^3(2), v_3) = \delta + e_i(2) \qquad (2.25)$$

where $$y_i(x_i^3(2), v_3) = y_i(2) - v_3 x_i^3(2)$$
for ($i = 1, 2, 3 \ldots\ldots\ldots\ldots\ldots n$-2).
This is also of the form (4.2).
Therefore, the estimate of δ becomes

$$\hat{\delta}_{(EM)} = \frac{1}{n-2}\sum_{i=1}^{n-2} y_i(x_i^3(2), v_3) \qquad (2.26)$$
= some value v_2, say

Again, substituting the estimate $\hat{\delta}_{(EM)}$ of δ in (4.21), one obtains that

$$y_i(1) = \phi + v_2 x_i^2(1) + v_3 x_i^3(1) + e_i(1)$$

i.e. $$y_i(x_i^2(1), x_i^2, x_i^3(1), v_2, v_3) = \phi + e_i(1) \qquad (2.27)$$

where

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 5 , May 2016

$$y_i(x_i^2(1), x_i^2, x_i^3(1), v_2, v_3) = y_i(1) - v_2 x_i^2(1) - v_3 x_i^3(1)$$

for (i = 1, 2, 3……….… n - 1).
This is also of the form (4.2).
Therefore, estimate of ϕ becomes

$$\hat{\phi}_{(EM)} = \frac{1}{n-1}\sum_{i=1}^{n-1} y_i(x_i^2(1), x_i^2, x_i^3(1), v_2, v_3) \tag{2.28}$$

= some value v_1, say

Again, substituting the estimate $\hat{\phi}_{(EM)}$ of ϕ in (4.20), one obtains that

$$y_i = \theta + v_1 x_i + v_2 x_i^2 + v_3 x_i^3 + e_i$$

i.e.
$$y_i(x_i, x_i^2, x_i^3, v_1, v_2, v_3) = \theta + e \tag{2.29}$$

where $y_i(x_i, x_i^2, v_1, v_2, v_3) = y_i - v_1 x_i - v_2 x_i^2 - v_3 x_i^3$

for (i = 1 , 2 , 3 , …………. , n).
This is also of the form (4.2).
Therefore, estimate of θ becomes

$$\hat{\theta}_{(EM)} = \frac{1}{n}\sum_{i=1}^{n} y_i(x_i, x_i^2, x_i^3, v_1, v_2, v_3) \tag{2.30}$$

= some value v_0, say

Now, the case of fitting of a polynomial (of degree k) curve is considered.
The mathematical curve considered here is of the form

$$Y = \alpha_0 + \alpha_1 X + \alpha_2 X^2 + \ldots\ldots\ldots\ldots\ldots + \alpha_k X^k \tag{2.31}$$

where $\alpha_0, \alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_k$ are the parameters to be estimated on the basis of observed data.
If there are more than (k + 1) observations, they need not necessarily satisfy the theoretical linear curve properly. Moreover, observations may suffer from error. Thus the observations satisfy the model

$$y_i = \alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 + \alpha_3 x_i^3 + ----+ \alpha_k x_i^k + e_i \tag{2.32}$$

for (i = 1, 2, 3………….. n).
where e_i is the deviation/error component.
This yield

$$\Delta y_i = \alpha_1 \Delta x_i + \alpha_2 \Delta x_i^2 + \alpha_3 \Delta x_i^3 + ----+ \alpha_k \Delta x_i^k + \Delta e_i$$

This can be expressed as

$$y_i(1) = \alpha_1 + \alpha_2 x_i^2(1) + \alpha_3 x_i^3(1) + ----+ \alpha_k x_i^k(1) + e_i(1) \tag{2.32}$$

where

$$y_i(1) = R(\Delta y_i : \Delta x_i),$$
$$x_i^2(1) = R(\Delta x_i^2 : \Delta x_i),$$
$$x_i^3(1) = R(\Delta x_i^3 : \Delta x_i)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ,$$
$$x_i^k(1) = R(\Delta x_i^k : \Delta x_i)$$
$$\& \; e_i(1) = R(\Delta e_i : \Delta x_i)$$

(i = 1, 2, 3……… n - 1).

