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Regular Synthesis Algorithms of Actuation Devices in Polynomial Control Systems

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ABSTRACT: Questions of construction the regularization of synthesis algorithms of actuation devices in the control systems of polynomial objects described by multidimensional functional ranks of Voltaire are considered. Synthesis algorithms on the basis of an operator method of regularization with use of variation inequalities are given. Regular algorithms of the solution of the considered task in the absence of monotony of display are considered. Questions of creation of discrete approximation of the regularization algorithms considered are analyzed. The provided computing schemes allow to regularization a problem of synthesis of polynomial control systems of dynamic objects and by that to stabilize procedure of formation of the law of control.

KEYWORDS: polynomial control system, actuation device, regularization method, variation inequality, regularization parameter.

I. INTRODUCTION

Various methods solutions of problems of optimization of nonlinear stochastic systems on the basis of polynomial representation of models of the operated objects are so far proposed [1-6]. Practical realization of the developed methods of identification and optimization of nonlinear stochastic systems faces need of their consideration from positions of the return problems of dynamics of the operated objects. It is connected by that tasks of this kind, in essence, are badly caused and belong to a class incorrectly of objectives [7-11]. In such situation it is expedient to consider a problem of synthesis of methods and algorithms of identification and optimization of polynomial systems from the point of view of the theory of regular estimation defining methodology of creation of steady synthesis algorithms of the specified class of dynamic systems.

II. STATEMENT OF A PROBLEM

The problem of synthesis of polynomial control systems, as we know, is reduced to determination of the parameters of the actuation device of the chosen structure providing the best value of criterion of an optimality when performing certain restrictions [1,2,6]. To a bowl of all it leads to system of the nonlinear integrated equations of a look

$$\sum_{i,m=0}^{s_1,s_2}\int_0^{\infty}\!\!\cdots\!\!\int a^0_{2i+1,m}(\beta_{2i+1},\gamma_m)\tilde{K}_{2q+1,2i+1,l,m}(\tau_{2q+1},\beta_{2i+1},\lambda_l,\gamma_m;\rho_1,\ldots,\rho_{\alpha_0+\beta_0})\,d\beta_{2i+1}d\gamma_m = \\ = R_{2q+1,l}(\tau_{2q+1},\lambda_l);\qquad \tau_{2q+1}\ge 0\quad \lambda_l\ge 0,\; q=0,\ldots,s_1,\; l=0,\ldots,s_2. \tag{1}$$

The system of the equations (1) can be written down in the following operator form

$$K(a)=R,\; a\in D(K), \tag{2}$$

where the right member of equation (1) is defined by expression

$$R=R_{2q+1,l}(\tau_{2q+1},\lambda_l),$$

$$K(a)\equiv\sum_{i,m=0}^{s_1,s_2}\int_0^{\infty}\!\!\cdots\!\!\int a^0_{2i+1,m}(\beta_{2i+1},\gamma_m)\tilde{K}_{2q+1,2i+1,l,m}(\tau_{2q+1},\beta_{2i+1},\lambda_l,\gamma_m;\rho_1,\ldots,\rho_{\alpha_0+\beta_0})d\beta_{2i+1}d\gamma_m$$

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the nonlinear integrated operator from H_1 in H_2, $R \in H_2$, H_1, H_2 – material Gilbert spaces, a – the vector of coefficients made of elements of pulse characteristics $a_{2i+1,m}(\tau_1,..., \tau_{2i+1}, \lambda_1,..., \lambda_m)$ represents.

III. SOLUTION OF A TASK

It is known [10,12-15] that at the above assumptions the task (2) is generally incorrect., existence of decisions (2) for all right parts $R \in H$, in the same cases when decisions exist isn't guaranteed, their set is unstable in relation to small indignations R. We will consider that instead of an element R its approach $R_\delta \in H$, where is available

$$\|R - R_\delta\| \le \delta, \quad \delta \ge 0.$$

Recently intensively the approach to creation of steady methods of the solution of a task (2) based on use of operator regularization [12-15] develops. According to it to a task (2) with it is inexact the set right part the regularization equation is compared

$$K(a) + \alpha a = R_\delta, \quad a \in D(K),\ \alpha = \alpha(\delta) > 0\,, \tag{3}$$

where $\alpha > 0$ – parameter of regularization.

