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A New Approach for Solving Deterministic Multi-Item Fuzzy Inventory Model Three Constraints

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ABSTRACT: This paper discusses an Economic Order Quantity (EOQ) model in which fuzzy multi-item inventory model together with three constraints. The setup costs, the holding costs, demands, storage area, investment amount and the maximum average number of units are considered as triangular fuzzy numbers. The fuzzy parameters in the constraints are then transformed into crisp using GMIR technique. The fuzzy parameters in the objective function are then transformed into corresponding interval numbers. Minimization of the interval objective function (obtained by using interval parameters) has been transformed into a classical multi-objective EOQ problem. The order relation that represents the decision maker's preference among the interval objective function has been defined by the right limit, left limit, and center which is the half –width of an interval. This concept is used to minimize the interval objective function. The problem has been solved by fuzzy programming technique. Finally, the proposed method is illustrated with a numerical example.

KEYWORDS: Inventory, space constraint, investment constraint, maximum average number of units constraint, EOQ, Interval number, Fuzzy sets, triangular fuzzy number, GMIR technique, Fuzzy optimization technique, Multi-objective Programming

I.INTRODUCTION

In traditional mathematical problems, the parameters are always treated as deterministic in nature. However, in practical problem, uncertainty always exists. In order to deal with such uncertain situations fuzzy model is used. In such cases, fuzzy set theory, introduced by Zadeh [15] is acceptable. There are several studies on fuzzy EOQ model. Lin et al. [7] have developed a fuzzy model for production inventory problem. Katagiri and Ishii [5] have proposed an inventory problem with shortage cost as fuzzy quantity.

The parameters in any inventory model are normally variable uncertain, imprecise and adoptable to the optimum decision making process and the determination of optimum order quality becomes a vague decision making process. The vagueness pertained in the above parameters analyze the inventory problem in a fuzzy environment.

On the basis of this idea the GMIR technique [10] has been adopted to transform the fuzzy constraints to a crisp one so that we can get the crisp constraints.

This paper discusses a deterministic multi-item fuzzy EOQ model together with three constraints. Demands, holding costs, ordering costs, storage space, investment and the maximum average number of units are taken as triangular fuzzy numbers, and expression for fuzzy cost is established. For minimizing the cost function we transformed the fuzzy objective function into interval objective function. Now, this single objective function is then converted to multi-objective problem by defining left limit, right limit and center of the objective function. This multi-objective is then solved by fuzzy optimization technique. Linear membership function is considered here. This model is illustrated by a numerical example.

The article is organized as follows: In section 1 preliminary definition of fuzzy set, interval number, and triangular fuzzy number, α-cut of a fuzzy number, basic arithmetic optimization in interval, GMIR technique and nearest interval approximation is briefly described. Section 2 contains model formulation. The fuzzy optimization technique is section 3. In section 4 the process is illustrated by a numerical example and in the last section the entire work is concluded.

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II.RELATED WORK

For several years, classical economic order quantity (EOQ) problems were solved by many researches and had been published since 1915**.** The main area in which research articles have been published , may be classified as crisp and fuzzy inventory models, relating to Economic order quantity, economic production quantity, optimization, defuzzyfication [4,7]. The research papers relating to the optimization problems and defuzzyfications are of special interest [4]. In the same way many research articles are published in multi-item inventory models [2,8].There are several articles in ranking the fuzzy numbers which are useful to order the fuzzy numbers and to defuzzify them [9,10,11].The main objective of this paper is to construct multi item fuzzy EOQ model subject to three constraints and to determine the optimum order quantity so as to minimize the average total cost.

III. PRELIMINARIES

Definition 1*:*

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i.e) A={(x, $\mu_A(x)$) ; x∈ X.}, here μ_A:X→ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A is called the membership value of x∈ X in the fuzzy set A.

Definition 2:

Let $\Re$ be the set of all real numbers. An interval, may be expressed as

$$\bar{a} = [a_L, a_R] = \{x : a_L \le x \le a_R, a_L \in \Re, a_R \in \Re\} \quad \textbf{(1)}$$

where a_L and a_R are called the lower and upper limits of the interval $\bar{a}$, respectively.

If $a_L = a_R$ then $\bar{a}$ = [a_L , a_R] is reduced to a real number a, where a= a_L= a_R. alternatively an interval $\bar{a}$ can be expressed in mean-width or center-radius form as $\bar{a} = \langle m(\bar{a}), w(\bar{a}) \rangle$,where $m(\bar{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\bar{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\bar{a}$ .the set of all interval numbers in $\Re$ is denoted by I($\Re$).

