ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 2 , February 2016

Theoretical Model and Model Satisfied by Observed Data: One Pair of Related Variables

Dhritikesh Chakrabarty

Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India.

ABSTRACT: This paper describes the appropriate model, corresponding to the theoretical model obeyed by the variable(s) under study, which are to be satisfied by the observed data (containing error) on the variable(s). The current study has been made confined in the situation where one variable depends upon a single other variable only. An attempt has also been made to express the parameter involved in the model, in some situations, in terms of the observed data. Moreover, attempt has been made to search for the error involved in the estimate of the parameter, obtained by the usual statistical method, in the situation where observed data contain the parameter itself and error.

I. INTRODUCTION

Observed data on any variable(s) under study normally suffer from errors. Error can occur due to many causes. They can be broadly classified as assignable (or intentional) and unintentional (chance). Assignable cause/causes of error are controllable. Even if all the assignable causes of error are controlled or eliminated, observations still do not become free from error; they still suffer from some error which occurs due to some unknown and unintentional (chance) cause which is unavoidable, uncontrollable (*Chakrabarty*, 2014).

When variable(s) under study follow some rule/law that can be described by some theoretical model, the observed data on the variable(s) may not satisfy the same model due to the association of errors with them. Consequently, findings obtained by analyzing the observations using this theoretical model are also subject to errors. This leads to think of searching for the appropriate model(s) to be satisfied by the observed data containing error corresponding to the theoretical model obeyed by the variable(s) under study. An attempt has here been made to search for the appropriate model (s), corresponding to some important and widely used theoretical model(s) (*Chakrabarty*, 2014), that are satisfied by the observations containing chance error. An attempt has also been made here to search for the error associated to the estimate(s) of parameter(s) in such situation.

In some cases/situations, one variable depends upon only one other variable. On the other hand, in some other cases/situations, one variable depends upon a number of other variables. The current study has been made confined in the situation where one variable depends upon a single variable only. An attempt has also been made to express the parameter involved in the model, in some situations, in terms of the observed data. Moreover, attempt has been made to search for the error involved in the estimate of the parameter, obtained by the usual statistical method, in the situation where observed data contain the parameter itself and error {*Chakrabarty* (2014 , 2015)}.

II. SOME COMMONLY USED MODELS

Here X and Y are treated as the variables under study where the variable X theoretically depends upon the variable X. The following models have been found to be significant so far as their follower phenomenon is taken into account: However, these are not the exhaustive models. There exist more models.

Model–1 (Proportionality Model): Y is directly proportional to X i.e.

$$Y = \mu X \tag{2.1}$$

where μ is the parameter (the constant of proportionality).

Model–2 (Inverse Proportionality Model): Y is inversely proportional to X i.e.

$$Y = \mu . X^{-1} \quad \text{or} \quad X.Y = \mu \tag{2.2}$$

where μ is the parameter (the constant of inverse proportionality).
Model–3: Y is directly proportional to the square of X i.e.

$$Y = \mu . X^{2} \tag{2.3}$$

where μ is the parameter (the constant of proportionality).
Model–4: Y is inversely proportional to the square of X i.e.

$$Y = \mu . X^{-2} \quad \text{or} \quad X . Y^{2} = \mu \tag{2.4}$$

where μ is the parameter (the constant of proportionality).
Model–5: Y and X are connected by the relationship

$$f(Y) = \mu \,.\, f(X) \tag{2.5}$$

where (i) $f(.)$ is some function
and (ii) μ is the parameter.
Model–6: Y and X are connected by the relationship

$$f(Y) = \mu \,.\, f(X^{-1}) \tag{2.6}$$

where (i) $f(.)$ is some function
and (ii) μ is the parameter.
Model–7: Y and X are connected by the relationship

$$f(Y) = \mu \,.\, g(X) \tag{2.7}$$

where (i) $f(.)$ & $g(.)$ are some functions
and (ii) μ is the parameter.
Model–8: Y and X are connected by the relationship

$$f(Y) = \mu \,.\, g(X^{-1}) \tag{2.8}$$

where (i) $f(.)$ & $g(.)$ are some functions
and (ii) μ is the parameter.
Model–9 (Linear Model): Y is a linear function of X i.e.

