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Observations on the Non-Homogeneous Sextic Equation with Six Unknowns $x^6 - y^6 - 2z^3 = (k^2 + s^2)^{2n} T^4 (w^2 - p^2)$

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ABSTRACT: The sextic non-homogeneous equation with six unknowns represented by the Diophantine equation $x^6 - y^6 - 2z^3 = (k^2 + s^2)^{2n} T^4 (w^2 - p^2)$ is analyzed for its patterns of non-zero distinct integral solutions are illustrated. Various interesting relations between the solutions and special numbers, namely polygonal numbers, Pyramidal numbers, Jacobsthal numbers, Jacobsthal-Lucas number are exhibited.

KEYWORDS: Integral solutions, sextic, non-homogeneous equation.

I. Introduction

The theory of Diophantine equations offers a rich variety of fascinating problems [1-4]. Particularly, in [5-7], sextic equations with three unknowns are studied for their integral solutions. [8-14] analyze sextic equations with four unknowns for their non-zero integer solutions [15-17] analyze sextic equations with five unknowns for their non-zero solutions. This communication concerns with yet another interesting a non-homogeneous sextic equation with six unknowns given by $x^6 - y^6 - 2z^3 = (k^2 + s^2)^{2n} T^4 (w^2 - p^2)$. Infinitely many non-zero integer tuple (x,y,z) satisfying the above equation are obtained. Various interesting properties among the values of x,y,z are presented

II .NOTATIONS

KY_n **:** Polygonal number of rank n with size m
Pr_n : Pyramidal number of rank n with size m
j_n : Jacobsthal lucas number of rank n
J_n : Jacobsthal number of rank n

III. METHOD OF ANALYSIS

The equation under consideration is $x^6 - y^6 - 2z^3 = (k^2 + s^2)^{2n} T^4 (w^2 - p^2)$ (1)
Where k and s are given non-zero integers. Different patterns of solutions to (1) are illustrated below:

A. Pattern:1

Introduction of the transformations

$x = u + v,\ y = u - v,\ z = 2uv,\ w = uv + 3,\ p = uv - 3$ (2)

in (1) leads to $u^2 + v^2 = (k^2 + s^2)^n T^2 * 1$ (3)

Let $T = a^2 + b^2$ (4)

write (1) as $1 = \frac{[(1+i)(1-i)]^{2n}}{2n}$ (5)

Substituting (4) and (5) in (3) and employing the method of factorization, define

$$u + iv = (k + is)^n (a + ib)^2 \frac{(1+i)^{2n}}{2^n} \quad (6)$$

Since the complex number raised to any positive integer power is also a complex number, we write

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$$(k+is)^n = \alpha + i\beta \quad (7)$$

where $\alpha = \frac{1}{2}[(k+is)^n + (k-is)^n]$

$\beta = \frac{1}{2}[(k+is)^n - (k-is)^n]$

Using (7) in (6) and equating real and imaginary parts, we have

$$\left.\begin{aligned} u &= cosn\tfrac{\pi}{2}[\alpha(a^2-b^2)-2\beta ab] - sinn\tfrac{\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)] \\ v &= sin\tfrac{\pi}{2}[\alpha(a^2-b^2)-2\beta ab] - cosn\tfrac{\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)] \end{aligned}\right\} \quad (8)$$

Using (8) in (2) we get

$$\left.\begin{aligned} x(a,b) &= [\alpha(a^2-b^2)-2\beta ab]\left[\cos\frac{n\pi}{2}+\sin\frac{n\pi}{2}\right] + \left(\alpha(2ab)+\beta(a^2-b^2)\right)\left(\cos\frac{n\pi}{2} - \mathrm{sinn}\frac{n\pi}{2}\right) \\ y(a,b) &= [\alpha(a^2-b^2)-2\beta ab]\left[\cos\frac{n\pi}{2}-\sin\frac{n\pi}{2}\right] - [\alpha(2ab)+\beta(a^2-b^2)]\left[\sin\frac{n\pi}{2}+\cos\frac{n\pi}{2}\right] \\ z(a,b) &= 2[\cos\frac{n\pi}{2}(\alpha(a^2-b^2)-2\beta ab) - \sin\frac{n\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)]\sin\frac{n\pi}{2}[\alpha(a^2-b^2)-2\beta ab] + \\ &\quad \mathrm{cosn}\frac{n\pi}{2}(\alpha(2ab)+\beta(a^2-b^2))] \\ w(a,b) &= \left\{\cos\frac{n\pi}{2}[\alpha(a^2-b^2)-2\beta ab] - \sin\frac{n\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)]\right\} \\ &\quad \left\{\sin\frac{n\pi}{2}[\alpha(a^2-b^2)-2\beta ab] + \cos\frac{n\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)]\right\} + 3 \\ p(a,b) &= \left\{\cos\frac{n\pi}{2}[\alpha(a^2-b^2)-2\beta ab] - \sin\frac{n\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)]\right\} \\ &\quad \left\{\sin\frac{n\pi}{2}[\alpha(a^2-b^2)-2\beta ab] + \cos\frac{n\pi}{2}[\alpha(2ab)+\beta(a^2-b^2)]\right\} - 3 \end{aligned}\right\}$$