ISSN: 2350-0328
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Vol. 3, Issue 5 , May 2016

This yields

$$\Delta y_i(1) = \alpha_2 \Delta x_i^{\,2}(1) + \alpha_3 \Delta x_i^{\,3}(1) + - - - - - + \alpha_k \Delta x_i^{\,k}(1) + \Delta e_i(1)$$

i.e.
$$y_i(2) = \alpha_2 + \alpha_3 x_i^{\,3}(2) + - - - - - + \alpha_k x_i^{\,k}(2) + e_i(2) \qquad (2.34)$$

where

$$y_i(2) = R(\Delta y_i(1) : \Delta x_i^{\,2}(1)),$$
$$x_i^{\,3}(2) = R(\Delta x_i^{\,3}(1) : \Delta x_i^{\,2}(1))$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots ,$$
$$x_i^{\,k}(2) = R(\Delta x_i^{\,k}(1) : \Delta x_i^{\,2}(1))$$
$$\& \;\; e_i(2) = R(\Delta e_i(1) : \Delta x_i^{\,2}(1))$$
$$(i = 1, 2, 3 \ldots\ldots\ldots n - 2).$$

This yield

$$\Delta y_i(2) = \alpha_3 \Delta x_i^{\,3}(2) + - - - - - + \alpha_k \Delta x_i^{\,k}(2) + \Delta e_i(2)$$

i.e.
$$y_i(3) = \alpha_3 + - - - - - + \alpha_k x_i^{\,k}(3) + e_i(3) \qquad (2.35)$$

where

$$y_i(3) = R(\Delta y_i(2) : \Delta x_i^{\,3}(2)),$$
$$x_i^{\,4}(3) = R(\Delta x_i^{\,4}(2) : \Delta x_i^{\,3}(2)),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots ,$$
$$x_i^{\,k}(3) = R(\Delta x_i^{\,k}(2) : \Delta x_i^{\,3}(2))$$
$$\& \;\; e_i(3) = R(\Delta e_i(2) : \Delta x_i^{\,3}(2))$$
$$(i = 1, 2, 3 \ldots\ldots\ldots n - 3).$$

Continuing the process, one can arrive at the $(k-1)^{th}$ step that

$$\Delta y_i(k-2) = \alpha_{k-1} \Delta x_i^{\,k-1}(k-2) + \alpha_k \Delta x_i^{\,k}(k-2) + \Delta e_i(k-2)$$

i.e.
$$y_i(k-1) = \alpha_{k-1} + \alpha_k x_i^{\,k}(k-1) + e_i(k-1) \qquad (2.36)$$

where

$$y_i(k-1) = R(\Delta y_i(k-2) : \Delta x_i^{\,k-1}(k-2))$$
$$\& \;\; e_i(k-1) = R(\Delta e_i(k-2) : \Delta x_i^{\,k-1}(k-2))$$
$$(i = 1, 2, 3 \ldots\ldots\ldots n - k + 1).$$

At the k^{th} step, one obtain hat

$$\Delta y_i(k-1) = \alpha_k \Delta x_i^{\,k}(k-1) + \Delta e_i(k-1)$$

i.e.
$$y_i(k) = \alpha_k + e_i(k) \qquad (2.37)$$

where

$$y_i(k) = R(\Delta y_i(k-1) : \Delta x_i^{\,k}(k-1))$$
$$\& \;\; e_i(k) = R(\Delta e_i(k-1) : \Delta x_i^{\,k}(k-1))$$
$$(i = 1, 2, 3, \ldots\ldots\ldots\ldots, n - k).$$

This is of the form (4.2).

Therefore, estimate of α_k becomes

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 5 , May 2016

$$\hat{\alpha}_k = \frac{1}{n-k}\sum_{i=1}^{n-k} y_i\,(k) \qquad (2.38)$$

= some value v_k, say

Substituting the estimate $\hat{\alpha}_k$ of α_k in the similar equation obtained at the earlier step, one can obtain the estimate of α_{k-1}. Then substituting the estimate $\hat{\alpha}_{k-1}$ of α_{k-1} and $\hat{\alpha}_k$ of α_k respectively in the similar equation obtained at the earlier step, one can obtain the estimate of α_{k-2}. Continuing the process, one can obtain the estimates of α_{k-2}, α_{k-1}, ………… , α_2 , α_1, α_0 stepwise. The estimates will be as follow

$$\hat{\alpha}_{k-1} = \frac{1}{n-k+1}\sum_{i=1}^{n-k+1} y_i\,(x_i^{\,k}(2):\ v_k) \qquad (2.39)$$

= some value v_{k-1}, say

where $y_i\,(k-1) - \alpha_k\ x_i^{\,k}(k-1)$

$y_i\,(x_i^{\,2}(1),\ x_i^{\,2},\ x_i^{\,3}(1), v_2, v_3)$

$$\hat{\alpha}_{k-2} = \frac{1}{n-k+2}\sum_{i=1}^{n-k+2} y_i\,(x_i^{\,k}(2)\ :\ v_{k-1}, v_k) \qquad (2.40)$$

= some value v_{k-1}, say

$$y_i\,(k-1) = \alpha_{k-1} + v_k\ x_i^{\,k}(k-1) + e_i\,(k-1)$$

i.e. $$y_i\,(x_i^{\,k}, v_k) = \alpha_{k-1} + e_i\,(k-1) \qquad (2.41)$$

where $y_i\,(x_i^{\,k}, v_k) = y_i(k-1) - v_k\ x_i^{\,k}(k-1)$

$(i = 1, 2, 3\ldots\ldots\ldots\ n-1)$.