$$Y = \alpha + \beta X \tag{2.9}$$

where α & β are parameters.
Model–10 (Quadratic Model): Y is a quadratic function of X i.e.

$$Y = \alpha + \beta X + \gamma X^{2} \tag{2.10}$$

where α, β & γ are parameters.
Model–11 (Exponential Model): Y is an exponential function of X i.e.

$$Y = \lambda \, exp\,(-\nu X) \tag{2.11}$$

where λ and ν are parameters.
Model–12: Y is an exponential function of the inverse of X i.e.

$$Y = \lambda \, exp\,(-\nu X^{-1}) \tag{2.12}$$

where λ and ν are parameters.
Model–13: Y is a modified exponential function of X i.e.

$$Y = \mu + \lambda \, exp\,(-\nu X) \tag{2.13}$$

where μ , λ and ν are parameters.
Model–14: Y is a modified exponential function of the inverse of X i.e.

$$Y = \mu + \lambda \, exp\,(-\nu X^{-1}) \tag{2.14}$$

where μ , λ and ν are parameters.

III. MODEL SATISFIED BY OBSERVED DATA

Let

$$Y_1 , Y_2 , \ldots\ldots\ldots\ldots , Y_n$$

be the observations on Y corresponding to the observations

$$X_1 , X_2 , \ldots\ldots\ldots\ldots , X_n$$

on X respectively.
In this case, each observation is influenced by error.
Thus the observations satisfy the model

$$X_i = T(X_i) + \varepsilon(X_i) \tag{3.1}$$

$$\& \; Y_i = T(Y_i) + \varepsilon(Y_i) \tag{3.2}$$

where (i) $T(X_i)$ is the errorless/true part of X_i,
(ii) $\varepsilon(X_i)$ is the error associated to X_i,

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 2 , February 2016

(iii) $T(Y_i)$ is the errorless/true part of Y_i
& (iv) $\varepsilon(Y_i)$ is the error associated to Y_i .

Model–1 (Proportionality Model)

In this case,

$$T(Y_i) = \mu . T(X_i) \tag{3.3}$$

which means,

$$\{Y_i - \varepsilon(Y_i)\} = \mu.\{X_i - \varepsilon(X_i)\}$$

$$\text{i.e.} \quad Y_i = \mu.X_i + \mathcal{E}_i \tag{3.4}$$

$$\text{where } \mathcal{E}_i = \mathcal{E}_i(Y_i\ ,\ X_i\ ,\ \mu) = \varepsilon(Y_i) - \mu.\ \varepsilon(X_i), \tag{3.5}$$

$$(i = 1\ ,2\ , \ldots\ldots\ldots.\ , n).$$

Equation (3.4) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.1).

Model–2 (Inverse Proportionality Model)

In this case,

$$T(Y_i) = \mu.\{T(X_i)\}^{-1} \quad \text{or} \quad T(X_i)\ .T(Y_i) = \mu \tag{3.6}$$

which means,

$$\{Y_i - \varepsilon(Y_i)\} = \mu.\{X_i - \varepsilon(X_i)\}^{-1} \quad \text{or} \quad \{X_i - \varepsilon(X_i)\}.\{Y_i - \varepsilon(Y_i)\} = \mu$$

$$\text{i.e.} \quad X_i\, Y_i - X_i\, \varepsilon(Y_i) - Y_i\, \varepsilon(X_i) + \varepsilon(X_i).\varepsilon(Y_i) = \mu$$

$$\text{i.e.} \quad Y_i = \mu\, X_i^{-1} + \mathcal{E}_i \tag{3.7}$$

$$\text{where } \mathcal{E}_i = \varepsilon(Y_i) + Y_i\, X_i^{-1}\, \varepsilon(X_i) - \varepsilon(X_i).\varepsilon(Y_i)\, X_i^{-1}\ , \tag{3.8}$$

$$(i = 1\ ,2\ , \ldots\ldots\ldots.\ , n).$$

Equation (3.7) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.2).

