



# Impact of Error Contained in Observed Data on Theoretical Model: Study of Some Important Situations

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**ABSTRACT:** This paper describes the impact of error, contained in observed data on a variable under study, on the theoretical model followed by the variable and on the estimate(s) of parameter(s) involved in the theoretical model. Appropriate model, to be satisfied by the observed data (containing error) on a variable under study, has been identified corresponding to the theoretical model obeyed by the variable in the case of linear model. Moreover, attempt has been made to search for the error associated to the estimate of parameter in such situation. The attempt of searching for the error involved in the estimate has been made in the simplest situation.

**KEYWORDS:** Observation, error, theoretical simplest model, parameter, error in estimate.

## I. INTRODUCTION

Observations or data collected from experiments or survey normally suffer from various errors. Error occurs due to cause/causes which is/are assignable (or intentional). Assignable cause/causes of error are controllable. Even if all the assignable causes of error are controlled or eliminated, observations still do not become free from error; they still suffer from some error which occurs due to some unknown and unintentional (chance) cause which is unavoidable, and uncontrollable as in [6] & [7].

If the variable(s) under study follow some rule/law that can be described by some theoretical model, the observations on the variable(s) may not satisfy the same model due to the association of errors with the observations on the variable(s). Consequently, findings obtained by analysing the observations using this theoretical model are also subject to errors. This leads to think of searching for the appropriate model satisfied by the observations containing chance error corresponding to the theoretical model obeyed by the variable(s) under study. The current study is based on this necessity. An attempt has been made here to search for the appropriate model (models), corresponding to some important and widely used theoretical model(s) [6], that are satisfied by the observations containing chance error. An attempt has also been made here to search for the error associated to the estimate(s) of parameter(s) in such situation. The attempt of searching for the error involved in the estimate in such situation has been made in the case of the simplest model as in [8], [9], [10] & [11].

## II. OBSERVED DATA AND LINEAR MODEL

Let

$$X_1, X_2, \dots, X_n$$

be  $n$  observations on a variable  $X$  under study.

The observations normally suffer from assignable errors (which are intentional, controllable and avoidable) and chance error (which is unintentional, uncontrollable and unavoidable) as in [6] & [7]. If all the causes of assignable errors are controlled, even then the observations are not free from error due to the involvement of chance error.

Thus if the observations are influenced by chance error only, they are composed of two components namely true component and the error component.

That means,

$$X_i = T(X_i) + \varepsilon(X_i) \quad , \quad (i = 1, 2, \dots, n) \tag{2.1}$$

where (i)  $T(X_i)$  is the errorless/true component of  $X_i$

& (ii)  $\varepsilon(X_i)$  is the error component of  $X_i$  which is due to chance only.

The true component  $T(X_i)$  is due to the effect(s) of one or more factor(s) but not due to chance error. The theoretical model obeyed by the variable under study is framed of on the basis of the effective factor(s) only. Therefore, the theoretical model obeyed by the variable under study is satisfied by the true component  $T(X_i)$  of each observation  $X_i$ . Conversely, the model satisfied by the true component  $T(X_i)$  of each observation  $X_i$  is the theoretical model obeyed by the variable under study.

If the true component of the observations occur due to the effects of the  $p$  factors

$$\alpha_1, \alpha_2, \dots, \alpha_p$$

and a general effect  $\mu$  with additively law,

$$T(X_i) = \mu + \alpha_{1i} + \alpha_{2i} + \dots + \alpha_{pi} \tag{2.2}$$

where (i)  $\alpha_{ri}$  is the effect of the factor  $\alpha_r$ , which is present in  $X_i$  only  
& (ii)  $\mu$  is the general effect which is present in all  $X_i$ , ( $i = 1, 2, \dots, n$ ).

Thus, in this case, the variable under study obeys the model

$$X = \mu + \alpha_{1i} + \alpha_{2i} + \dots + \alpha_{pi} \tag{2.3}$$

The observations, in this case, thus satisfy the model

$$X_i = \mu + \alpha_{1i} + \alpha_{2i} + \dots + \alpha_{pi} + \varepsilon(X_i) \tag{2.4}$$

Thus if the variable under study obeys the theoretical model given by equation (2.3), the observations on it satisfy the model given by equation (2.4).

**Special Case (Simplest Model)**

If the effects of the factors

$$\alpha_1, \alpha_2, \dots, \alpha_p$$

are absent,

$$X_i = \mu + \varepsilon(X_i) \quad , (i = 1, 2, \dots, n) \tag{2.5}$$

This is the simplest model satisfied by the observations as in [7] & [8]. In this article, this model is recognized as **Model-I**.