Optimization in interval environment

Now we define a general non-linear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize } \bar{Z}(x) = \frac{\sum_{i=1}^{n} [a_{L_i}, a_{R_i}] \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} [b_{L_i}, b_{R_i}] \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(2)}$$

subject to $x_j > 0$, j=1,2,…,n and x ∈ S⊂$\Re$ where S is a feasible region of x, $0 < a_{L_i} < a_{R_i}$, $0 < b_{L_i} < b_{R_i}$ and r_i ,q_j are positive numbers. Now we exhibit the formulation of the original problem (2) as a multi-objective non-linear problem.

Now $\bar{Z}(x)$ can be written in the form $\bar{Z}(x) = [Z_L(x), Z_R(x)]$

$$\text{where } Z_L(x) = \frac{\sum_{i=1}^{n} a_{L_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{R_i} \prod_{j=1}^{n} x_j^{q_j}} \quad \textbf{(3)}$$

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$$Z_R(x) = \frac{\sum_{i=1}^{n} a_{R_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{L_i} \prod_{j=1}^{n} x_j^{q_j}} \quad (4)$$

The center of the objective function

$$Z_C(x) = \frac{1}{2}[Z_L(x) + Z_R(x)] \quad (5)$$

Thus the problem (2) is transformed in to

Minimize $\{Z_L(x), Z_R(x); x \in S\}$ **(6)**

Subject to the non-negativity constraints of the problem, where Z_L,Z_R are defined by (4) and (5).

Definition 3: *(Triangular fuzzy number)*:

For a triangular fuzzy number A(x), it can be represented by A(a,b,c:1) with membership function $\mu_A(x)$ given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}; a \le x \le b \\ 1 \quad ; x = b \\ \frac{(c-x)}{(c-b)}; b \le x \le c \\ 0 \quad ; otherwise \end{cases}$$

Definition 4:*(α-cut of a fuzzy number)***:**

The α-cut of a fuzzy number A(x) is defined as A(α)={x: μ(x) ≥ α, α ϵ [0,1]}

Nearest interval approximation:

According to Gregorzewski [3] we determine the interval approximation of a fuzzy number as: Let $\tilde{A}$ = (a_1, a2, a3) be an arbitrary triangular fuzzy number with a α-cuts [$A_L(\alpha)$,$A_R(\alpha)$].

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L = \int_0^1 A_L(\alpha) d\alpha = \int_0^1 [a_1 + (a_2 - a_1)\alpha] d\alpha = \frac{a_1 + a_2}{2}$$

$$C_R = \int_0^1 A_R(\alpha) d\alpha = \int_0^1 [a_3 - (a_3 - a_2)\alpha] d\alpha = \frac{a_2 + a_3}{2} \quad (7)$$

Therefore, the interval number considering $\tilde{A}$ as triangular fuzzy number is $\left[\frac{a_1 + a_2}{2}, \frac{a_2 + a_3}{2}\right]$.

Multi-item inventory problem

Assumptions:

A multi-item, multi-objective inventory model is developed under the following notations and assumptions.
i. The inventory system pertains to multi- items.
ii. Demand rate is deterministic.
iii. The inventory is replenished in single delivery for each order.
iv. Replenishment is instantaneous.
v. There is no lead time.
vi. Shortage are not allowed.

Notations**:**

n - total number of items being controlled simultaneously.

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Q_i	-	number of units ordered per order for i^{th} item(a decision variable)
D_i	-	Annual demand for i^{th} item
s_i	-	setup cost per order for i^{th} item.
h_i	-	holding cost per unit quantity per unit time for i^{th} item.
B	-	maximum available storage space to store all items.
F	-	maximum investment
N	-	maximum average number of units for all items
A_i	-	Storage space required per unit of item i (i=1,2,3,…..n)
c_i	-	Price per unit of item i (i=1,2,3……..n)
$\tilde{s}_i$	-	Fuzzy setup cost per order for i^{th} item.
$\tilde{h}_i$	-	Fuzzy holding cost per unit quantity per unit time for i^{th} item.
$\tilde{B}$	-	Fuzzy maximum average number of units for all items
$\tilde{F}$	-	Fuzzy maximum investment
$\tilde{N}$	-	Fuzzy maximum average number of units for all items