Model–3

In this case,

$$T(Y_i) = \mu.\{T(X_i)\}^2 \tag{3.9}$$

which means,

$$\{X_i - \varepsilon(X_i)\} = \mu.\{Y_i - \varepsilon(Y_i)\}^2 \tag{3.10}$$

$$(i = 1\ ,2\ , \ldots\ldots\ldots.\ , n).$$

Equation (3.10) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.3).

Model–4

In this case,

$$T(Y_i) = \mu.\{T(X_i)\}^{-2} \quad \text{or} \quad T(Y_i)\ .\{T(X_i)\}^2 = \mu \tag{3.11}$$

which means,

$$\{Y_i - \varepsilon(Y_i)\} = \mu.\{X_i - \varepsilon(X_i)\}^{-2} \quad \text{or} \quad \{X_i - \varepsilon(X_i)\}.\{Y_i - \varepsilon(Y_i)\}^2 = \mu \tag{3.12}$$

$$(i = 1\ ,2\ , \ldots\ldots\ldots.\ , n).$$

Equation (3.12s) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.4).

Model–5

In this case

$$f\{T(Y_i)\} = \mu\ .\ f\{T(X_i)\} \tag{3.13}$$

which means,

$$f\{Y_i - \varepsilon(Y_i)\} = \mu\ .\ f\{X_i - \varepsilon(X_i)\} \tag{3.14}$$

$$(i = 1\ ,2\ , \ldots\ldots\ldots.\ , n).$$

Equation (3.14) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.5).

Model–6

In this case

$$f\{T(Y_i)\} = \mu\ .\ f\,[\{T(X_i)\}^{-1}] \tag{3.15}$$

which means,

$$f\{Y_i - \varepsilon(Y_i)\} = \mu\ .\ f\,[\{X_i - \varepsilon(X_i)\}^{-1}] \tag{3.16}$$

$$(i = 1\ ,2\ , \ldots\ldots\ldots.\ , n).$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 2 , February 2016

Equation (3.16) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.6).

Model–7

In this case,

$$f\{T(Y_i)\} = \mu \, . \, g\{T(X_i)\} \tag{3.17}$$

which means,

$$f\{Y_i - \varepsilon(Y_i)\} = \mu . \, g\{X_i - \varepsilon(X_i)\} \tag{3.18}$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Equation (3.18) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.7).

Model–8

In this case

$$f\{T(Y_i)\} = \mu \, . \, g\,[\{T(X_i)\}^{-1}] \tag{3.19}$$

which means,

$$f\{Y_i - \varepsilon(Y_i)\} = \mu \, . \, g\,[\{X_i - \varepsilon(X_i)\}^{-1}] \tag{3.20}$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Equation (3.20) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.8).

Model–9 (Linear Model)

In this case, $T(X_i)$ and $T(Y_i)$ satisfy the theoretical relationship and thus

$$T(Y_i) = \alpha + \beta \, . \, T(X_i) \tag{3.21}$$

which implies $$Y_i = \alpha + \beta X_i + \{\varepsilon(Y_i) - \beta \, \varepsilon(X_i)\}$$

Thus the observed values satisfy the model

$$Y_i = \alpha + \beta X_i + \mathcal{E}_i \tag{3.22}$$

$$\text{where } \mathcal{E}_i = \varepsilon(Y_i) - \beta \, \varepsilon(X_i) , \tag{3.23}$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Here $\mathcal{E}_i$, being a linear function of chance error variables, is also a chance error variable.

Equation (3.22) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.9).