**III.ERROR IN ESTIMATE OF THE PARAMETER IN THE SIMPLEST MODEL**

important, and removing these is important in the context of removing indirect advertisements, and for aesthetic  
If

$$X_1, X_2, \dots, X_n$$

are  $n$  observations containing a parameter  $\mu$  and chance error  $\varepsilon_i$ , then the observations satisfy the model described by

$$X_i = \mu + \varepsilon_i \quad , (i = 1, 2, \dots, n) \tag{3.1}$$

Here,

$$\mu = T(X_i) \quad , (i = 1, 2, \dots, n) \tag{3.2}$$

The existing methods of estimation namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square as in [1], [2], [4], [5], [12], [13], [14], [15], & [16] provides

$$\bar{X} = n^{-1} \sum_{i=1}^n X_i \tag{3.3}$$

as estimator of the parameter  $\mu$  as in [6], [7] & [10].

On the other hand, equation (3.1) implies that

$$\sum_{i=1}^n X_i = n\mu + \sum_{i=1}^n \varepsilon_i$$

which further implies

$$\bar{X} = \mu + \bar{\varepsilon} \tag{3.4}$$

where

$$\bar{\varepsilon} = n^{-1} \sum_{i=1}^n \varepsilon_i \tag{3.5}$$

Thus, the estimator  $X$  of the parameter  $\mu$  suffers from an error  $\varepsilon$  given by (3.5).

This error however may not be zero .

Now  $T(X_i)$ , the errorless part of  $X_i$  (in other words the true part of  $X_i$ ) is nothing but the value of the parameter  $\mu$ .  
Moreover,

$$\mu = T(X_i) = X_i - \varepsilon_i , \quad (i = 1, 2, \dots, n)$$

Thus the neither the observed values nor the mean of them, whose values are known, can be the true value of the parameter  $\mu$ .

**IV. SOME EXAMPLE OF ERROR (IN CASE OF MODEL-1)**

**Example-4.1 (Annual Maximum of Ambient Air Temperature at Guwahati):**

Values of annual maximum of Ambient Air Temperature at Guwahati observed during the period from 1969 to 2013 have been presented in **Table-4.1**.

**Table-4.1**

Observed Value of Annual Maximum of Ambient Air Temperature (in Degree Celsius) at Guwahati occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Calendar Year, Month & Date of occurrence	TPY No	Observed Value	Calendar Year, Month & Date of occurrence
1	37.1	1969, May, 20	23	37.4	1991, July, 20
2	36.6	1970, April,01	24	39.4	1992, April, 16
3	36.0	1971, March, 27	25	36.4	1993, September, 08
4	35.7	1972, July, 14	26	38.1	1994, May, 07
5	39.0	1973, April,10	27	36.3	1995, May, 14
6	36.1	1974, August, 14	28	39.9	1996, April, 19
7	39.2	1975, April, 10	29	37.4	1998, May 21
8	39.0	1976, April,17	30	37.5	2000, May, 13
9	35.3	1977, August, 14	31	36.7	2001, April,07
10	36.8	1978, May, 19	32	35.5	2002, May, 17
11	38.6	1979, March, 27	33	37.4	2003, July, 26
12	35.1	1980, July, 01	34	38.0	2004, March, 28
13	35.8	1981, June, 21	35	36.6	2005, July, 30
14	36.5	1982, May, 26	36	38.0	2006, August, 11
15	36.7	1983, April, 06	37	37.3	2007, May, 06
16	37.2	1984, April, 06	38	37.3	2008, August, 08
17	36.5	1985, April,26	39	38.0	2009, May, 23
18	38.4	1986, April,03	40	37.2	2010, July, 03
19	37.2	1987, May, 19	41	37.3	2011, August, 30
20	36.4	1988, August, 03	42	37.4	2012, April,03
21	36.7	1989, July, 23	43	38.8	2013, June, 12
22	36.0	1990, September, 02			

Observation on annual maximum temperature at a location satisfies the model described by equation (3.1) as in [7], [9] & [11].

It has been found that the true value of the central tendency of annual maximum temperature at Guwahati is 37.25 Degree Celsius as in [9] & [11].

The estimated value of the central tendency computed from these observations by the existing statistical method of estimation i.e. by applying the formula given by equation (3.3) has been found to be 37.20465116 Degree Celsius. Thus, the estimate of the central tendency suffers from an amount 0.04534884 Degree Celsius of error.