One of effective ways of obtaining the approximate solution of the equation (2) is reduced to replacement of this equation with family the regularization of tasks by introduction of the smoothing operator $U(a)$ with positive coefficient α [12]:

$$K\left(a_\alpha^\delta\right) + \alpha U\left(a_\alpha^\delta\right) = R_\delta\,. \tag{4}$$

Often the operator $U(a)$ in (4) choose in a look:

$$U(a) = grad\,\|a\|^2 = \|a\|\, grad\,\|a\|\,.$$

Owing to monotony of the operator K it is possible to write down:

$$\left(K(a) - K(a_\beta^\delta), a - a_\alpha^\delta\right) \ge 0.$$

Following the considered regularization method, we will define an auxiliary variation inequality [13]

$$\left(K(a) + \alpha U(a) - R_\delta,\ a - q\right) \le 0,\ \ \forall q \in Q,\ \ Q \subset D(K)\,.$$

We will consider a variation inequality with exact data

$$\left(K(a) + \alpha U(a) - R, a - q\right) \le 0\ \ \forall q \in Q\,.$$

For the considered case following [12,15] it is possible to write down the following equation

$$\left(y_\sigma + \alpha U(a_\sigma - a^0) - R_\delta, q - a_\sigma\right) \ge 0,\ \ y_\sigma \in \overline{K} a_\sigma,\ \forall q \in Q,\ \ Q \subset \operatorname{int} D(K)\,, \tag{5}$$

where $\overline{K}$ – the maximum monotonous expansion K, a^0 – some the fixed element from H. We will designate the solution of this equation through a_σ, where $\sigma = (\delta, \alpha)$.

If $\delta/\alpha \to 0$ at $\alpha \to 0$, the sequence $\{a_\sigma\}$ meets on norm H to an element $a^* \in N$ which satisfies to a ratio

$$\left\|a^* - a^0\right\| = \min_{a \in N}\left\|a - a^0\right\|.$$

For a choice of the parameter of regularization α providing $a_\sigma \to a^*$, we will consider the following equation

$$\rho(\alpha) = \left\|y_\sigma - R_\delta\right\|. \tag{6}$$

Believing in the equation (5) $q = a^0$ we will have [15]:

$$\lim_{\alpha \to \infty} \rho(\alpha) \le \left\|y_0 - R_\delta\right\|,\ y_0 = \overline{K} a^0\,. \tag{7}$$

Taking into account a condition $y_\sigma \to y_0$ on the basis of (6) and (7) it is possible to write

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$$\lim_{\alpha\to\infty}\rho(\alpha)=\left\|y_0-R_\delta\right\|.$$

Then for any $0<\delta<\bar{\delta}<1$ the inequality is carried out

$$\left\|y_0-R_\delta\right\|>k\delta^p,\ k>1,\ 0<p<1.$$

Thus, at $\alpha,\delta\to 0$, it is possible to come to the following limit ratio

$$\| K(a_\alpha^\delta)-R_\delta \|\to 0,$$

wherein

$$\| a_\alpha^\delta \|=\| K(a_\alpha^\delta)-R_\delta \| /\alpha.$$

The above-stated algorithms are especially effective in a case, when display K monotonously. In such cases any of the indignant problems of type (5) has the only decision; the sequence $\{a_\varepsilon\}$ meets at $\{\varepsilon\}\to 0$ to the point from a set of decisions A having the minimum norm.

