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Square Harmonious Graphs

N. Adalin Beatress , P.B.Sarasija

Department of Mathematics, All Saint College of Education, Kaliyakkavilai, Tamil Nadu.
Department of Mathematics, Noorul Islam Centre for Higher Education, Kumaracoil, Tamil Nadu

ABSTRACT: In this paper we have introduced a new harmonious labeling called square harmonious labeling. A graph G(V,E) with n vertices and m edges is said to be a square harmonious graph if there exists an injection $f : V(G) \to \{1,2,\ldots, m^2+1\}$ such that the induced mapping. $f^*:E(G) \to \{1,4,9,\ldots,m^2\}$ defined by $f^*(uv) = (f(u) + f(v)) \bmod (m^2+1)$ is a bijection. the resulting edge labels and vertex labels are distinct. The function f is called a square harmonious labeling of G. Here we prove that path graph, star graph, bistar graph, corona graph $P_n \odot pk_1$, the graph $C_3 @ pk_1$ and the comb graph $P_n \odot k_1$ are Square harmonious graph.

KEYWORDS: Harmonious labeling, Bistar, Comb graph

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I. Introduction

In this paper, we consider finite, undirected, simple graph G(V,E) with n vertices and m edges.
For notations and terminology we follow Bondy and Murthy [1].Harmonious graphs naturally arose in the study by Graham and Sloane [3] of modular versions of additive base problems. Square graceful graphs were introduced in [4]. For a detailed survey on graph labeling we refer to Gallian [2] . We also refer [5,6,7]
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Definition : A graph G(V,E) with n vertices and m edges is said to be a square harmonious graph if there exists an injection $f : V(G) \to \{1,2,\ldots,(m^2+1)\}$ such that the induced mapping $f^*:E(G) \to \{1,4,9,\ldots,m^2\}$ defined by $f^*(uv) = (f(u) + f(v)) \bmod (m^2+1)$ is a bijection, the resulting edge labels and vertex labels are distinct. The function f is called a square harmonious labeling of G.

In this paper, we prove that the path graph, star graph, bistar graph, the graph $C_3 @ pk_1$ and the comb graph $P_n \odot k_1$ are square harmonious graphs.

II Main Results

Theorem 2.1. Every path P_n ($n \geq 3$) is a square harmonious graph.

Proof: Let P_n be a path with n vertices and m = (n-1) edges. Let $V(P_n) = \{ v_1,v_2,\ldots,v_n\}$and

$E(P_n) = \{ v_iv_{i+1}, 1 \leq i \leq n-1\}$. Define an injection function $f : V(P_n) \to \{1,2,3,\ldots, m^2+1\}$ by

$$f(v_1) = 3,\ f(v_2) = 1,\ f(v_3) = m^2+1, \text{ and } f(v_4) = m^2,\quad f(v_5) = m^2 -2n + 4,$$

$$f(v_{2i}) = v_{2i-1} + (2i-5),\ 3 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor,\quad f(v_{2i+1}) = V_{2i} - 2n+2i,\ 3 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

f induces a bijection $f^* : E(P_n) = \{1,4,9,\ldots,(n-1)^2\}$. $f^*(uv) = (f(u) + f(v)) \bmod (m^2+1)$.

The edge labels are distinct. Hence every path P_n, $n \geq 3$ is a square harmonious graph.

Theorem 2.2. The star graph $k_{1,n}$ is a square harmonious graph for all $n \geq 2$.

Proof: Let $k_{1,n}$ be a star graph with (n+1) vertices and m = n edges.

Let $V(k_{1,n}) = \{v_1, v_2, \ldots, v_{n+1}\}$. Let v_{n+1} be the centre vertex. Let $E(k_{1,n}) = \{v_i v_{n+1}, 1 \leq i \leq n\}$.

Define an injective function $f : V(k_{1,n}) \rightarrow \{1,2,\ldots,m^2+1\}$ by

$f(v_{n+1}) = 2m-3$, $\quad f(v_i) = (m-i+1)^2 - 2m + 3, 1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$, $f\left(v_{\left\lceil \frac{m}{2} \right\rceil+1}\right) = m^2 - 2m + 5,$

$f\left(v_{\left\lceil \frac{m}{2} \right\rceil+j}\right) = v_{\left\lceil \frac{m}{2} \right\rceil+j-1} + 2j - 1, 2 \leq j \leq \left\lfloor \frac{m}{2} \right\rfloor$. $f^*(uv) = (f(u) + f(v)) \bmod (m^2+1)$.

Hence the star graph $K_{1,n}$ is a square harmonious graph.

Theorem 2.3. The bistar graph $B_{p,q}$ is a square harmonious labelling graph.