This can be obtained by equation (3.3) from the amounts of errors involved in the observations as shown in the following table **Table-4.2**:

**Table-4.2**

Error associated to Observed Value of Annual Maximum of Ambient Air Temperature (in Degree Celsius) at Guwahati occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Amount of Error Involved = Observed Value – 37.25	TPY No	Observed Value	Amount of Error Involved = Observed Value – 37.25
1	37.1	- 0.15	23	37.4	0.15
2	36.6	- 0.65	24	39.4	2.15
3	36.0	- 1.25	25	36.4	- 0.85
4	35.7	- 1.55	26	38.1	0.85
5	39.0	1.75	27	36.3	- 0.95
6	36.1	- 1.15	28	39.9	2.65
7	39.2	1.95.0	29	37.4	0.15
8	39.0	1.75	30	37.5	0.25
9	35.3	- 1.95	31	36.7	- 0.55
10	36.8	- 0.45	32	35.5	- 1.75
11	38.6	1.35	33	37.4	0.15
12	35.1	- 2.15	34	38.0	0.75
13	35.8	- 1.45	35	36.6	- 0.65
14	36.5	- 0.45	36	38.0	0.75
15	36.7	- 0.55	37	37.3	0.05
16	37.2	- 0.05	38	37.3	0.05
17	36.5	- 0.75	39	38.0	0.75
18	38.4	1.15	40	37.2	- 0.05
19	37.2	- 0.05	41	37.3	0.05
20	36.4	- 0.85	42	37.4	0.15
21	36.7	- 0.55	43	38.8	1.55
22	36.0	- 1.25			

The amount of the said error computed from this table by the formula (3.5) is found to be 0.04534884 Degree Celsius.

**Example-4.2 (Annual Minimum of Ambient Air Temperature at Guwahati):**

Values of annual minimum Temperature at Guwahati observed during the period from 1969 to 2013 have been presented in **Table-4.3**.

**Table-4.3**

Observed Value of Annual Minimum of Ambient Air Temperature (in Degree Celsius) at Guwahati occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Calendar Year, Month & Date of occurrence	TPY No	Observed Value	Calendar Year, Month & Date of occurrence
1	6.6	1969, January,16	23	7.4	1991, January, 20
2	6.6	1969, December, 27	24	5.9	1992, January, 05
3	5.9	1971, January, 31 & February, 01	25	8.4	1993, February, 23
4	8.2	1972, January, 21	26	7.8	1993, December, 27
5	5.0	1973, February, 03	27	7.5	1995, January, 22
6	6.3	1974, February, 07	28	9.4	1996, January,19
7	7.4	1975, January, 19	29	NA	NA
8	6.6	1976, January, 22	30	NA	NA
9	6.2	1977, January, 30	31	NA	NA
10	7.3	1978, January, 12	32	8.9	2001, January, 08
11	6.2	1979, January,09	33	8.6	2002, January, 26
12	6.4	1980, February, 08	34	8.0	2003, January, 16
13	7.5	1981, January,10	35	7.9	2004, February, 04
14	8.3	1982, February, 07	36	6.7	2004, December, 27
15	4.9	1983, January, 06	37	9.6	2006, January, 12
16	6.1	1984, January, 30	38	6.4	2007, January, 18
17	7.8	1985, January, 19	39	7.8	2008, February, 03
18	8.6	1986, January, 20	40	9.9	2009, January, 07
19	7.7	1987, January, 05	41	8.6	2010, January, 03
20	9.2	1988, January, 01	42	7.0	2011, January, 21
21	6.7	1989, January, 14	43	6.4	2012, January, 15
22	8.6	1989, December, 31	44	5.6	2013, January, 11

Observation on annual minimum temperature at a location satisfies the model described by equation (3.1) as in [7], [9] & [11].

It has been found that the true value of the central tendency of annual maximum temperature at Guwahati is 7.44 Degree Celsius as in [9] & [11].

The estimated value of the central tendency computed from these observations by the existing statistical method of estimation i.e. by applying the formula given by equation (3.3) has been found to be 7.385714286 Degree Celsius.

Thus the estimate of the central tendency suffers from an amount  
– 0.054285714 Degree Celsius.  
of error.