This is of the form (4.2).

Therefore, the estimate of α_{k-1} becomes

$$\hat{\alpha}_{k-1} = \frac{1}{n-1}\sum_{i=1}^{n-1} y_i\,(x_i^{\,k}, v_k) \qquad (2.42)$$

Again substituting the estimate $\hat{\alpha}_{k-1}$ of α_{k-1} in (4.35), one obtains that

$$y_i\,(k-1) = \alpha_{k-1} + v_k\ x_i^{\,k}(k-1) + e_i\,(k-1)$$

i.e. $$y_i\,(x_i^{\,k}, v_k) = \alpha_{k-1} + e_i\,(k-1) \qquad (2.43)$$

where $y_i\,(x_i^{\,k}, v_k) = y_i\,(k-1) - v_k\ x_i^{\,k}(k-1)$

$(i = 1, 2, 3\ldots\ldots\ldots\ n-1)$.

This is of the form (4.2).

Therefore, the estimate of α_{k-1} becomes

$$\hat{\alpha}_{k-1} = \frac{1}{n-1}\sum_{i=1}^{n-1} y_i\,(x_i^{\,k}, v_k) \qquad (2.44)$$

Substituting the estimate $\hat{\alpha}_k$ of α_k and $\hat{\alpha}_{k-1}$ of α_{k-1} respectively in the earlier equation and applying the same logic one can get the estimate of α_{k-2}.

Continuing the process, one can get the estimates of the remaining parameters. At the last two stages, one will get that

$$\hat{\alpha}_1 = \frac{1}{n-1}\sum_{i=1}^{n-1} y_i\,(x_i^{\,2},\ x_i^{\,3}, \ldots\ldots, x_i^{\,k} : v_2, v_3, \ldots\ldots, v_k) \qquad (2.45)$$

= some value v_1, say.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 5 , May 2016

$$\hat{\alpha}_0 = \frac{1}{n}\sum_{i=1}^{n} y_i\,(x_i\,,\ x_i^{2}, \ldots\ldots, x_i^{k} : v_1, v_2, \ldots\ldots, v_k) \qquad (2.46)$$

= some value v_0, say.

Steps in the Method:

In this method, the estimates of the parameters are to be computed in the following order:
First, estimate v_k of the parameter a_k.
Then, estimate v_{k-1} of the parameter a_{k-1} using the estimate v_k already obtained.
……………………………………………………………………………………..
Next, estimate v_s of the parameter a_s using the estimates v_k, v_{k-1}, ….. , v_{s+1} already obtained.
Next, estimate v_{s-1} of the parameter a_{s-1} using the estimates v_k, v_{k-1}, ….. , v_s already obtained.
…………………………………..
Next, estimate v_1 of the parameter a_1 using the estimates v_k, v_{k-1}, ….. , v_2 already obtained.
Finally, estimate v_0 of the parameter a_0 using the estimates v_k, v_{k-1}, ….. , v_1 already obtained.

III. Numerical Application:

Example 3.1: Consider the following observations on X and Y:

Table-2.1

x_i :	0	0.5	0.7	1.0	1.6	2.0	2.1	2.5	3.1	4.0
y_i :	0.2	1.7	2.4	3.3	5.1	6.3	6.4	7.6	9.4	12.3

In order to fit the linear curve

$$Y = a_0 + a_1 X$$

(where a_0 & a_1 are the parameters)

to these data, the following table is to be constructed:

Table-2.2

x_i	y_i	Δx_i	Δy_i	y_i (1)	$y_i(x_i, v_1)$	x_i	y_i	Δx_i	Δy_i	$y_i(1)$	$y_i(x_i, v_1)$
0	0.2	0.5	1.5	3.0	-1.001234568	2.0	6.3	0.1	0.2	2	2.975308640
0.5	1.7	0.2	0.7	3.5	1.993827160	2.1	6.5	0.4	1.2	3	1.974074072
0.7	2.4	0.3	0.9	3	2.991358024	2.5	7.7	0.6	1.8	3	1.969135800
1.0	3.3	0.6	1.8	3	2.987654320	3.1	9.5	0.9	2.8	3.1111	1.961728392
1.6	5.1	0.4	1.2	3	2.980246912	4.0	12.3				2.950617280
						Total = 17.5	Total = 55.0			Total = 27.0111111	Total = 24.78305064

Then by the formula (2.7),

$$\text{Estimate of } a_1 = \frac{1}{9}\sum_{i=1}^{9} y_i(1) = 3.001234568 = v_1\ , \ \text{say}$$