Multi- item crisp EOQ model with three constraints:

Minimize $T(Q_i) = \sum_{i=1}^{n}\left[\frac{D_i}{Q_i}s_i + \frac{Q_i}{2}h_i\right]$

S.to:
$$\left.\begin{array}{l}\sum_{i=1}^{n} A_i Q_i \le B \\ \sum_{i=1}^{n} C_i Q_i \le F \\ \frac{1}{2}\sum_{i=1}^{n} Q_i \le N\end{array}\right\}$$

Multi- item Fuzzy EOQ model with three constraints

$$\min \tilde{T}(Q_i) = \sum_{i=1}^{n}\left[\frac{D_i}{Q_i}\tilde{s}_i + \frac{Q_i}{2}\tilde{h}_i\right]$$

S.to:
$$\left.\begin{array}{l}\sum_{i=1}^{n} A_i Q_i \le \tilde{B} \\ \sum_{i=1}^{n} C_i Q_i \le \tilde{F} \\ \frac{1}{2}\sum_{i=1}^{n} Q_i \le \tilde{N}\end{array}\right\} \qquad \textbf{(8)}$$

Here $\tilde{h}_i, \tilde{s}_i, \tilde{B}, \tilde{F}$ and $\tilde{N}$ are triangular fuzzy numbers represented as $\tilde{h}_i = (h_{1i}, h_{2i}, h_{3i})$; $\tilde{s}_i = (s_{1i}, s_{2i}, s_{3i})$; $\tilde{B} = (B_1, B_2, B_3)$; $\tilde{F} = (F_1, F_2, F_3)$; $\tilde{N} = (N_1, N_2, N_3)$

RANKING OF TRIANGULAR FUZZY NUMBERS

The graded mean integration representation (GMIR)

A triangular fuzzy number $\tilde{A} = (a, b, c)$ defined by its membership function as follows:

$L(x) = \frac{x-a}{b-a}; a \le x \le b$ and $R(x) = \frac{c-x}{c-b}; b \le x \le c$

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The inverse functions L^{-1} and R^{-1} can be analytically expressed as given below:
$L^{-1}(h) = a + (b-a)h$; $R^{-1}(h) = c + (c-b)h$

Then the graded mean integration representation (GMIR) of membership function of $\tilde{A}$ is

$$R(\tilde{A}) = \frac{\int_0^1 h\left[\alpha L^{-1}(h) + (1-\alpha)R^{-1}(h)\right]dh}{\int_0^1 hdh}$$

$$= \frac{\int_0^1 h\left[\alpha(a+(b-a)h) + (1-\alpha)(c-(c-b)h)\right]dh}{\int_0^1 hdh}$$

$$= \frac{\left[\frac{a\alpha h^2}{2} + \frac{\alpha(b-a)h^3}{3} + \frac{(1-\alpha)ch^2}{2} - \frac{(1-\alpha)(c-b)h^3}{3}\right]_0^1}{\left[\frac{h^2}{2}\right]_0^1}$$

$$= \frac{\alpha(a-c)+c+2b}{3}$$

Now applying the ranking function GMIR to the constraints

$$\text{S.to:}\quad \left.\begin{aligned} &R\left[\sum_{i=1}^{n} A_i Q_i \le \tilde{B}\right] \\ &R\left[\sum_{i=1}^{n} C_i Q_i \le \tilde{F}\right] \\ &R\left[\frac{1}{2}\sum_{i=1}^{n} Q_i \le \tilde{N}\right] \end{aligned}\right\} \quad \text{(i.e)} \quad \left.\begin{aligned} &\sum_{i=1}^{n} A_i Q_i \le R\left(\tilde{B}\right) \\ &\sum_{i=1}^{n} C_i Q_i \le R\left(\tilde{F}\right) \\ &\frac{1}{2}\sum_{i=1}^{n} Q_i \le R\left(\tilde{N}\right) \end{aligned}\right\}$$

Where $R(\tilde{s}_i) = \frac{\alpha(s_{1i} - s_{3i}) + s_{3i} + 2s_{2i}}{3}$

$$R(\tilde{h}_i) = \frac{\alpha(h_{1i} - h_{3i}) + h_{3i} + 2h_{2i}}{3}$$

$$R(\tilde{B}) = \frac{\alpha(B_1 - B_3) + B_3 + 2B_2}{3}$$

$$R(\tilde{F}) = \frac{\alpha(F_1 - F_3) + F_3 + 2F_2}{3}$$

$$R(\tilde{N}) = \frac{\alpha(N_1 - N_3) + N_3 + 2N_2}{3}$$

Up to this stage, we are assuming that the demands, ordering costs, holding costs as real numbers i.e .of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we approach with fuzzy variables, where demands and other cost components are considered as triangular fuzzy numbers.