However, the monotony of the condition rather restrictive and is not performed for a large number of applications. Therefore naturally there is a question of applicability of a method of regularization in the absence of monotony. It is caused by that at the solution of the considered task chances when K is the monotonous multiple-valued operator set on some the convex closed set.

In these conditions methods of iterative regularization and averaging can provide convergence [10,16]. These methods define a new iteration point explicitly and therefore simply realizable. Besides, in these conditions it is possible to use also implicit methods, in particular, "proximal" [17]. Proximal iterations consist in the consecutive solution of a task on finding of a point $a^{k+1}\in Q$ such that:

$$\exists w^{k+1}\in K\left(a^{k+1}\right):\left\langle w^{k+1}+\theta^{-1}\left(a^{k+1}-a^k\right),v-a^{k+1}\right\rangle\geq 0\ \ \forall v\in Q. \qquad (8)$$

The task (8) of complexity is approximately equivalent to an initial task therefore creation of effective methods of proximal type is connected with any certain way of simplification or replacement of an auxiliary task (8).

Now some approaches to simplification of "proximal" iteration (8) due to use of specifics of an initial task [16] are known, namely for considerable part of applied tasks the operator K we will present in the form

$$K=L+M,$$

where L – unambiguous, a M – the multiple-valued operator. We will note that in the considered applied problem of synthesis of polynomial control systems the operator M can be subdifferentsialy some convex function defined that is solutions of the corresponding problem of optimization [18].

Based on these properties it is expedient to use implicit "proximal" iteration only for the operator M, i.e. to define a new iterative point a^{k+1} as the solution of a task [16,19]:

$$\exists m^{k+1}\in M\left(a^{k+1}\right):\left\langle L\left(a^k\right)+\theta^{-1}\left(a^{k+1}-a^k\right),v-a^{k+1}\right\rangle+\left\langle m^{k+1},v-a^{k+1}\right\rangle\geq 0 \qquad \forall v\in Q, \qquad (9)$$

The approach [20] based on a basis of the two-level scheme appears more effective. In this case iteration of type (9) is applied to determination of the dividing hyper plane, i.e. parameters of the main process. According to this method the sequence of matrixes $\{A_k\}$ about n such that gets out

$$\xi'\| p\|^2\leq\left\langle A_k p,p\right\rangle\leq\xi''\| p\|^2\ \ \forall p\in R^n, \qquad 0<\xi'\leq\xi''<\infty,$$

also the solution of an auxiliary variation inequality of a look is found

$$\exists m^k\in M\left(v^k\right):\left\langle L\left(a^k\right)+\theta^{-1}A_k\left(v^k-a^k\right),v-v^k\right\rangle+\left\langle m^k,v-v^k\right\rangle\geq 0\ \ \forall v\in Q,\ \ \theta>0.$$

If $a^k=v^k$, the stop of iterative process is made. Otherwise it is necessary

$$l^k=L\left(v^k\right)-L\left(a^k\right)-\theta^{-1}A_k\left(v^k-a^k\right),\quad \omega_k=\left\langle l^k,a^k-v^k\right\rangle,$$

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$$a^{k+1} = \pi_V\left(a^k - \gamma\,\omega_k\, l^k / \left\| l^k \right\|^2\right),\quad \gamma \in (0,2),$$

where $\pi_V(\cdot)$ – the operator of design on V.

Following [16,19] it is possible to show that if in a method there is a stop on the k-th iteration, $a^k \in Q^*$ and if the sequence $\{a^k\}$ is infinite, exists $\tilde{\theta} > 0$ such that at any $\theta \in (0, \tilde{\theta})$ the limit ratio of a look is carried out:

$$\lim_{k\to\infty} a^k = a^* \in Q^*.$$

At the numerical solution of an incorrect task (2) there is a problem of sampling of an initial task, i.e. replacement of continuous mathematical model some its finite-dimensional analog. However at realization of these methods it is necessary to carry out transition from an initial "continuous" infinite-dimensional task to finite-dimensional analog. Here it is already not enough to know regularization properties of only one "continuous" option of a method, and it is necessary to find out the conditions providing for finite-dimensional the regularization decisions convergence to exact solutions of an initial task.
The most common ways of sampling the infinite-dimensional tasks are course the difference and projective methods [8]. Course the difference method consists in replacement of the initial continuous equation with the final equation. Thus finding of the approximate decision is usually reduced to the decision of system of the linear or nonlinear algebraic equations. Narrowing of the initial operator on a finite-dimensional subspace that allows to reduce also a task to system of the algebraic equations is the basis for a projective method. In works [21,22] the general approach to finite-dimensional approximation the regularization of decisions including projective methods and methods of course the difference approximation as special cases is developed.
We approximate initial pair of spaces (H_1, H_2) by means of systems of the linear connecting projectors $p = \{p_n : H_1 \to H_{1,n}\}$ and $q = \{q_n : H_2 \to H_{2,n}\}$, we will believe thus that

$$\| q_n \| \le T \quad \forall n, \tag{10}$$

where $n \in N$, $n \to \infty$ - natural numbers.
For regularization of a task (2) we will consider the following variation inequality

$$\left(K_n(a_n) + \alpha U_n(a_n) - q_n f^\delta, x_n - a_n\right)_n \ge 0 \qquad \forall x_n \in D_n \tag{11}$$

with the parameter of regularization $\alpha > 0$. Owing to monotony of the sum $(K_n + \alpha U_n)$ at any $\alpha > 0$ variation inequality (11) has a nonempty set of decisions $A^n \subset D_n$ at everyone n.

Considering monotony of the operator K_n taking into account expression (10) and conditions of approximation $\| f^\delta - f \| \le \delta$ right member of equation (2) it is possible to write [22]:

$$\begin{aligned}\left(K_n x'_n - q_n f,\, x'_n - a^n\right)_n &\ge \left(K_n a^n - q_n f,\, x'_n - a^n\right)_n \ge \\ &\ge \left(K_n(a_n) + \alpha U_n(a^n) - q_n f^\delta,\, x'_n - a^n\right)_n - \\ &- \alpha\left(U_n a^n,\, x'_n - a^n\right)_n - T\delta \left\| x'_n - a^n \right\|_n, \quad x'_n \in D_n \ \ \forall n.\end{aligned} \tag{12}$$

At convergence of operators $\{K_n\}$ to the exact operator K expression (12) can be written down in a look

$$\left(K_n x'_n - q_n f,\, x'_n - a^n\right)_n \to \left(Kx - f,\, x - a^0\right). \tag{13}$$

Therefore, limit transition in an inequality (12) leads to a variation inequality $\left(Kx - f,\, x - a^0\right) \ge 0$ for any element $x \in D$. Thus the variation inequality (13) in view of monotony of the exact operator K and camber of the closed set D is equivalent to a variation inequality

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$$\langle K(a)-f,\ x-a\rangle \geq 0 \quad \forall x \in D,\ x_n \in D_n. \tag{14}$$

Therefore the limit point a^0 is the exact solution of a task (14).

On the basis above the given expressions and methods of discrete approximation [21-23] it is possible to show that if parameters δ, α, n are connected by a ratio

$$(\delta + r_n)/\alpha \to 0, \qquad \delta,\ \alpha,\ 1/n \to 0,$$

that at any choice the regularization decisions $a^n \in A^n$ a limit set $\{a^n\}^*$ lies in a subset normal, i.e. minimum on norm, exact decisions $A_* \subset A_0$ and thus convergence of norms $\|a^n\|_n \to \|a_*\|$, $a_* \in A_*$, $\delta,\ \alpha,\ 1/n \to 0$ takes place.

IV. CONCLUSION

The expressions given above allow to provide convergence of the regularization algorithms considered and by that to stabilize procedure of formation of the law of control in problems of synthesis of actuation devices in polynomial control systems.

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