Model–10 (Quadratic Model)

In this case, $T(X_i)$ and $T(Y_i)$ satisfy the theoretical relationship and thus

$$T(Y_i) = \alpha + \beta \, T(X_i) + \gamma . \{T(X_i)\}^2 \tag{3.24}$$

which implies

$$Y_i = \alpha + \beta X_i + \gamma X_i^2 + \{\varepsilon(Y_i) - \beta \, \varepsilon(X_i) - 2 \, \gamma X_i . \varepsilon(X_i) - \gamma . \varepsilon(X_i)^2\}$$

which further implies

$$Y_i = \alpha + \beta X_i + \gamma X_i^2 + \mathcal{E}_i \tag{3.25}$$

$$\text{where} \quad \mathcal{E}_i = \varepsilon(Y_i) - \beta \, \varepsilon(X_i) - 2 \, \gamma X_i . \varepsilon(X_i) - \gamma . \varepsilon(X_i)^2 , \tag{3.26}$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Here $\mathcal{E}_i$ being a linear function of chance error variables is also a chance error variable.

Equation (3.26) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.10).

Model–11 (Exponential Model)

In this case, $T(X_i)$ and $T(Y_i)$ satisfy the theoretical relationship and thus

$$T(Y_i) = \lambda \, exp\{- \nu . T(X_i)\} \tag{3.27}$$

which implies

$$Y_i = \varepsilon(Y_i) + \lambda \, exp\{- \nu . \varepsilon(X_i)\} \, . \, exp\{- \nu X_i\}$$

Therefore

$$Y_i = \varepsilon(Y_i) + \mathcal{E}_i \, . \, exp\{- \nu X_i\} \tag{3.28}$$

$$\text{where} \quad \mathcal{E}_i = \lambda \, exp\{- \nu . \varepsilon(X_i)\} , \tag{3.29}$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Here $\mathcal{E}_i$ being a exponential function of chance error variables is also a chance error variable.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 2 , February 2016

Equation (3.28) is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.11).

Model–12

In this case, $T(X_i)$ and $T(Y_i)$ satisfy the theoretical relationship and thus

$$T(Y_i) = \lambda\ exp\ [-\ v.\{T(X_i)\}^{-1}] \tag{3.30}$$

which implies

$$Y_i = \varepsilon(Y_i) + \lambda\ exp\ [-\ v.\{X_i - \varepsilon(X_i)\}^{-1}] \tag{2.31}$$
$$(i = 1\ ,2\ , \ldots\ldots\ldots\ , n).$$

This is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.12).

Model–13

In this case, $T(X_i)$ and $T(Y_i)$ satisfy the theoretical relationship and thus

$$T(Y_i) = \mu + \lambda\ exp\{-\ v\ T(X_i)\} \tag{3.32}$$

which implies

$$Y_i = \mu + \varepsilon(Y_i) + \lambda\ exp[-\ v.\{X_i - \varepsilon(X_i)\}]$$

which further implies

$$Y_i = \mu_i + \mathcal{E}_i\ .\ exp\{-\ vX_i\} \tag{3.33}$$

where $\mu_i = \mu + \varepsilon(Y_i)$ & $\mathcal{E}_i = \lambda\ .\ exp\{v.\ \varepsilon(X_i)\}$, (3.34)

$$(i = 1\ ,2\ , \ldots\ldots\ldots\ , n).$$

Here $\mathcal{E}_i$ being a exponential function of chance error variables is also a chance error variable.

This is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.13).

Model–14

In this case,

$$T(Y_i) = \mu + \lambda\ exp\ [-\ v.\{T(X_i)\}^{-1}] \tag{3.35}$$

which implies

$$Y_i - \varepsilon(Y_i) = \mu + \lambda\ exp\ [-\ v.\{\ X_i - \varepsilon(X_i)\}^{-1}]$$

which further implies,

$$Y_i = \varepsilon(Y_i) + \mu + \lambda\ exp\ [-\ v.\{\ X_i - \varepsilon(X_i)\}^{-1}]\ , \tag{3.36}$$
$$(i = 1\ ,2\ , \ldots\ldots\ldots\ , n).$$

This is the model satisfied by the observations containing errors if the associated variables obey the theoretical model described by equation (2.14).