Let us assume the fuzzy demands $\tilde{D}_i = (D_i - \alpha, D_i, D_i + \beta)$ fuzzy holding costs $\tilde{h}_i = (h_i - \alpha, h_i, h_i + \beta)$,fuzzy ordering costs $\tilde{S}_i = (S_i - \alpha, S_i, S_i + \beta)$, fuzzy storage space $\tilde{B} = (B_1, B_2, B_3)$,fuzzy investment $\tilde{F} = (F_1, F_2, F_3)$ and

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fuzzy maximum average number of units $\tilde{N}=(N_1,N_2,N_3)$.Replacing the real valued variables D_i, S_i, h_i, B, F & N by the triangular fuzzy variables $\tilde{D}_i,\tilde{S}_i,\tilde{h}_i,\tilde{B},\tilde{F}$ and $\tilde{N}$ we get,

$$\min T(Q_i)=\sum_{i=1}^{n}\left[\frac{\tilde{D}_i}{Q_i}\tilde{s}_i+\frac{Q_i}{2}\tilde{h}_i\right]$$

$$\text{S.to:}\quad \left.\begin{aligned}&\sum_{i=1}^{n}A_iQ_i\le R(\tilde{B})\\&\sum_{i=1}^{n}C_iQ_i\le R(\tilde{F})\\&\frac{1}{2}\sum_{i=1}^{n}Q_i\le R(\tilde{N})\end{aligned}\right\}\qquad \textbf{(9)}$$

Now we represent the fuzzy EOQ model to a deterministic form so that it can be easily tackled. Following Grzegorzewski [3] , the fuzzy numbers are transformed into interval numbers as

$\tilde{D}_i=(D_i-\alpha,D_i,D_i+\beta)$ =[D_{iL},D_{iR}]

$\tilde{S}_i=(S_i-\alpha,S_i,S_i+\beta)$ = [S_{iL},S_{iR}]

$\tilde{h}_i=(h_i-\alpha,h_i,h_i+\beta)$=[$h_{iL}$,$h_{iR}$].

Using the above expression (9) becomes

$$\tilde{T}(Q_i)=\left[f_{L,}f_R\right]$$

Where,

$$f_L=\sum_{i=1}^{n}\left[\frac{D_{iL}}{Q_{iL}}s_{iL}+\frac{Q_{iL}}{2}h_{iL}\right]$$

$$\text{Subject to :}\quad \left.\begin{aligned}&\sum_{i=1}^{n}A_iQ_i\le R(\tilde{B})\\&\sum_{i=1}^{n}C_iQ_i\le R(\tilde{F})\\&\frac{1}{2}\sum_{i=1}^{n}Q_i\le R(\tilde{N})\end{aligned}\right\}\qquad \textbf{(10)}$$

$$f_R=\sum_{i=1}^{n}\left[\frac{D_{iR}}{Q_{iR}}s_{iR}+\frac{Q_{iR}}{2}h_{iR}\right]$$

$$\text{Subject to :}\quad \left.\begin{aligned}&\sum_{i=1}^{n}A_iQ_i\le R(\tilde{B})\\&\sum_{i=1}^{n}C_iQ_i\le R(\tilde{F})\\&\frac{1}{2}\sum_{i=1}^{n}Q_i\le R(\tilde{N})\end{aligned}\right\}\qquad \textbf{(11)}$$

The composition rules of intervals are used in these equations.

Hence the proposed model can be stated as

Minimize $\{f_L(Q),\,f_R(Q)\}$, **(12)**

Generally, the multi-objective optimization problem(12), in case of minimization problem, can be formulated in a conservative sense from (6)as

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Minimize $\{f_L(Q), f_R(Q)\}$,

$$\text{Subject to : } \left.\begin{array}{l} \sum_{i=1}^{n} A_i Q_i \le R(\tilde{B}) \\ \sum_{i=1}^{n} C_i Q_i \le R(\tilde{F}) \\ \frac{1}{2}\sum_{i=1}^{n} Q_i \le R(\tilde{N}) \end{array}\right\}$$

Q≥0. **(13)**

Where $f_C = \dfrac{f_L + f_R}{2}$.