This can be obtained by equation (3.3) from the amounts of errors involved in the observations as shown in the following table **Table-4.4**:

**Table-4.4**

Error associated to Observed Value of Annual Minimum of Ambient Air Temperature (in Degree Celsius) at Guwahati occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Amount of Error Involved = Observed Value – 37.25	TPY No	Observed Value	Amount of Error Involved = Observed Value – 37.25
1	6.6	- 0.84	23	7.4	- 0.04
2	6.6	- 0.84	24	5.9	- 1.54
3	5.9	- 1.54	25	8.4	0.96
4	8.2	0.76	26	7.8	0.36
5	5.0	- 2.44	27	7.5	0.06
6	6.3	- 1.14	28	9.4	1.96
7	7.4	- 0.04	29	8.3	0.86
8	6.6	- 0.84	30	8.9	1.46
9	6.2	- 1.24	31	8.6	1.16
10	7.3	- 0.14	32	8.0	0.56
11	6.2	- 1.24	33	7.9	0.46
12	6.4	- 1.04	34	6.7	- 0.74
13	7.5	0.06	35	9.6	2.16
14	8.3	0.86	36	6.4	- 1.04
15	4.9	- 2.54	37	7.8	0.36
16	6.1	- 1.34	38	9.9	2.46
17	7.8	0.36	39	8.6	1.16
18	8.6	1.16	40	7.0	- 0.44
19	7.7	0.26	41	6.4	- 1.04
20	9.2	1.76	42	5.6	- 1.84
21	6.7	- 0.74	43		
22	8.6	1.16	44		

The amount of the said error computed from this table by the formula (3.5) is found to be - 0.054285714 Degree Celsius.

**Example-4.3 (Annual Maximum of Ambient Air Temperature at Tezpur):**

Values of annual maximum Temperature at Tezpur observed during the period from 1969 to 2013 have been presented in **Table-4.5**.

**Table-4.5**

Observed Value of Annual Maximum of Ambient Air Temperature (in Degree Celsius) at Tezpur occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Calendar Year, Month & Date of occurrence	TPY No	Observed Value	Calendar Year, Month & Date of occurrence
1	36.5	1969, June 3	23	36.1	1991, August 22
2	35.8	1970, July 9	24	36.4	1992, June 10
3	35.9	1971, May 22	25	35.9	1993, June 16



4	45.7	1972, April 18	26	37.1	1994, July 16
5	37.2	1973, July 10	27	36.0	1995, June 12
6	36.6	1974, May 29	28	36.9	1996, July 26
7	39.5	1975, July 17 & August 02	29	36.4	1998, May 21
8	35.1	1976, April 22	30	37.5	2000, May 13
9	38.2	1977, March 5 & May 26	31	36.4	2001, June 12
10	36.4	1978, May 19	32	35.7	2002, May 14
11	38.1	1979, April 28	33	37.2	2003, July 24
12	35.1	1980, September 3	34	35.7	2004, August 12
13	37.1	1981, June 20	35	36.1	2005, September 18
14	36.6	1982, August 14	36	38.2	2006, May 25
15	38.4	1983, October 6	37	37.7	2007, August 6
16	36.5	1984, June 23	38	36.9	2008, July 8
17	35.4	1985, June 24	39	37.8	2009, September 7
18	37.2	1986, August 11	40	36.8	2010, August 5
19	36.8	1987, August 25	41	37.1	2011, August 30
20	36.6	1988, June 6	42	37.0	2012, September 8
21	38.5	1989, October 16	43	37.8	2013, June 11
22	35.7	1990, August 21			

Observation on annual maximum temperature at a location satisfies the model described by equation (3.1) as in [7], [9] & [11].

It has been found that the true value of the central tendency of annual maximum temperature at Guwahati is 36.8 Degree Celsius as in [3].

The estimated value of the central tendency computed from these observations by the existing statistical method of estimation i.e. by applying the formula given by equation (3.3) has been found to be 37.01395349 Degree Celsius. Thus, the estimate of the central tendency suffers from an amount

$$- 0.2139534884 \text{ Degree Celsius}$$

of error.