IV. PARAMETER DETERMINATION IN PROPORTIONALITY MODEL

Model–1 (Proportionality Model)

Equations (3.1), (3.2) and (3.3) together imply that

$$\mu = T(Y_i)\ /\ T(X_i) = \{Y_i - \varepsilon(Y_i)\}\ /\ \{X_i - \varepsilon(X_i)\}\ , \tag{4.1}$$
$$(i = 1\ ,2\ , \ldots\ldots\ldots\ , n).$$

Thus, the ratios

$$T(Y_i)\ /\ T(X_i)\ \ ,\ \ (i = 1\ ,2\ , \ldots\ldots\ldots\ , n) \tag{4.2}$$

or is equivalently the ratios

$$\{Y_i - \varepsilon(Y_i)\ \}\ /\ \{X_i - \varepsilon(X_i)\}\ \ ,\ \ (i = 1\ ,2\ , \ldots\ldots\ldots\ , n) \tag{4.3}$$

are equal and this common value is nothing but the true value of the parameter μ.

However, the values of these ratios are unknown.

On the other hand, neither the observed ratios

$$Y_i\ /\ X_i\ \ ,\ \ (i = 1\ ,2\ , \ldots\ldots\ldots\ , n) \tag{4.4}$$

nor the mean of these observed ratios, whose values are known can be the true value of the parameter μ.

Model–2 (Inverse Proportionality Model)

Equations (3.1), (3.2) and (3.6) together imply that

$$\mu = T(Y_i)\ .T(X_i\ ) = \{Y_i - \varepsilon(Y_i)\}.\{X_i - \varepsilon(X_i)\} \tag{4.5}$$

Thus, the products

$$T(Y_i)\ .T(X_i\ )\ \ ,\ \ (i = 1\ ,2\ , \ldots\ldots\ldots\ , n) \tag{4.6}$$

or is equivalently the products

$$\{Y_i - \varepsilon(Y_i)\}.\{X_i - \varepsilon(X_i)\}\ \ ,\ \ (i = 1\ ,2\ , \ldots\ldots\ldots\ , n) \tag{4.7}$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 2 , February 2016

are equal and this common value is nothing but the true value of the parameter μ.
However, the values of these products are unknown.
On the other hand, neither the observed products

$$Y_i . X_i \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.8)$$

nor the mean of these observed products, whose values are known can be the true value of the parameter μ.

Model–3

Equations (3.1), (3.2) and (3.9) together imply that

$$\mu = T(Y_i) \,/\, \{T(X_i)\}^2 = \{Y_i - \varepsilon(Y_i)\} \,/\, \{Y_i - \varepsilon(Y_i)\}^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n). \qquad (4.9)$$

Thus, the ratios

$$T(Y_i) \,/\, \{T(X_i)\}^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.10)$$

or is equivalently the ratios

$$\{Y_i - \varepsilon(Y_i)\} \,/\, \{Y_i - \varepsilon(Y_i)\}^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.11)$$

are equal and this common value is nothing but the true value of the parameter μ.
However, the values of these ratios are unknown.
On the other hand, neither the observed ratios

$$Y_i \,/\, X_i^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.12)$$

nor the mean of these observed ratios, whose values are known can be the true valu e of the parameter μ.