Here the interval valued problem (12) is represented as

Minimize $\{f_L(Q), f_C(Q), f_R(Q)\}$,

$$\text{Subject to : } \left.\begin{array}{l} \sum_{i=1}^{n} A_i Q_i \le R(\tilde{B}) \\ \sum_{i=1}^{n} C_i Q_i \le R(\tilde{F}) \\ \frac{1}{2}\sum_{i=1}^{n} Q_i \le R(\tilde{N}) \end{array}\right\}$$

Q≥0 **(14)**

The expression (14) gives a better approximation than those obtained from (12).

Fuzzy programming technique for solution:

To solve multi-objective minimization problem given by (14), we have used the following fuzzy programming technique.

For each of the objective functions $f_L(Q), f_C(Q), f_R(Q)$,subject to the space constraint we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $f_L(Q), f_C(Q), f_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for objective $f_k(Q)$, k=L,C,R. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of fuzzy programming technique are given below.

Step 1: Solve the multi-objective cost function subject to the constraint as a single objective cost function subject to the constraint using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k value corresponding to the set of solutions. The initial fuzzy model of (10) can then be stated as, in terms of the aspiration levels for each objective, as follows: find Q satisfying $f_k \tilde{\le} L_k, k = L, C, R$ subject to the space constraint non negativity conditions.

Step 4: Define fuzzy linear membership function $\left(\mu_{f_k}; k = L, C, R\right)$ for each objective function is defined by

$$\mu_{f_k} = \begin{cases} 1 & ; f_k \le L_k \\ 1 - \dfrac{f_k - L_k}{d_k} & ; L_k \le f_k \le U_k \\ 0 & ; f_K \ge U_k \end{cases} \qquad \textbf{(15)}$$

Step 5: After determining the linear membership function defined in(15) for each objective functions following the problem (14) can be formulated an equivalent crisp model

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Max α,

$$\alpha \le \mu_{f_k}(x); k = L, C, R.$$

$$\left.\begin{array}{l} \sum_{i=1}^{n} A_i Q_i \le R(\tilde{B}) \\ \sum_{i=1}^{n} C_i Q_i \le R(\tilde{F}) \\ \frac{1}{2}\sum_{i=1}^{n} Q_i \le R(\tilde{N}) \end{array}\right\},$$

α ≥ 0, Q ≥ 0.

Numerical example:

In this section, the above mentioned algorithm is illustrated by a numerical example.

Here the parameters demands, ordering costs, holding costs are considered as triangular fuzzy numbers (TFN). After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation following [3].

Let h_1= Rs.2100; h_2=1600; D_1=7; D_2=17; S_1=2100; S_2=1300; A_1=25 sq.units; A_2=25 sq.units; C_1=4500; C_2=2000; B=250; F=17,000; N=5

Taking these as triangular fuzzy numbers we have,

$\tilde{D}_1$ =(5,7,9), $\tilde{D}_2$ =(14,17,20), $\tilde{h}_1$ =(1950,2075,2150), $\tilde{h}_2$ = (1400,1550,1700), $\tilde{S}_1$ =(1800,2100,2175), $\tilde{S}_2$ =(1125,1200,1300), $\tilde{F}$ =(15000,18000,20000), $\tilde{B}$ =(175,225,300), $\tilde{N}$ =(3,5,8)

The deterministic multi item fuzzy inventory model subject to three constraints using the above values now becomes

$$\tilde{\min} T(Q_i) = \sum_{i=1}^{2}\left[\frac{\tilde{D}_i}{Q_i}\tilde{s}_i + \frac{Q_i}{2}\tilde{h}_i\right]$$

$$= \left[\frac{\tilde{D}_1}{Q_1}\tilde{s}_1 + \frac{Q_1}{2}\tilde{h}_1\right] + \left[\frac{\tilde{D}_2}{Q_2}\tilde{s}_2 + \frac{Q_2}{2}\tilde{h}_2\right]$$

Subject to :

$$25Q_1 + 25Q_2 \le R(\tilde{B})$$

$$4500Q_1 + 2000Q_2 \le R(\tilde{F})$$

$$\frac{1}{2}Q_1 + \frac{1}{2}Q_2 \le R(\tilde{N})$$

Applying GMIR technique to the constraints and taking α =0 we get,

$R(\tilde{B})$=R(175,225,300)= $R(\tilde{B}) = \frac{[\alpha(-125)+750]}{3} = 250$