This can be obtained by equation (3.3) from the amounts of errors involved in the observations as shown in the following table **Table-4.6**:

**Table-4.6**

Error associated to Observed Value of Annual Maximum of Ambient Air Temperature (in Degree Celsius) at Tezpur occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Amount of Error Involved = Observed Value - 37.25	TPY No	Observed Value	Amount of Error Involved = Observed Value - 37.25
1	36.5	- 0.3	23	36.1	- 0.7
2	35.8	- 1.0	24	36.4	- 0.4
3	35.9	- 0.9	25	35.9	- 0.9
4	45.7	- 8.9	26	37.1	0.3
5	37.2	0.4	27	36.0	- 0.8
6	36.6	- 0.2	28	36.9	0.1
7	39.5	2.7	29	36.4	- 0.4
8	35.1	- 1.7	30	37.5	0.7

9	38.2	1.4	31	36.4	- 0.4
10	36.4	0.0	32	35.7	- 1.1
11	38.1	1.8	33	37.2	0.4
12	35.1	- 1.7	34	35.7	- 1.1
13	37.1	- 1.0	35	36.1	- 0.7
14	36.6	- 0.3	36	38.2	1.4
15	38.4	- 0.1	37	37.7	0.9
16	36.5	0.4	38	36.9	0.1
17	35.4	- 0.3	39	37.8	1.0
18	37.2	1.6	40	36.8	0.0
19	36.8	0.4	41	37.1	0.3
20	36.6	- 0.4	42	37.0	0.2
21	38.5	- 0.1	43	37.8	1.0
22	35.7	- 0.8			

The amount of the said error computed from this table by the formula (3.5) is found to be - 0.2139534884 Degree Celsius.

**Example-4.4 (Annual Minimum of Ambient Air Temperature at Tezpur):**

Values of annual minimum Temperature at Tezpur observed during the period from 1969 to 2013 have been presented in **Table-4.7**.

**Table-4.7**

Observed Value of Annual Minimum of Ambient Air Temperature (in Degree Celsius) at Tezpur occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Calendar Year, Month & Date of occurrence	TPY No	Observed Value	Calendar Year, Month & Date of occurrence
1	8.8	1970, January 6	23	7.6	1992, January 5 & February 9
2	8.5	1970, January 31	24	8.1	1992, December 24
3	8.5	1972, February 10	25	9.5	1994, January 19
4	10.0	1973, January 20	26	8.5	1995, January 24
5	9.3	1974, January 20	27	9.8	1996, January 19
6	9.3	1974, December 31	28	NA	NA
7	10.5	1975, December 31	29	NA	NA
8	8.6	1977, February 9	30	NA	NA
9	8.2	1978, January 13	31	7.6	2000, January 8
10	8.5	1979, January 9	32	9.0	2001, January 7
11	9.2	1980, January 10	33	9.4	2002, January 8
12	9.6	1981, January 11	34	9.3	2003, January 16
13	10.2	1982, February 11	35	9.9	2004, January 6 & 30
14	8.1	1983, February 7	36	8.5	2004, December 29
15	8.6	1984, January 31	37	9.2	2006, January 8 & 16
16	8.6	1984, December 31	38	8.0	2007, January 18
17	9.3	1986, January 1	39	7.0	2008, February 3
18	8.5	1987, January 7	40	8.5	2009, January 7 & 8





19	9.1	1987, December 31	41	9.6	2010, February 7
20	7.5	1989, January 14	42	8.0	2011, January 21
21	8.1	1989, December 31	43	6.8	2011, December 25
22	7.8	1991, January 23	44	7.4	2013, January 12

Observation on annual minimum temperature at a location satisfies the model described by equation (3.1) as in [7], [9] & [11].

It has been found that the true value of the central tendency of annual maximum temperature at Guwahati is 8.6 Degree Celsius as in [3].

The estimated value of the central tendency computed from these observations by the existing statistical method of estimation i.e. by applying the formula given by equation (3.3) has been found to be 8.670731707 Degree Celsius.

Thus the estimate of the central tendency suffers from an amount

0.070731707 Degree Celsius

of error.

This can be obtained by equation (3.3) from the amounts of errors involved in the observations as shown in the following table **Table-4.8**:

**Table-4.8**

Error associated to Observed Value of Annual Minimum of Ambient Air Temperature (in Degree Celsius) at Guwahati occurred during Temperature Periodic Year (TPY)