Model–4

Equations (3.1), (3.2) and (3.11) together imply that
In this case,

$$\mu = T(X_i) . \{T(Y_i)\}^2 = \{X_i - \varepsilon(X_i)\} . \{Y_i - \varepsilon(Y_i)\}^2 \qquad (4.13)$$

Thus, the products

$$T(X_i) . \{T(Y_i)\}^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.14)$$

or is equivalently the products

$$\{X_i - \varepsilon(X_i)\} . \{Y_i - \varepsilon(Y_i)\}^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.15)$$

are equal and this common value is nothing but the true value of the parameter μ.
However, the values of these products are unknown.
On the other hand, neither the observed products

$$X_i . Y_i^2 \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (4.16)$$

nor the mean of these observed products, whose values are known can be the true value of the parameter μ.

V. ERROR INVOLVED IN ESTIMATE OF PARAMETER IN AN SPECIAL MODEL

If

$$X_1 , X_2 , \ldots\ldots\ldots , X_n$$

are n observations containing a parameter μ and chance error ε_i , then the observations satisfy the model described by

$$X_i = \mu + \varepsilon_i \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (5.1)$$

Here,

$$\mu = T(X_i) \ , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (5.2)$$

The existing methods of estimation namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square {Aldrich (1930) , Anders (1999) , Barnard (1949) , Birnbaum (1962) , Ivory (1825) , *Kendall* & *Stuart* (1977) , Lehmann & Casella George(1998) , Lucien (1990) , *Walker* & *Lev* (1965)} provides

$$\bar{X} = n^{-1} \sum_{i=1}^{n} X_i \qquad (5.3)$$

as estimator of the parameter μ.
This estimator suffers from an error

$$\bar{\varepsilon}_i = n^{-1} \sum_{i=1}^{n} \varepsilon_i \qquad (5.4)$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 3, Issue 2 , February 2016

which may not be zero *Chakrabarty* (2014, 2015)}.
This is the amount of error involved in the estimate of the parameter obtained by the existing statistical method of estimation.

VI. CONCLUSION

The current study has been made confined in the situation where one variable depends upon a single variable only. However, there exist more theoretical models under this situation. Some of them are as follows:

(1) $h(X, Y) = \mu$ (5.1)
where μ is a parameters and $h(X, Y)$ is some function of two variables X and Y.

(2) $Y = a_0 + a_1 X + a_2 X^2 + \ldots\ldots\ldots\ldots + a_p X^p$ (5.2)
where $a_0, a_1, a_2, \ldots\ldots\ldots\ldots + a_p$ are parameters.

(3) $Y = \mu + \lambda X^{-1}$ (5.3)
where μ and λ are parameters.

(4) $Y = f(X : \mu_1, \mu_2, \ldots\ldots, \mu_k$ (5.4)
where $\mu_1, \mu_2, \ldots\ldots\ldots, \mu_k$ are parameters and $f(X : \mu_1, \mu_2, \ldots\ldots, \mu_k)$ is some function of X.

(5) $Y = A\ exp\ \{g(X : \mu_1, \mu_2, \ldots\ldots\ldots, \mu_k)\}$ (5.5)
where $\mu_1, \mu_2, \ldots\ldots\ldots, \mu_k$ are parameters and $g(X : \mu_1, \mu_2, \ldots\ldots, \mu_k)$ is some function of X.

There is necessity of searching for the appropriate model(s), corresponding to these theoretical models, to be satisfied by the respective observed data.

An attempt has also been made in this study to express the parameter involved in the model, in some situations, in terms of the observed data. There is necessity of searching for the expressions of the parameters involved in the other models for which such expressions have not been obtained till now.

Another necessity is to search for the error involved in the estimates of the parameters, obtained by the usual statistical method, involved in the models for observed data that have already been searched for as well as for the models to be searched for.

The situation discussed here corresponds to theoretically known relationships between two variables. There is necessity for studying the associations of errors in the situations where more than two variables are related by known theoretical relationships.