$R(\tilde{F})$=R(15000,18000,20000)= $R(\tilde{F}) = \frac{[\alpha(-5000)+56000]}{3} = 18667$

$R(\tilde{N})$=R(3,5,8)= $R(\tilde{N}) = \frac{[\alpha(-5)+18]}{3}$ = 6

Now applying nearest interval approximation and fuzzy programming technique to the objective, the fuzzy numbers $\tilde{D}_i, \tilde{S}_i, \tilde{h}_i$ are transformed into interval numbers as,

$\tilde{D}_1$ = [D_{1L},D_{1R}]=[6,8]

$\tilde{D}_2$ = [D_{2L},D_{2R}]=[16,19]

Individual minimum and maximum of objective functions f_L, f_C, f_R are given below:

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Table 1

Objective functions	Optimize f_L	Optimize f_C	Optimize f_R
f_L	f_L'=14922.15	f_L'' =14922.77	f_L''' =14928.21
f_C	f_C' =17007.68	f_C'' =17006.96	f_C''' =17011.13
f_R	f_R' =18898.38	f_R'' =18894.88	f_R''' =18890.15

Now we calculate

$L_L = \min\left(f_L', f_L'', f_L'''\right)$=14922.15 $U_L = \max\left(f_L', f_L'', f_L'''\right)$=14928.21

$L_C = \min\left(f_C', f_C'', f_C'''\right)$=17006.96 $U_C = \max\left(f_C', f_C'', f_C'''\right)$=17011.13

$L_R = \min\left(f_R', f_R'', f_R'''\right)$=18890.15 $U_R = \max\left(f_R', f_R'', f_R'''\right)$=18898.38

Using the equation (18), we formulate the following problem as:

Max α

$$\frac{11700}{Q_1} + 1006.5Q_1 + \frac{18608}{Q_2} + 737.5Q_2 + 6.06\alpha \le 14928.21$$

$$\frac{17104}{Q_1} + 1056.5Q_1 + \frac{23750}{Q_2} + 812.5Q_2 + 8.23\alpha \le 18898.38$$

$$\frac{14308}{Q_1} + 1031.5Q_1 + \frac{21717}{Q_2} + 775Q_2 + 4.17\alpha \le 17011.13$$

$$25Q_1 + 25Q_2 \le 250$$
$$4500Q_1 + 2000Q_2 \le 18667$$
$$\frac{1}{2}(Q_1 + Q_2) \le 6 \qquad \textbf{(16)}$$

Results and Discussions:

The solutions obtained from (16) are given in table 2 and 3.

Table 2: optimum value of α

Maximum α
0.7879

Table 3:optimum results

f_L^*	f_C^*	f_R^*	Q^*	Q_1^*
14923.47	17007.08	18890.013	2	4

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Table 4: comparison table

Model	Crisp	Crisp	Crisp	Crisp	Crisp	Fuzzy
h_1	2100	2000	1900	2100	2100	[2013,2113]
h_2	1600	1500	1500	1500	1600	[1475,1625]
s_1	2100	2000	1900	2000	1800	[1950,2138]
s_2	1300	1200	1100	1200	1000	[1163,1250]
D_1	7	7	7	7	7	[6,8]
D_2	17	17	17	17	17	[16,19]
B	250	260	250	260	250	(175,225,300)
F	17000	18000	17500	17000	17500	(15,000,18,000,20,000)
N	5	6	7	5	6	(3,5,8)
Q_1*	2	2	2	2	2	2
Q_2*	4	4	4	4	4	4
Min C(Q)	18055.55	16585.83	15879.50	17081.75	15506.86	[14923,18890]
A	---	---	---	---	---	0.7879

IV.CONCLUSION

In this paper, we have presented an inventory model with three constraints, where carrying costs, ordering or setup costs, demands, investment amount are assumed as triangular fuzzy numbers instead of crisp values so as to make the inventory model more realistic. At first, the fuzzy storage constraint and investment constraint are converted into crisp using GMIR technique. The expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantity is approximated by interval number. After that the problem of minimizing the cost function subject to the constraints is transformed into a multi-objective inventory problem subject to the constraints, where the objective functions are left limit, right limit and the center of the interval function. Fuzzy optimization technique is then used to found out the optimal results. A numerical example illustrates the proposed method.

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