Next, we are to compute the values of $y_i\,(x_i\,, v_1)$ where

$$y_i\,(x_i\,, v_1) = y_i - v_1\, x_i$$

$$= y_i - 3.001234568\, x_i$$

$$(i = 1, 2, 3\ldots\ldots\ 10).$$

The values, computed, have been shown in **Table-2.2**.
Then by the formula (2.9),

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International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 5 , May 2016

$$\text{Estimate of } a_0 = \frac{1}{10}\sum_{i=1}^{10} y_i\,(x_i\,, v_1) = 2.478305064$$

Thus the linear curve fitted (by the principle of least squares) to the observations becomes

$$Y = 2.478305064 + 3.001234568\ X$$

Example 3.2: Consider the following observations on X and Y:

Table-2.3

x_i :	0	1	3	4	7	8	10	12
y_i :	5	9	19	30	69	84	124	175

In order to fit the quadratic curve

$$Y = a_0 + a_1 X + a_2 X^2$$

(where a_0 . a_1 & a_2 are parameters)

to these data, the following table is to be constructed:

Table-2.4

x_i	y_i	Δx_i	Δx_i^2	Δy_i	x_i^2 (1)	y_i(1)	Δx_i^2 (1)	Δy_i(1)	y_i(2)	$y_i(x_i^2$ (1) , $v_2)$	$y_i(x_i , x_i^2, v_1 , v_2)$
0	5	1	1	4	1	4	3	1	0.3333	2.937	5
1	9	2	8	10	4	5	3	6	2	0.748	6.425
3	19	1	7	11	7	11	4	2	0.5	3.559	4.897
4	30	3	33	39	11	13	4	2	0.5	1.307	6.944
7	69	1	15	15	15	15	3	5	1.6666	- 0.945	6.329
8	84	2	36	40	18	20	4	5.5	1.375	0.866	3.872
10	124	2	44	51	22	25.5				2.114	2.588
12	175										3.784
Total = 45									Total = 6.375	Total = 10.586	Total = 39.839

Then by the formula (2.14),

$$\text{Estimate of } a_2 = \frac{1}{6}\sum_{i=1}^{6} y_i\,(2)$$

$$= 1.063 = v_2, \text{ say}$$

Next, we are to compute the values of $y_i(x_i^2\,(1), v_2)$ where

$$y_i\,(x_i^2\,(1), v_2) = y_i\,(1) - v_2\ x_i^2\,(1)$$

$$= y_i\,(1) - 1.062\ x_i^2\,(1)$$

$$(i = 1, 2, 3\ldots\ldots\ldots\ldots 7).$$

The values, computed, have been shown in **Table-2.4**.

Then by the formula (2.16),

$$\text{Estimate of } a_1 = \frac{1}{7}\sum_{i=1}^{7} y_i(x_i^2\,(1)\,, v_2) = 1.512 = v_1\,,\text{ say}$$

Next, we are to compute the values of $y_i(x_i\,,\ x_i^2, v_1, v_2)$ where

$$y_i\,(x_i\,,\ x_i^2, v_1, v_2) = y_i - v_1\,x_i - v_2\,x_i^2$$

$$(i = 1, 2, 3\ldots\ldots\ldots..\ldots 8).$$

The values, computed, have been shown in **Table-2.4**.

Then by the formula (2.18),

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 3, Issue 5 , May 2016

$$\text{Estimate of } \boldsymbol{a_0} = \frac{1}{8}\sum_{i=1}^{8} y_i\,(x_i\,,\ x_i^2\,,\ v_1\,,\ v_2) = 4.98 = v_0\text{, say}$$

Thus the quadratic curve fitted (by the principle of least squares) to the observations becomes

$$\boldsymbol{Y = 4.98 + 1.512\,X + 1.063\,X^2}$$

IV. CONCLUSION:

(1) The approach has been found suitable for finding of method of fitting of a polynomial curve of any finite order to numerical data. .

(2) It is yet to be investigated whether this approach can be applicable in finding of suitable method of fitting of a curve other than polynomial curve to numerical data.

(3) It is yet to be search for whether the estimates of parameter obtained by this method and those obtained by usual method of least squares are identical.

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International Journal of Advanced Research in Science, Engineering and Technology

Vol. 3, Issue 5 , May 2016

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International Journal of Advanced Research in Science, Engineering and Technology

Vol. 3, Issue 5 , May 2016

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(3) Indian Statistical Association (IAS)
(4) Indian Society for Probability & Statistics (ISPS)
(5) Forum for Interdisciplinary Mathematics (FIM)
(6) Electronics Scientists & Engineers Society (ESES)
(7) International Association of Engineers (IAENG)
Moreover, he is a Referee of the Journal of Assam Science Society (JASS) and a Member of the Editorial Board of the Journal of Environmental Science, Computer Science and Engineering & Technology (JECET).
Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.