Proof: Let $B_{p,q}$ be a bistar graph with n = p+q+2 vertices and m = p+q+1 edges.

Let $V(B_{p,q}) = \{u_i, 1 \leq i \leq p+1, v_j, 1 \leq j \leq q+1\}$,

Let $E(B_{p,q}) = \{u_i u_{p+1}, 1 \leq i \leq p, v_j v_{q+1}, 1 \leq j \leq q, u_p v_q\}$.

Define an injection function $f : V(B_{p,q}) \rightarrow \{1,2,3,\ldots..(p+q+1)^2+1\}$ by

$f(u_{p+1}) = 1$, $f(u_1) = 3$, $f(u_i) = u_{i-1}+2i+1$, $2 \leq i \leq p$, $(v_{q+1}) = m^2+1$, $f(v_i) = (m-i+1)^2$, $1 \leq i \leq q$.

The edge labels are distinct.

Theorem : 2.4. The graph C_3 @ pK_1, $(p \geq 2)$ is a square harmonious graph.

Proof : Let u_1, u_2, u_3 be the vertices of C_3 and $v_1, v_2, \ldots, v_p$ be the new vertices.

Let $V(C_3@ pk_1) = \{u_1,u_2,u_3, v_1,v_2,\ldots,v_p\}$. Let $E(C_3@ pk_1) = \{u_1u_2,u_2u_3, u_3u_1, u_1p_1, u_1p_2,\ldots,u_1p_p\}$.

Here u_1 is adjacent to $v_1, v_2, \ldots, v_p$. Define $f : V(C_3 @ pK_1) \rightarrow \{1,2,3,\ldots,(P+3)^2+1\}$ by

$f(u_1) = m^2+1$, $f(u_2) = 9$, $f(u_3) = 16$, $f(v_i) = (m-i+1)^2$, $1 \leq i \leq p-2$, $f(v_{p-1}) = 4$, $f(v_p) = 1$

The induced function $f^* : E(C_3@pK_1) \rightarrow \{1,4,\ldots,(P+3)^2\}$ is bijective.

Theorem 2.5. The comb graph $P_n \odot K_1$, $(n \geq 2)$ is a square harmonious graph.

Proof: Let $\{u_1,u_2,\ldots,u_n\}$ be the vertices of path P_n and $\{v_1,v_2,\ldots,v_n\}$ be the n pendant vertices of $u_1,u_2,\ldots,u_n$ respectively. Here m = 2n-1.

Define an injection $f : V(P_n \odot K_1) \rightarrow \{1,2,3, \ldots.. (2n-1)^2+1\}$ by

$f(u_{2i-1}) = i(2i-1)$, $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$, $f(u_{2i}) = i(2i+1)$, $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$, $f(v_1) = m^2+1$,

$f(v_{2i-1}) = (m-2i+3)^2 - i(2i-1)$, $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$, $f(v_{2i}) = (m-2i+2)^2 - i(2i+1)$, $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$,

The induced function $f^* : E(P_n \odot K_1) \rightarrow \{1,4,9, \ldots, (2n-1)^2\}$ is bijective.

Theorem 2.6. The corona graph $P_n \odot pK_1$ $(n \geq 2)$ is a square harmonious graph.

Proof: Let $\{u_1,u_2,\ldots,u_n\}$ be the vertices of the path P_n and $u_{j1},u_{j2},\ldots,u_{jp}$ be the p pendent vertices of the vertex u_j of the path P_n for $1 \leq j \leq n$. Here m = mp+n-1.

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Define an injection $f : V(P_n \odot pK_1) \rightarrow \{1,2,3,\ldots,(np+n-1)^2+1\}$ by

$f(u_{2i-1}) = i(2i-1)$, $1 \le i \le \lceil \frac{n}{2} \rceil$, $f(u_{2i}) = i(2i+1)$, $1 \le i \le \lfloor \frac{n}{2} \rfloor$, $f(u_{11}) = m^2+1$,

$f(u_{2i-1,j}) = [m-p(2i-2) - j+2]^2 - i(2i-1)$, $2 \le i \le \lceil \frac{n}{2} \rceil$, $1 \le j \le p$.

$f(u_{2i,j}) = [m-p(2i-1) - j+2]^2 - i(2i+1)$, $1 \le i \le \lfloor \frac{n}{2} \rfloor$, $1 \le j \le p$. $u_{ij} = (m-j+2)^2 - 1$, $2 \le j \le p$.

The induced function $f^* : E(P_n \odot pK_1) \rightarrow \{1,4,\ldots (np+n-1)^2\}$ is bijective.

Hence the corona graph $P_n \odot pK_1$ is square harmonious.

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