It is yet to search for the appropriate model, to be satisfied by observed data, corresponding to the theoretical models in the situation where one variable may theoretically depend upon more than one variable. Suppose, the variable Y depends upon the variables $X_1, X_2, \ldots\ldots\ldots\ldots, X_k$. Then some theoretical models to be studied are as follows:

(1) $f(X_1, X_2, \ldots\ldots\ldots, X_n) = \mu$
where μ is the parameter.

(2) $Y = f(X_i : \mu_1, \mu_2, \ldots\ldots\ldots, \mu_k)$
where $\mu_1, \mu_2, \ldots\ldots\ldots, \mu_k$ are the parameters.

(3) $Y = A.exp\{f(X_1, X_2, \ldots\ldots\ldots, X_n : \mu_1, \mu_2, \ldots\ldots\ldots, \mu_k)\}$
where $A, \mu_1, \mu_2, \ldots\ldots\ldots, \mu_k$ are the parameters.

(4) $f(X_1, X_2, \ldots\ldots\ldots, X_n) = \mu$
where μ is the parameter.

(5) $f(Y, X_1, X_2, \ldots\ldots\ldots\ldots, X_k) = \mu$
where μ is the parameter.

Thus one problem for researcher at this stage is to search for the appropriate model corresponding to each of these theoretical models to be satisfied by the corresponding observed data and to find out the estimate(s) of the parameter(s) involved as well as the error(s) involved in the respective estimate(s).

There is also necessity for studying the associations of errors in the situations where two and /or more variables are related by unknown theoretical relationships.

REFERENCES

[1] Aldrich John, " Fisher's Inverse Probability of 1930 ", International Statistical Review, Vol. 68, pp. 155 – 172. 2000.
[2] Anders Hald, " On the History of Maximum Likelihood in Relation to Inverse Probability and Least Squares", Statistical Science, Vol. 14, pp. 214 – 222, 1999.
[3] Barnard G. A., " Statistical Inference", Journal of the Royal Statistical Society, Series B, Vol. 11, pp. 115 – 149, 1949.
[4] Birnbaum Allan, " On the Foundations of Statistical Inference" Journal of the American Statistical Association, Vol. 57, pp. 269 – 306, 1962
[5] Chakrabarty D., " Analysis of Errors Associated to Observations of Measurement Type ", International Journal Electronics and Applied Research (ISSN: 2395-0064), 1(1), 15 – 28, also available in http://eses.co.in//ESESJuornal , 2014.
[6] Chakrabarty D., "Observation Composed of a Parameter and Chance Error: An Analytical Method of Determining the Parameter", International Journal of Electronics and Applied Research, 1(2), (ISSN : 2395 – 0064), 20 – 38, also available in http://eses.co.in//ESESJuornal , 2014.
[7] Chakrabarty D., " Observation Composed of a Parameter and Chance Error: Determination of the Parameter as Stable Range ", International Journal Electronics and Applied Research (ISSN: 2395-0064), 1(1), 35 – 47, also available in http://eses.co.in//ESESJuornal , 2015.
[8] Chakrabarty D., "Observation Consisting of Parameter and Error: Determination of Parameter ", Proceedings of the World Congress on Engineering 2015, (WCE 2015, July 1 - 3, 2015, London, U.K.), ISBN: 978-988-14047-0-1, ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online), Vol. II, 680 – 684., 2015.
[9] Ivory, " On the Method of Least Squares ", Phil. Mag., vol. LXV, pp. 3 – 10, 1825.
[10] Kendall M. G. and Stuart A., "Advanced Theory of Statistics ", Vol. 1 & 2, 4th Edition, New York, Hafner Press,1977.
[11] Lehmann Erich L. & Casella George, " Theory of Point Estimation", 2nd ed. Springer. ISBN 0 – 387 – 98502 – 6, 1998.
[12] Lucien Le Cam (1990) : " Maximum likelihood — An introduction ", ISI Review, Vol. 58, no. 2, pp. 153 –171, 1990.
[13] Walker Helen M. and Lev J. , "Statistical Inference ", Oxford & IBH Publishing Company, 1965.