TPY No	Observed Value	Amount of Error Involved = Observed Value - 37.25	TPY No	Observed Value	Amount of Error Involved = Observed Value - 37.25
1	8.8	0.2	23	7.6	- 1.0
2	8.5	- 0.1	24	8.1	- 0.5
3	8.5	- 0.1	25	9.5	0.9
4	10.0	1.4	26	8.5	- 0.1
5	9.3	0.7	27	9.8	1.2
6	9.3	0.7	28	NA	NA
7	10.5	1.9	29	NA	NA
8	8.6	0.0	30	NA	NA
9	8.2	0.4	31	7.6	- 1.0
10	8.5	0.1	32	9.0	0.4
11	9.2	0.6	33	9.4	0.8
12	9.6	1.0	34	9.3	0.7
13	10.2	1.6	35	9.9	1.3
14	8.1	- 0.5	36	8.5	- 0.1
15	8.6	0.0	37	9.2	0.6
16	8.6	0.0	38	8.0	- 0.6
17	9.3	0.7	39	7.0	- 1.6
18	8.5	- 0.1	40	8.5	- 0.1
19	9.1	0.5	41	9.6	1.0
20	7.5	- 1.1	42	8.0	- 0.6
21	8.1	- 0.5	43	6.8	- 1.8
22	7.8	- 0.8	44	7.4	- 2.2

The amount of the said error computed from this table by the formula (3.5) is found to be 0.07073170732 Degree Celsius.

**V. CONCLUSION**

In this study, error has been determined in the case of the simplest model only. The same is to be determined in the case of linear model whose corresponding model has here been identified for the observed data.

There is also necessity for studying the associations of errors in the situations where two and /or more variables are connected by theoretical relationships.

It is yet to search for the appropriate model, to be satisfied by observations containing error, corresponding to many theoretical models. Some important theoretical models have been outlined below:

- (1) Two variables  $X$  and  $Y$  are connected by the theoretical relationship

$$f(X) = \mu \cdot f(Y) \quad (5.1)$$

where (i)  $f(\cdot)$  is some function  
and (ii)  $\mu$  is the parameter.

- (2) The two variables  $X$  and  $Y$  are connected by the theoretical relationship

$$f(X) = \mu \cdot g(Y) \quad (5.2)$$

where (i)  $f(\cdot)$  &  $g(\cdot)$  are some functions  
and (ii)  $\mu$  is the parameter.

- (3) The variable  $X$  is proportional to the variable  $Y$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$X = \mu \cdot Y \quad (5.3)$$

where  $\mu$  is the parameter (the constant of proportionality).

- (4) The variable  $X$  is inversely proportional to the variable  $Y$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$X = \mu \cdot Y^{-1} \quad \text{or} \quad X \cdot Y = \mu \quad (5.4)$$

where  $\mu$  is the parameter (the constant of proportionality).

- (5) The variable  $X$  is proportional to the square of the variable  $Y$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$X = \mu \cdot Y^2 \quad (5.5)$$

where  $\mu$  is the parameter (the constant of proportionality).

- (6) The variable  $X$  is inversely proportional to the square of the variable  $Y$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$X = \mu \cdot Y^{-2} \quad \text{or} \quad X \cdot Y^2 = \mu \quad (5.6)$$

where  $\mu$  is the parameter (the constant of proportionality).

- (7) The variable  $Y$  theoretically depends linearly upon the variable  $X$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$Y = \alpha + \beta X \quad (5.7)$$

where  $\alpha$  &  $\beta$  are parameters.

- (8) The variable  $Y$  theoretically depends linearly upon the variable  $X$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$Y = \alpha + \beta X + \gamma X^2 \quad (5.8)$$

where  $\alpha$ ,  $\beta$  &  $\gamma$  are parameters.

- (9) The variable  $Y$  theoretically depends exponentially upon the variable  $X$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$Y = \lambda \exp(-\nu X) \quad (5.9)$$

where  $\lambda$  and  $\nu$  are parameters.

- (10) The variable  $Y$  theoretically depends negatively exponentially upon the variable  $X$  i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$Y = \lambda \exp(-\nu X^{-1}) \quad (5.10)$$

where  $\lambda$  and  $\nu$  are parameters.

- (11) The variable  $Y$  theoretically depends upon the variable  $X$  by modified exponentially law i.e.  $X$  and  $Y$  are connected by the theoretical relationship

$$Y = \mu + \lambda \exp(-\nu X) \quad (5.11)$$

where  $\mu$ ,  $\lambda$  and  $\nu$  are parameters.



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(12) In **Model–11**, it may be so that the variable  $Y$  depends theoretically upon the variable  $X$  by the theoretical relationship

$$Y = \mu + \lambda \exp(-vX^{-1}) \quad (5.12)$$

where  $\mu$ ,  $\lambda$  and  $v$  are parameters.

Thus the current problem for researcher at this stage is to search for the appropriate models, corresponding to these theoretical models, which are satisfied by the observed data containing error and also to search for the error associated to the estimates of the corresponding parameters involved in the models.

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