--------------------(9)

Thus (4) and (9) represent the non-zero integer solutions to (1)

For illustration and clear understanding, substituting n=1, in (9), the corresponding non-zero distinct integral solutions to (1) are given by

$x(a,b) = \alpha(a^2-b^2+2ab) + \beta(a^2-b^2-2ab)$

$y(a,b) = \alpha(a^2-b^2-2ab) - \beta(a^2-b^2+2ab)$

$w(a,b) = [\alpha(a^2-b^2)-2\beta ab][\alpha(2ab)+\beta(a^2-b^2)] + 3$

$p(a,b) = [\alpha(a^2-b^2)-2\beta ab][\alpha(2ab)+\beta(a^2-b^2)] - 3$

$T(a,b) = a^2+b^2$

B. Properties

(i) $x(s,s+1) + y(s,s+1) = 2\alpha[t_{3,s} - 2t_{4,s} + 1] - 4\beta Pr_s$

$(ii) x(2^s,1) = (\alpha+\beta)(3J_{2s}) + 2(\alpha-\beta)(j_s - (-1)^s)$

(iii) $y(2^s,1) = \alpha(3J_{2s} - j_{s+1} + (-1)^{s+1}) - \beta(KY_s)$

Note:

Suppose, we choose k and s such that $k^2+s^2=\sigma^2$. Then (3) becomes

$$u^2+v^2 = (\sigma^n T)^2 \quad (10)$$

which is in the form of the Pythagorean equation. For this choice, the sextuple (x,y,z,w,p,T) satisfying (1) is given by

$x = \sigma^{2n}[p^2-q^2+2pq]$

$y = \sigma^{2n}[-p^2+q^2+2pq]$

$z = 4\sigma^{4n}[p^2-q^2]pq$

$w = 2pq[p^2-q^2] + 3$

$p = 2pq[p^2-q^2] - 3$

$T = \sigma^n(p^2+q^2)$

It is observed that the above values are different from (4) and (9)

C. Pattern.2

Note that (10) is written in the form of ratio as

$$\frac{\sigma^n T+v}{u} = \frac{u}{\sigma^n T - v} = \frac{A}{B},\ B \neq 0 \quad (11)$$

which is equivalent to the system of equation

$$\left.\begin{aligned} (\sigma^n B)T + Bv - Au &= 0 \\ (\sigma^n A)T - Av - Bu &= 0 \end{aligned}\right\} \quad (12)$$

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Applying the method of cross multiplication to the above system, we obtain

$$\left.\begin{array}{l} u = -2\sigma^n AB \\ v = \sigma^n (B^2 - A^2) \end{array}\right\} \quad (13)$$

$$T = -(A^2 + B^2) \quad (14)$$

Thus from (13) and (2), we get

$$\left.\begin{array}{l} x = \sigma^n (B^2 - A^2 - 2AB) \\ y = \sigma^n (B^2 - A^2 + 2AB) \\ z = -4\sigma^{2n} (B^2 - A^2) AB \\ w = -2\sigma^{2n} AB (B^2 - A^2) + 3 \\ p = -2\sigma^n AB (B^2 - A^2) - 3 \end{array}\right\} \quad (15)$$

Hence, (15) and (14) represent the integral solutions of (1)

D. Properties:

(i) $w^3 + p^3 + 3zpw = z^3$

(ii) $x^2 - y^2 = 2(w + p)$

(iii) $x^2 - y^2 - 4p \equiv 0 \ (mod 12)$

(iv) $(x^2 - y^2)^2 = z^2 (w - p - 2)$

(v) $z^2 - 4w^2 + 24w \equiv 0 \ (mod\ 9)$

IV. REMARKABLE OBSERVATIONS

(i) The triple (x,y,z) satisfies the hyperbolic paraboloid $x^2 - y^2 = 2z$

(ii) If $\alpha > \beta$ and $a^2 - b^2 > 2ab$, then $u > v$. Let (α, β, γ) be the Pythagorean triangle with u, v as generators. Set $\alpha = 2uv,\ \beta = u^2 - v^2,\ \gamma = u^2 + v^2$ and A,P represent its area and perimeter respectively. Note that

$$(i)\ xyz = 2A$$

$$(ii)\ \frac{4A}{P} = (x - y)y$$

V. CONCLUSION

It is worth to mention here that, the values of w and p in (2) may be considered as (i) w= 3uv+1, p=3u-v and (ii) w=3uv+1, p=3uv-1. Further, in addition to (5), one may also write 1 as $1 = \frac{(p^2-q^2+2ipq)(p^2-q^2-2ipq)}{(p^2+q^2)^2}$ Following the analysis presented above, one may obtain other patterns of non-zero integer solutions to (1)

To conclude, one may search for other choice of transformations to analyze (1) for its non-zero distinct integral solutions

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